

Phase constant  $\beta$  for an isolated waveguide

$$a = a_0 e^{-j\beta z}$$

$$b = b_0 e^{-j\beta z}$$

$$\frac{da}{dz} = -j\beta a_0 e^{-j\beta z} = -j\beta a$$

$$\frac{db}{dz} = -j\beta b_0 e^{-j\beta z} = -j\beta b$$

Coupling coefficient  $\kappa$   
(overlap integral)

$$\kappa = f(d, s, n_1, n_2, \lambda)$$

$$\frac{da}{dz} = -j\beta a - j\kappa b$$

$$\frac{db}{dz} = -j\beta b - j\kappa a$$

$$\frac{d^2 a}{dz^2} = -j\beta \frac{da}{dz} - j\kappa \frac{db}{dz}$$

$$= -j\beta \frac{da}{dz} - j\kappa (-j\beta b - j\kappa a)$$

$$= -j\beta \frac{da}{dz} - j\kappa b (-j\beta) - \kappa^2 a$$

$$= -j\beta \frac{da}{dz} + \left( \frac{da}{dz} + j\beta a \right) (-j\beta) - \kappa^2 a$$

$$\frac{d^2 a}{dz^2} = -j2\beta \frac{da}{dz} + (\beta^2 - \kappa^2) a$$

$$\frac{d^2 a}{dz^2} + j2\beta \frac{da}{dz} + (\kappa^2 - \beta^2) a = 0$$

$$\frac{d^2 b}{dz^2} + 2j\beta \frac{db}{dz} + (\kappa^2 - \beta^2) b = 0$$

$$a(z) = \{ A_1 \cos kz + A_2 \sin kz \} e^{-j\beta z}$$

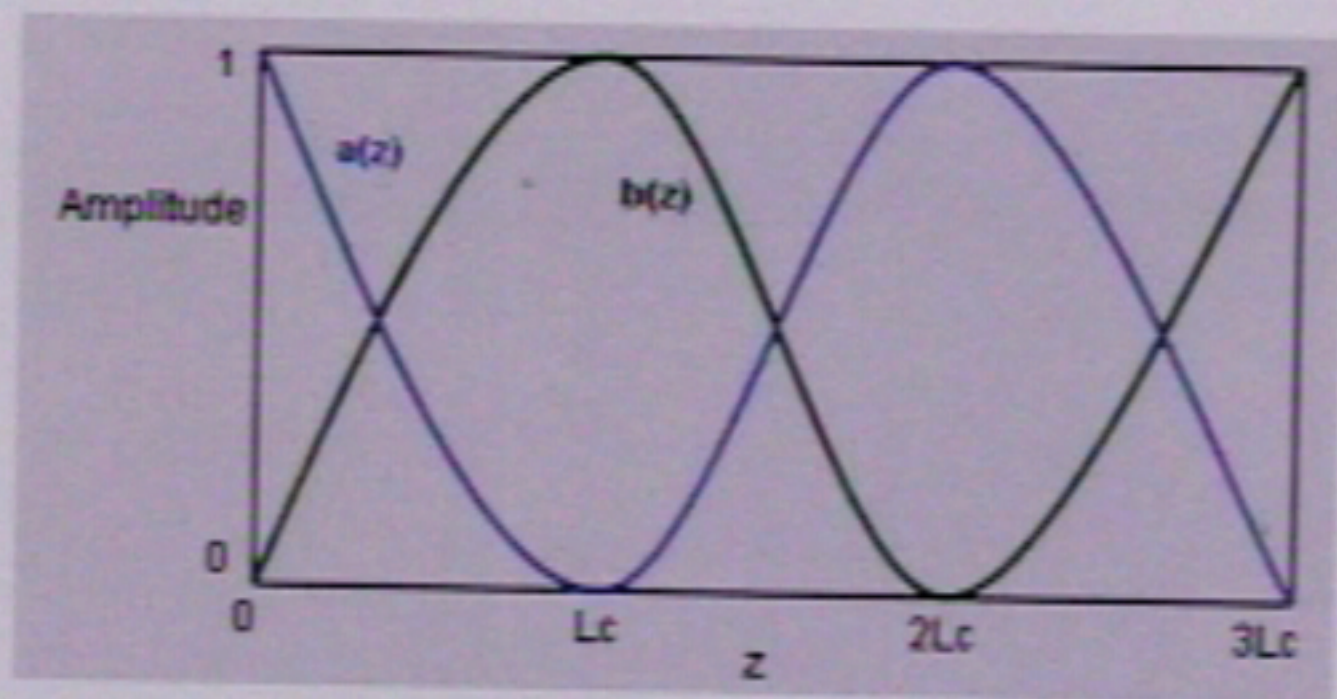
$$b(z) = \{ B_1 \cos kz + B_2 \sin kz \} e^{-j\beta z}$$

