Prof Gadre
Date 15/1/10

LECTURE 6

THE HAAR FILTER BANK

Consider

$y(t) \in V_1$

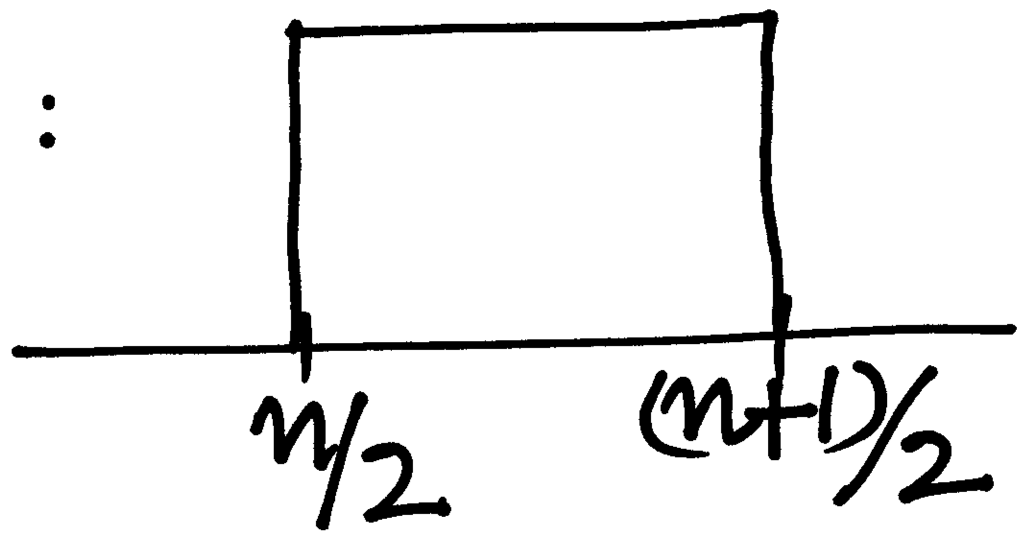
$y[-2] = 4 \quad y[-1] = 7$ and so on

$4 \quad 7 \quad 10 \quad 16 \quad 14 \quad 11 \quad 3 \quad -1$

$-1 \quad -\frac{1}{2} \quad 0 \quad 1/2 \quad 1 \quad 3/2 \quad 2 \quad 5/2$

$$y(t) = \sum_{n=-\infty}^{+\infty} y[n]\,\phi(2t-n)$$

$\phi(2t-n)$:

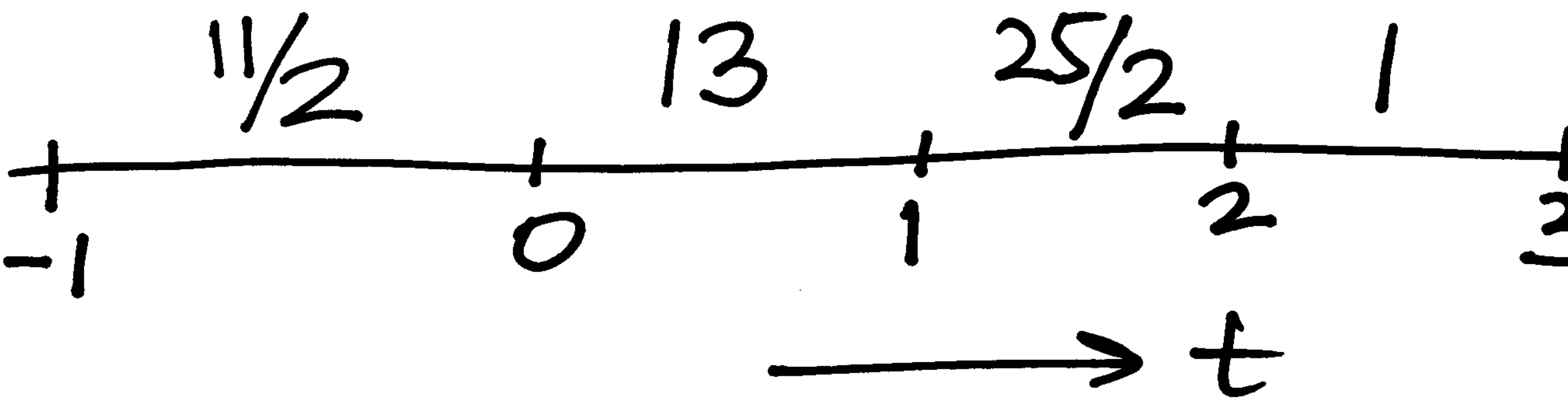


$$v_1 = v_0 \oplus w_0$$

$$y = y_{v_0} + y_{w_0}$$

$y_{V_0}(t) =$ sequence

$$\frac{11}{2} \quad \underset{\substack{\uparrow \\ 0}}{13} \quad \frac{25}{2} \quad 1$$

