$$\rho(f) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$= \frac{A_{21} / B_{21}}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1}$$

$$= \frac{A_{21} / B_{21}}{\frac{B_{12}}{B_{21}} e^{hf/kT} - 1}$$

BLACK BODY RADIATION

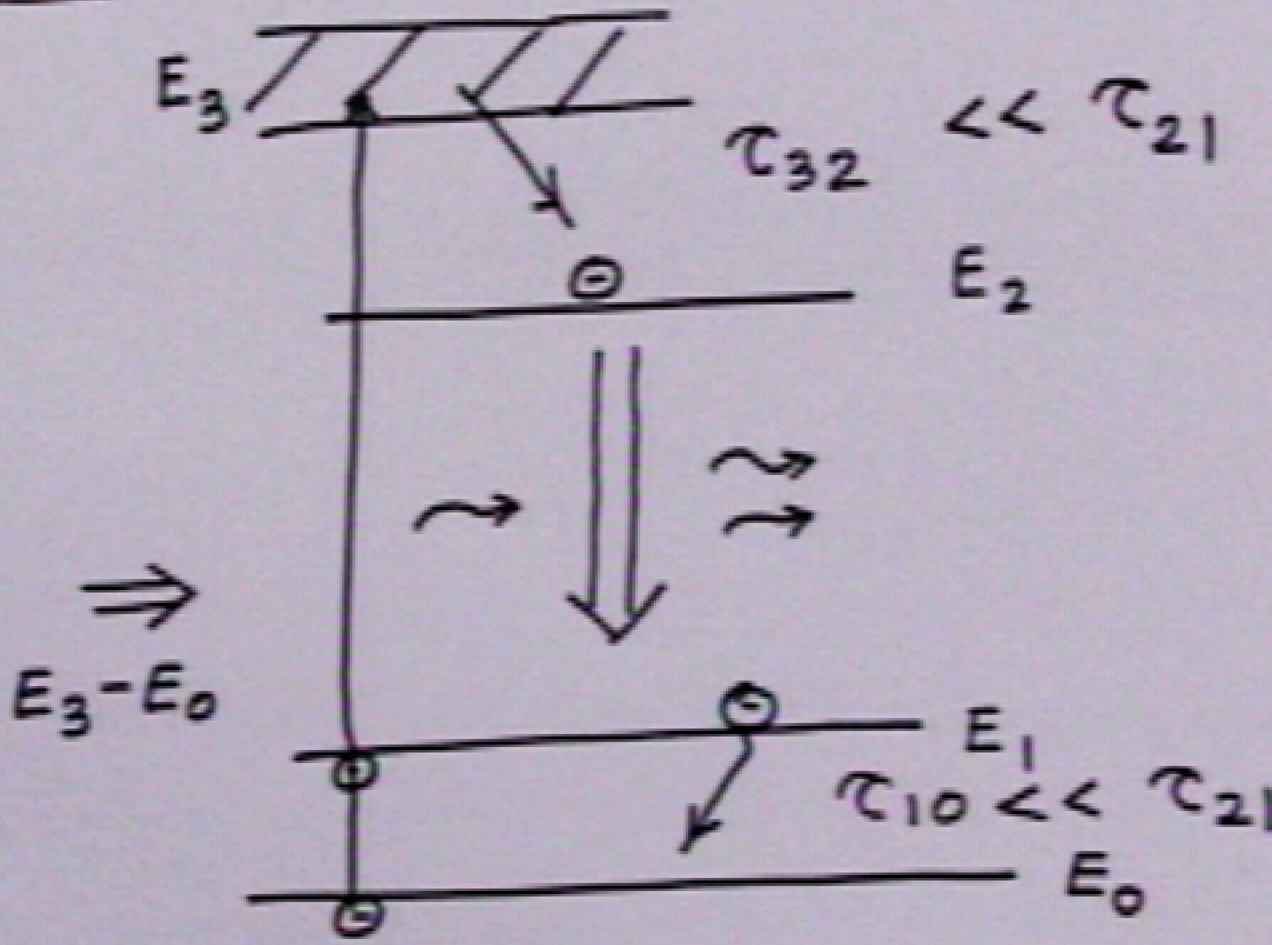
$$\rho(f) = \frac{8\pi n^3 f^3}{c^3} \frac{1}{e^{hf/kT} - 1}$$

$$\rho(f) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} e^{hf/kT} - 1}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 f^3}{c^3}$$

$$\boxed{B_{12} = B_{21}}$$

Three - Energy Level



Four Level

Net downward Transitions

$$-\frac{dN_2}{dt} = B_{21}\,\rho(f)\,N_2 - B_{12}\,\rho(f)\,N_1$$

$$= B_{21}\,\rho(f)\,(N_2 - N_1)$$

No. of photons N_p

$$-\frac{dN_2}{dt} = \frac{dN_p}{dt}$$

$$\frac{dN_p}{dt} = B_{21}\,\rho(f)\,(N_2 - N_1)$$

$$hf\,\frac{dN_p}{dt} = \frac{d}{dt}(\underbrace{hf\,N_p}_{\rho(f)})$$

$$\frac{d\rho(f)}{dt} = B_{21}\,\rho(f)\,(N_2 - N_1)$$

$$= \frac{c^3 h f}{8\pi f^3 n^3 \tau_{sp}}\,\rho(f)\,(N_2 - N_1)$$

$$x = \frac{c}{n} t$$

$$dx = \frac{c}{n} dt$$

$$\frac{d\rho(f)}{dx} \cdot \frac{c}{n} = \frac{c^3 h f}{8\pi f^3 n^3 \tau_{sp}}\,\rho(f)\,(N_2 - N_1)$$

$$\frac{d\rho(f)}{dx} = \left(\frac{c^2 h}{8\pi f^2 n^2 \tau_{sp}}(N_2 - N_1)\right)\rho(f)$$

$G \leftarrow$ Gain Const.

$$\frac{d\rho(f)}{dx} = G\,\rho(f)$$

$$\rho(f, x) = \rho(f, x=0)\, e^{Gx}$$

If $N_2 < N_1$: G is $-ve$

$G = -\alpha \leftarrow$ Attenuation Constant

If $N_2 = N_1$: $G = 0$ Transparency

If $N_2 > N_1$

Population Inversion : G is $+ve$

Growth of photon flux.