Initial conditions :

$$\text{At } z=0, \quad a(0) = 1, \quad b(0) = 0$$

$$a(z) = \cos kz e^{-j\beta z}$$

$$b(z) = -j \sin kz e^{-j\beta z}$$



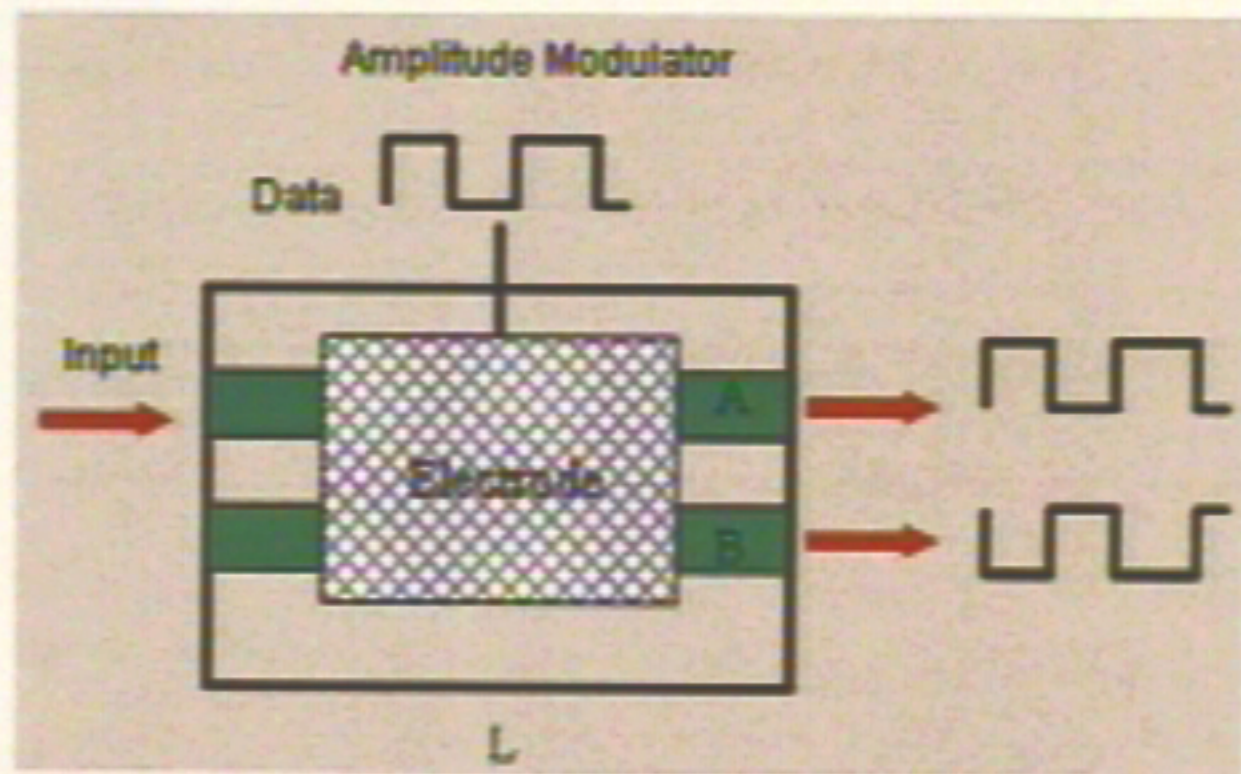