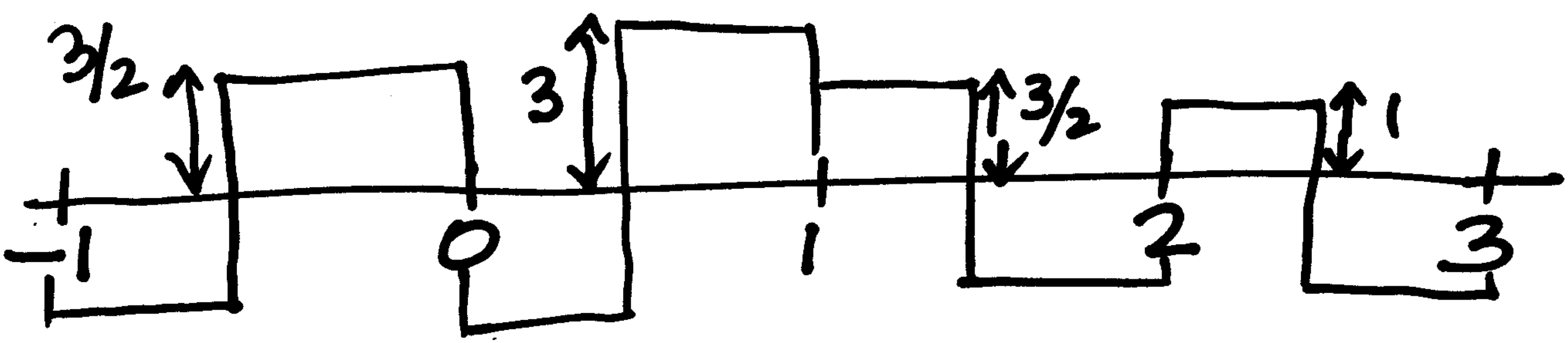


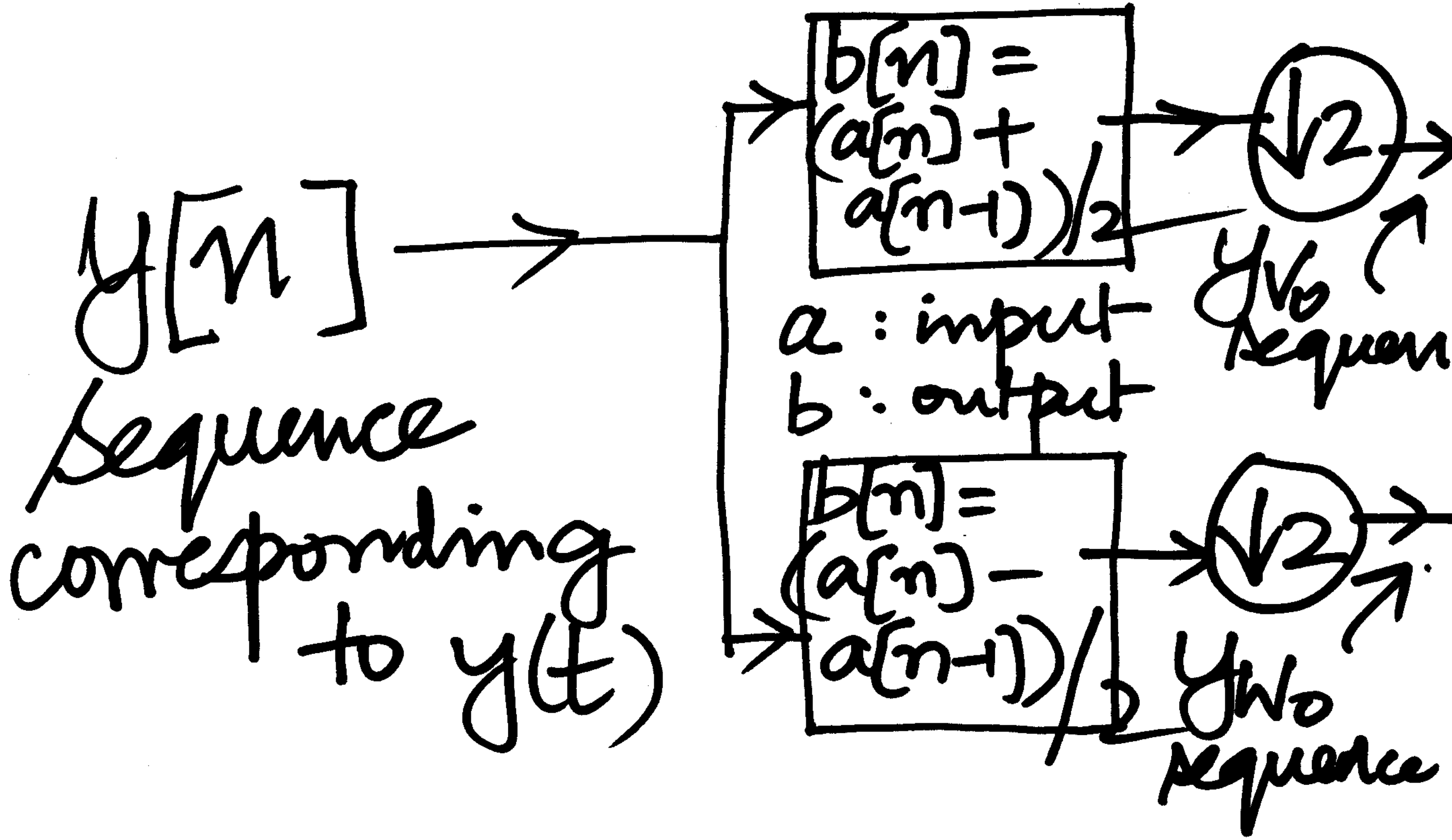
Sequence

$y_{W_0}(t)$ $\quad -3/2 \quad -3 \quad 3/2 \quad 1$

$\uparrow$ 0

$\longrightarrow t$




y[n]
sequence
corresponding
to y(t)
b[n] =
(a[n] +
a[n-1])/2
a : input
b : output
↓2
yV0
sequen
b[n] =
(a[n] −
a[n-1])/2
↓2
yW0
sequence

$y[n] \xrightarrow{\text{Z transform}}$

$$Y(z) = \sum_{n=-\infty}^{+\infty} y[n]\, z^{-n}$$

Converges in a region of complex z.

$$y[n] \xrightarrow{\mathcal{Z}} Y(z), \; R_Y$$

$$y[n-D] \xrightarrow{\mathcal{Z}} z^{-D} Y(z), \; \text{atleast } R_Y$$

$$b[n] = \frac{1}{2}\{a[n] + a[n-1]\}$$

$a[\cdot]$ = input,

$b[\cdot]$ = output

Taking Z-transform

$$B(z) = \frac{1}{2}\left\{A(z) + z^{-1}A(z)\right\}$$

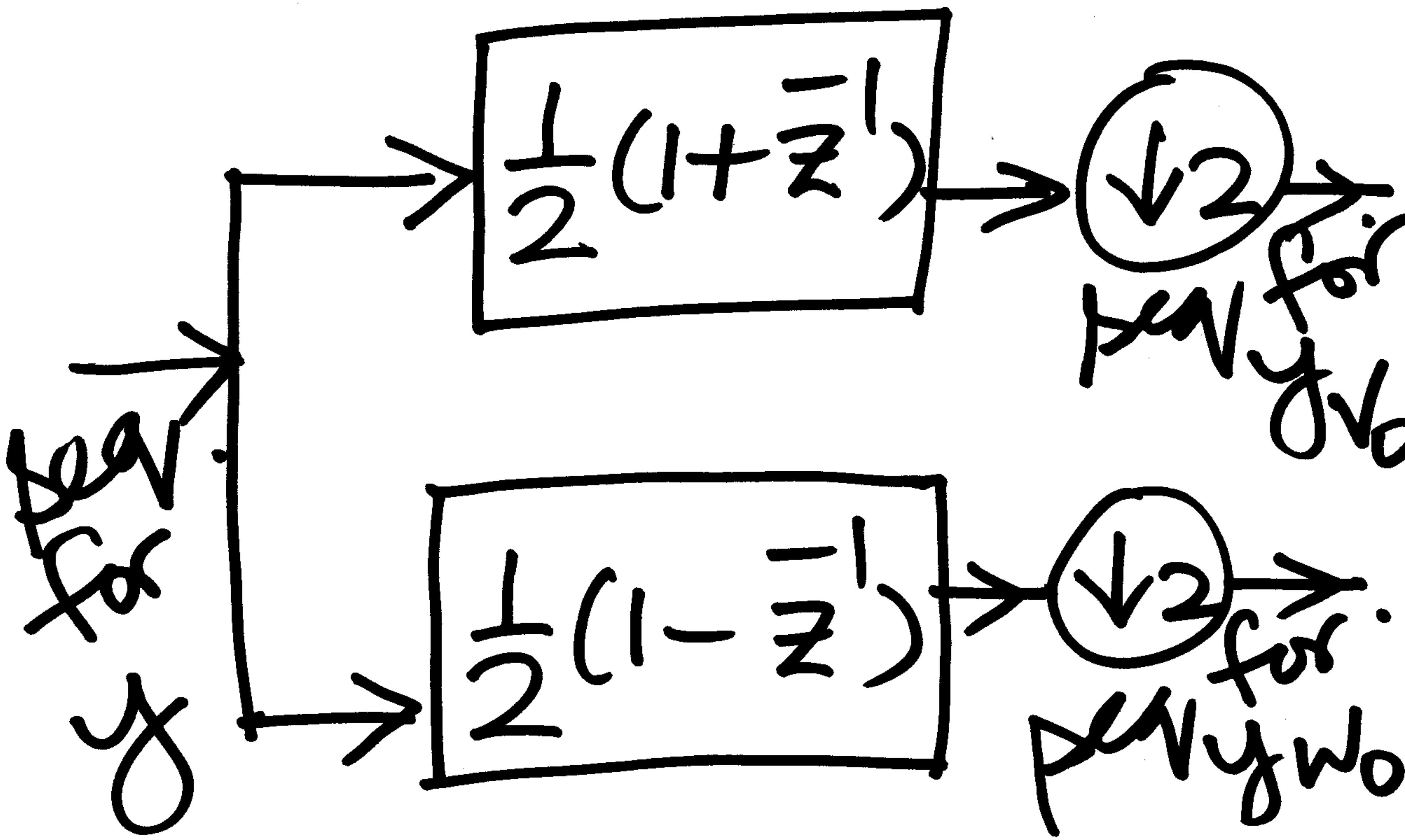
$$\frac{B(z)}{A(z)} = \text{System function}$$

$$= \frac{1}{2}(1 + z^{-1})$$

$$b[n] = \frac{1}{2}\{a[n] - a[n-1]\}$$

System function

$$\frac{B(z)}{A(z)} = \frac{1}{2}(1 - z^{-1})$$



$\frac{1}{2}(1+z^{-1})$
↓2
for
$\frac{1}{2}(1-z^{-1})$
↓2
for
for

To synthesize $y(t)$ from $y_{v_0}(t)$ and $y_{w_0}(t)$

In continuous time,
this is simply

$$y(t) = y_{v_0}(t) + y_{w_0}(t)$$

Simple !!