$$k_c z = \pi/2$$

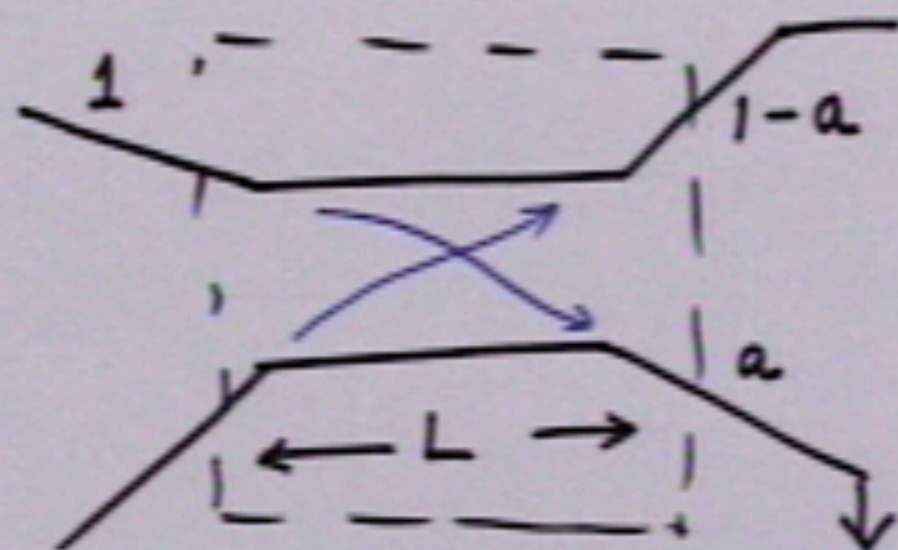
$$\downarrow$$

$$= L_c \leftarrow \text{coupling length}$$

$$k_c = \pi/2L_c$$

### Amplitude Modulator