$y =$ 4 7 10 16 14 11 3 −

↑

0

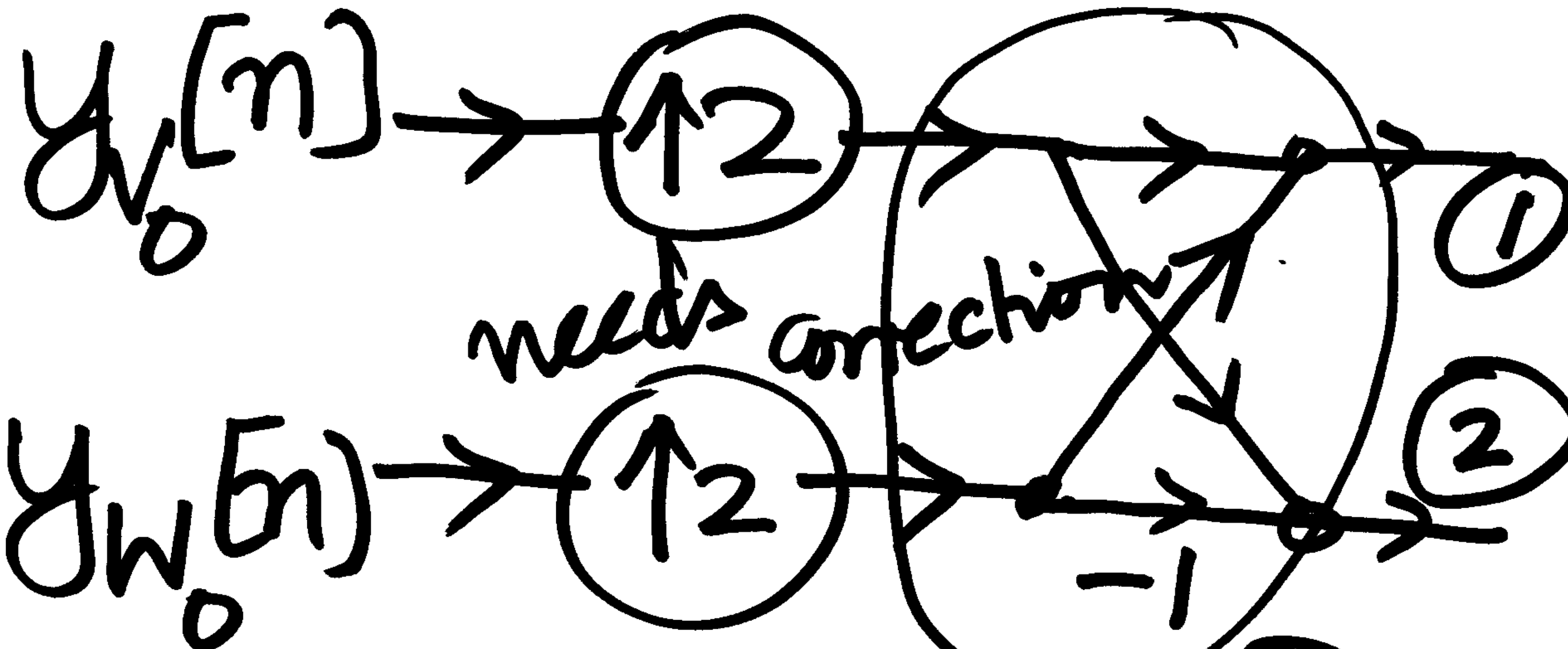
$y_{V_0}[n] = \frac{11}{2} \quad \underset{\underset{0}{\uparrow}}{13} \quad \frac{25}{2} \quad 1$

$y_{W_0}[n] \quad -3/2 \quad \underset{\underset{0}{\uparrow}}{-3} \quad 3/2 \quad 1$

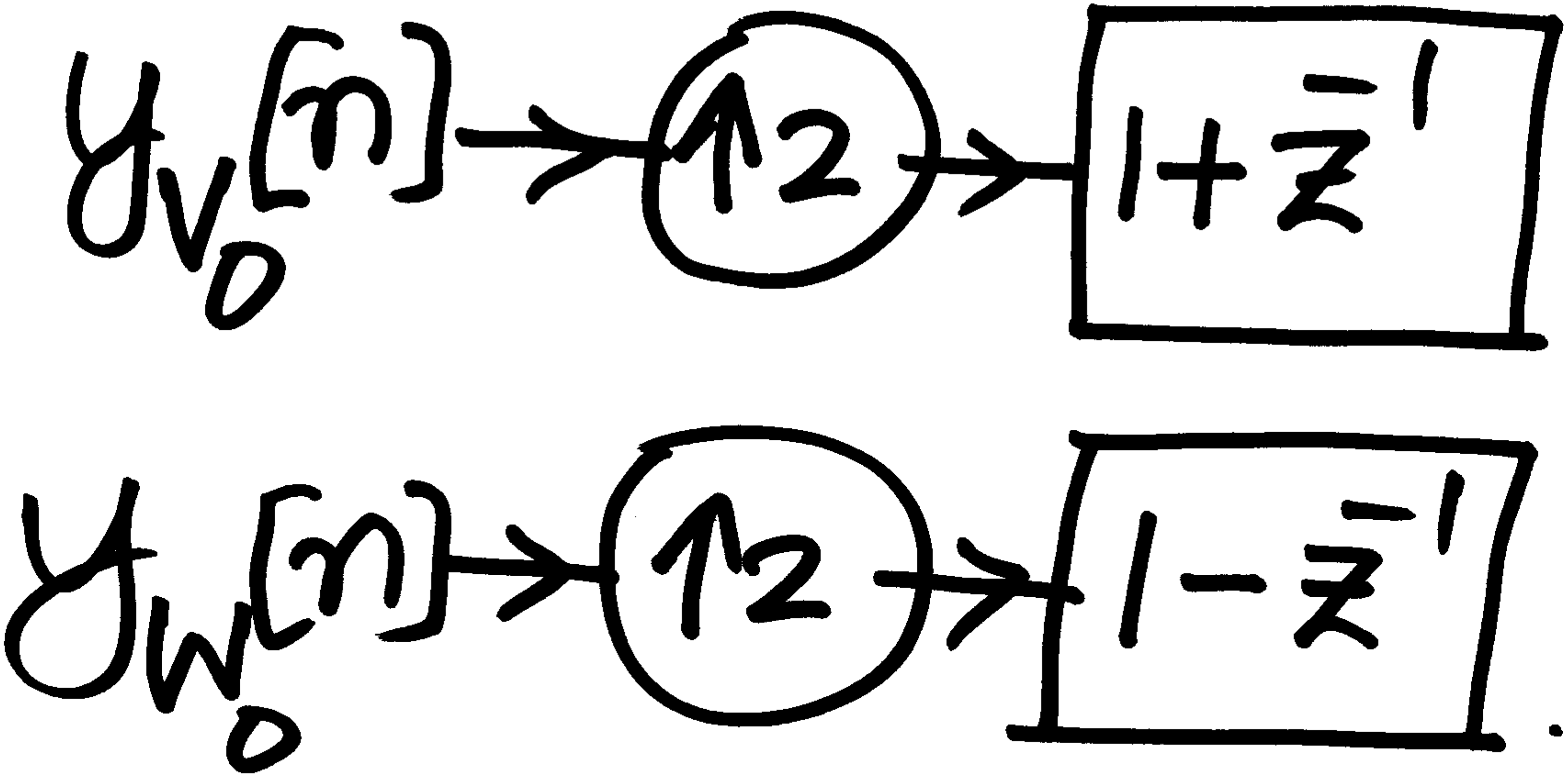
To 'outdo' or 'overcome' decimation let us define $\textcircled{\uparrow 2}$