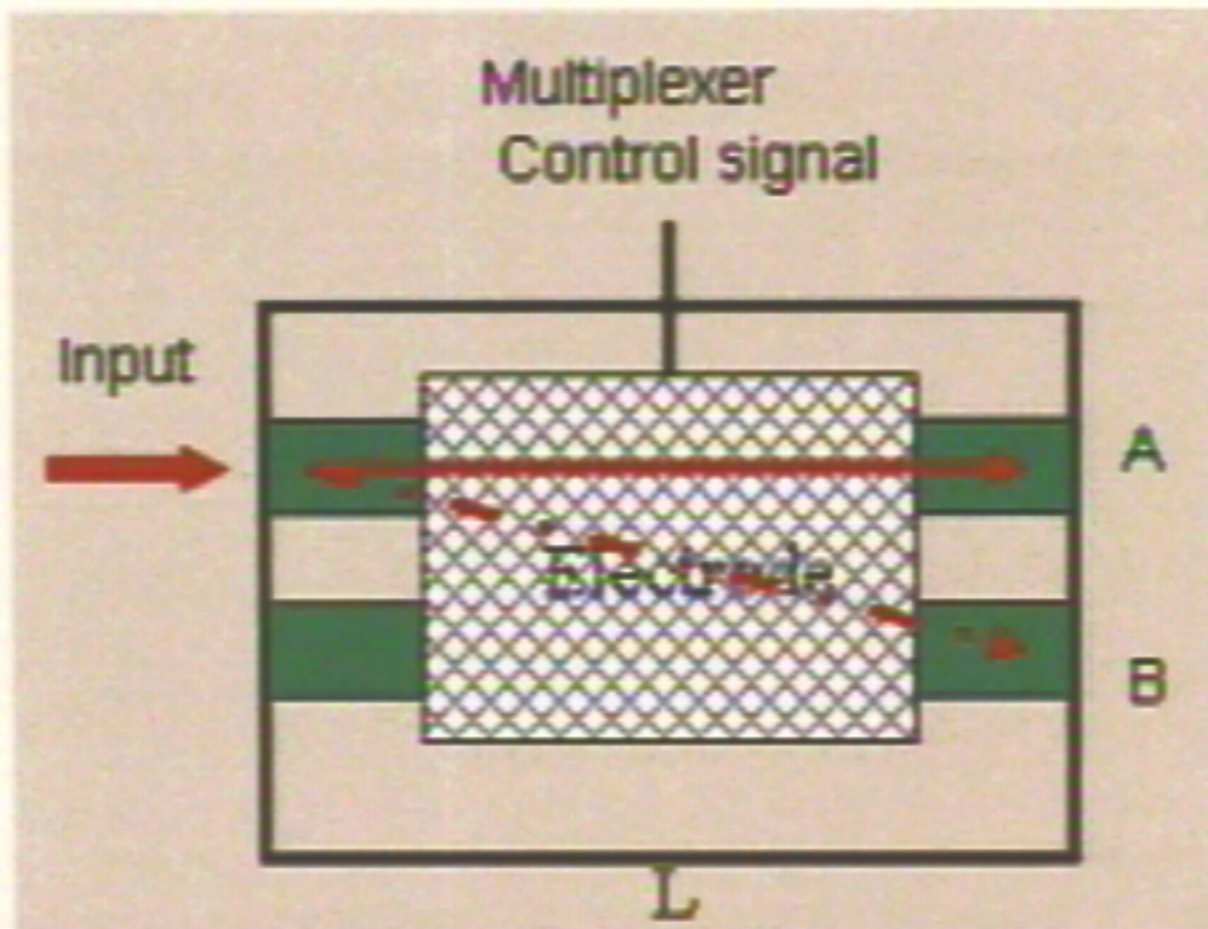




1.  $L \ll L_c \rightarrow$  Tapping Signals.

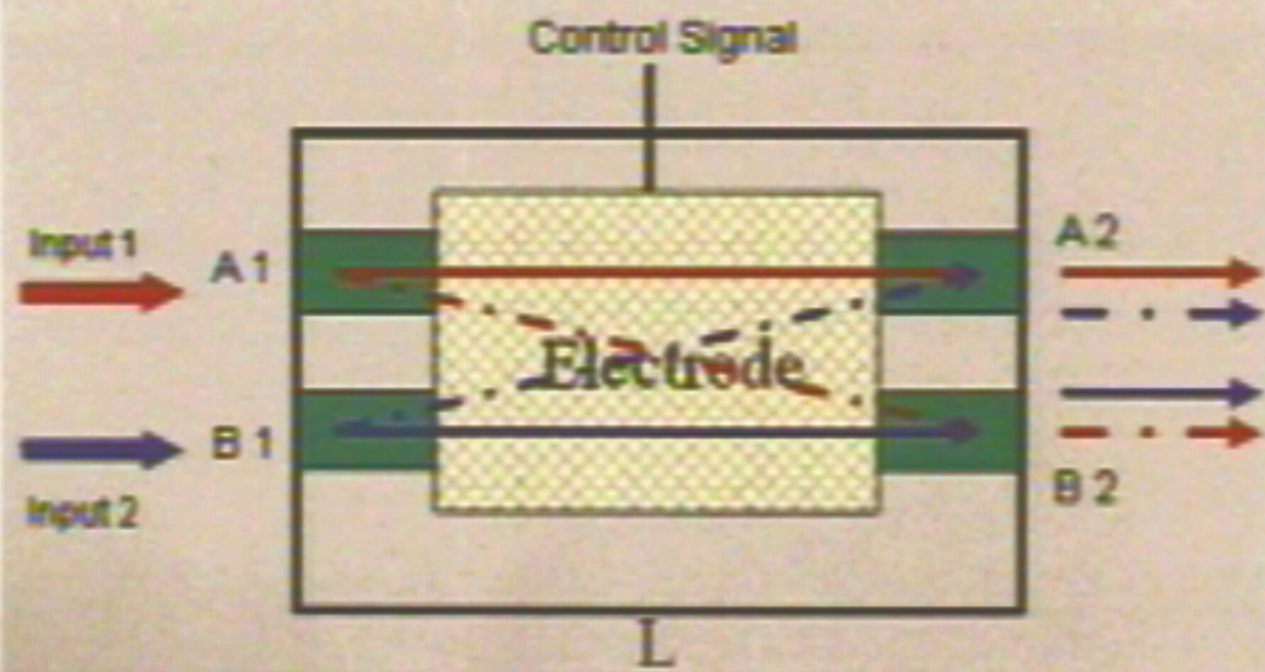
2.  $L = L_c/2 \rightarrow$  3-dB Power Divider.