$$y_{w_0}[n] \longrightarrow \boxed{\uparrow 2} \longrightarrow$$

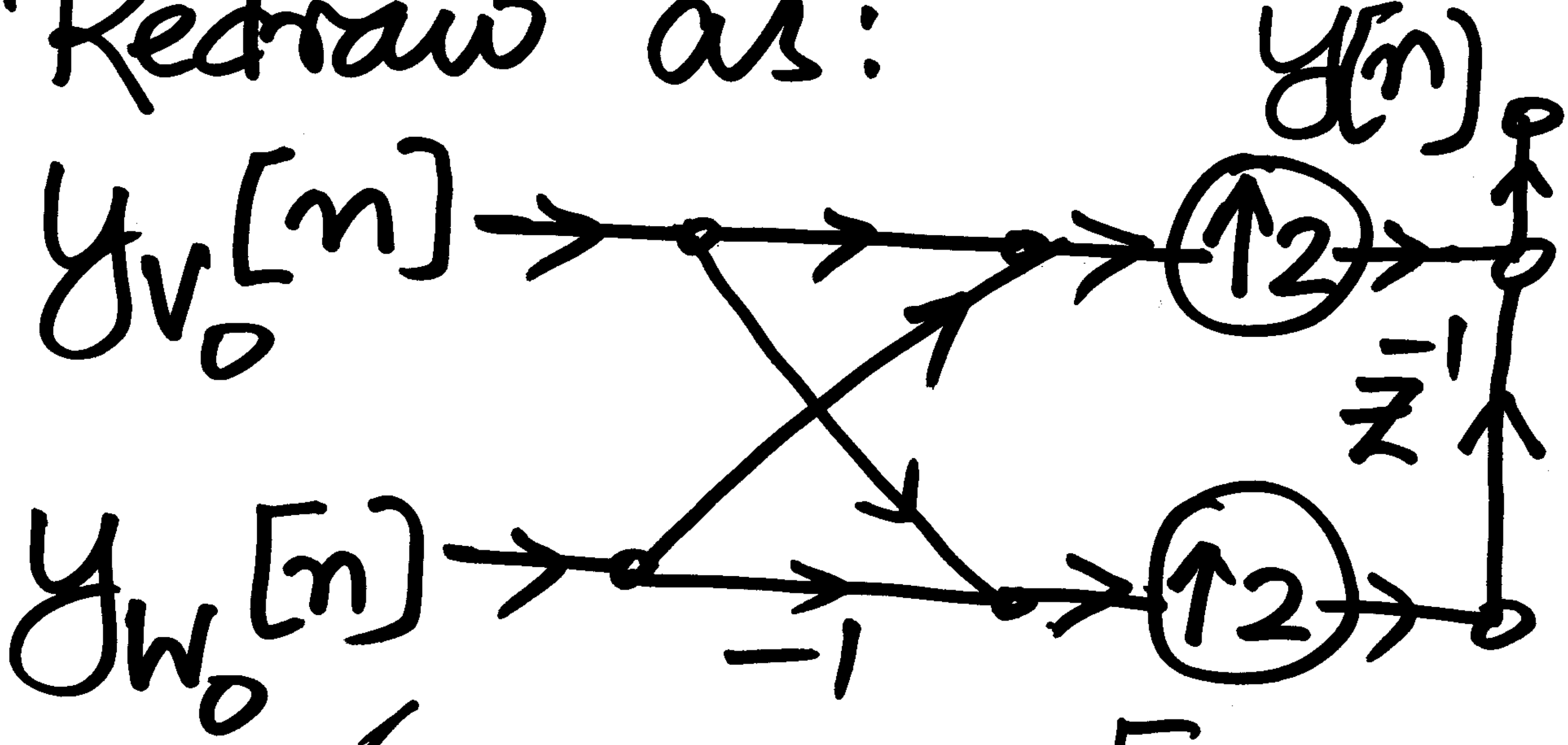
$$-\frac{3}{2} \quad 0 \quad \underset{\underset{0}{\uparrow}}{-3} \quad 0 \quad \frac{3}{2} \quad 0 \quad 1 \quad 0$$



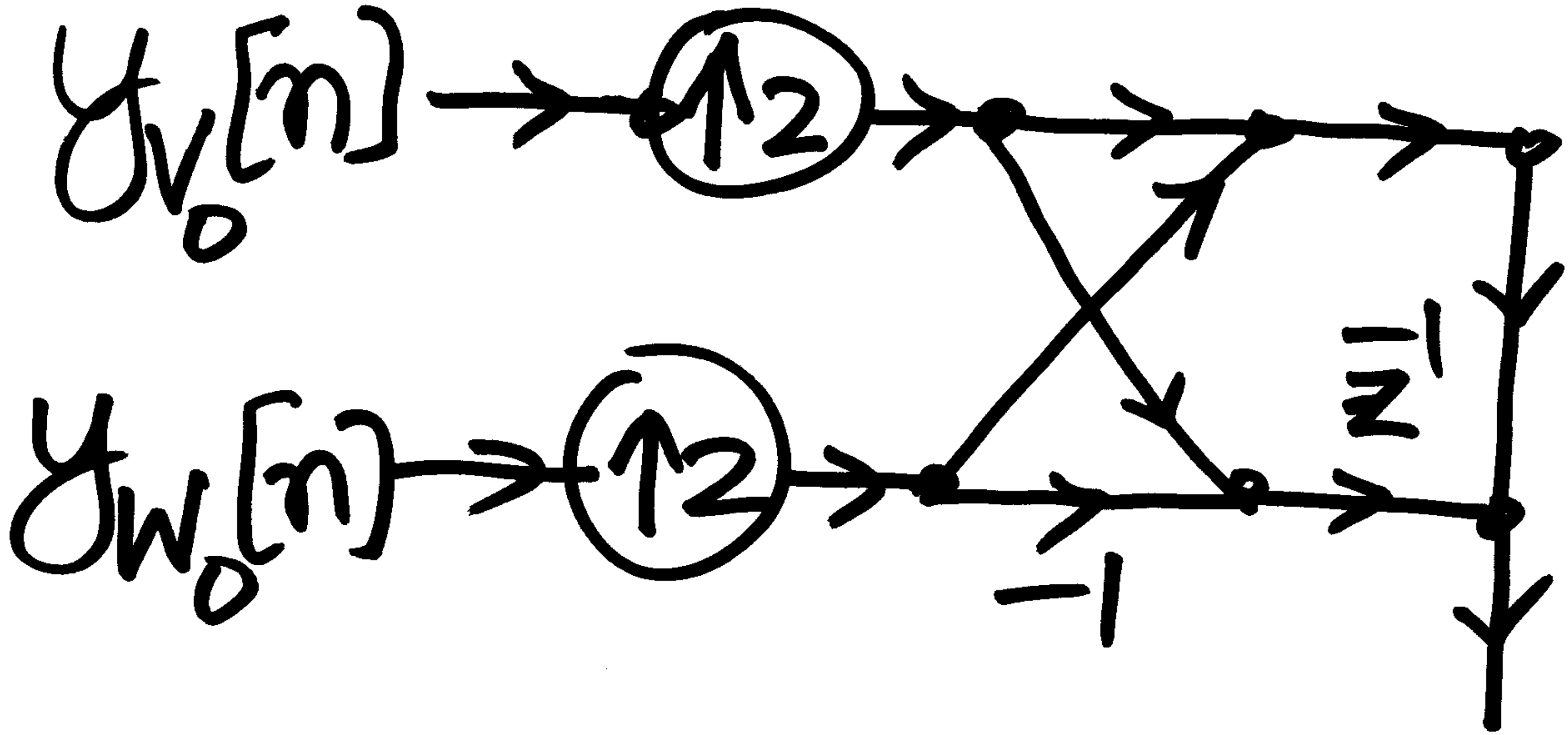
Alternately take ① and ②.