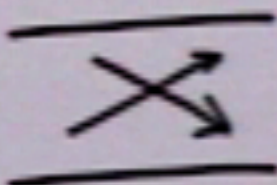
3.  $L = L_c \rightarrow$  Cross-over.

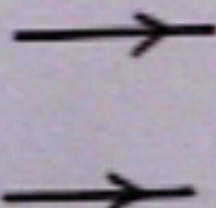


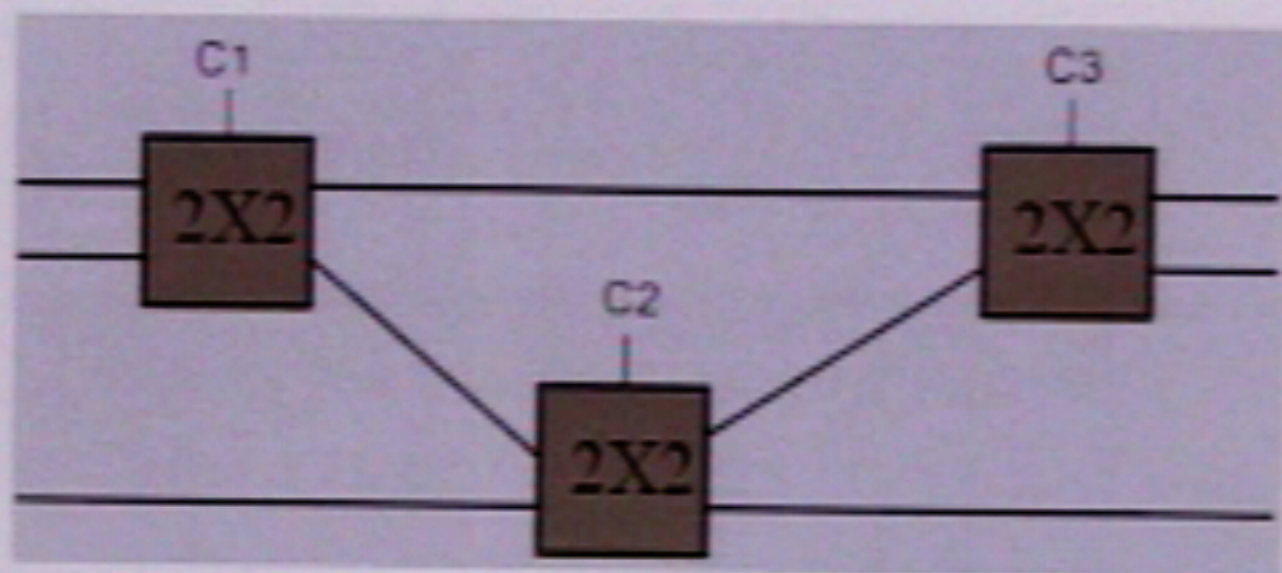


### Cross-connect (2 x 2 Switch)



 = cross - state

 = Bar - state



$a_1$   
 $b_1$

$a_2$   
 $b_2$

$c_1$

$c_2$

$$\left\{ \begin{array}{l} c_1 \rightarrow = \\ c_2 \rightarrow = \\ c_3 \rightarrow = \end{array} \right. \quad \begin{array}{l} a_1 \rightarrow a_2 \\ b_1 \rightarrow b_2 \\ c_1 \rightarrow c_2 \end{array}$$

$$\begin{array}{l} c_1 \rightarrow \times \\ c_2 \rightarrow = \\ c_3 \rightarrow = \end{array} \quad \begin{array}{l} a_1 \rightarrow b_2 \\ b_1 \rightarrow a_2 \\ c_1 \rightarrow c_2 \end{array}$$