y_{v_0}[n]
↑2
1+z^{-1}
y_{w_0}[n]
↑2
1−z^{-1}

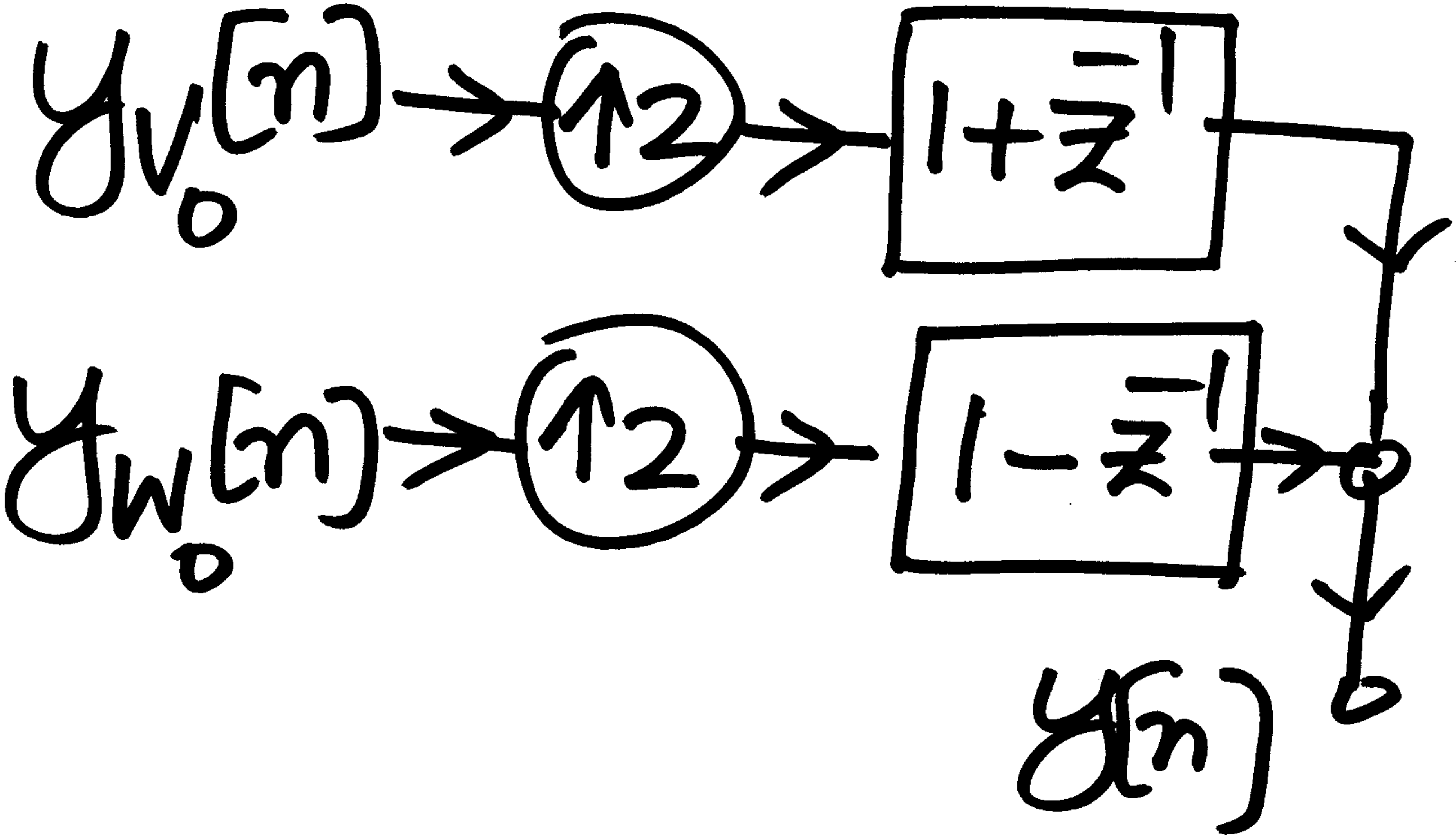
Redraw as:



SYNTHESIS FILTER BANK



$y_{v_0}[n]$
↑2
$y_{w_0}[n]$
↑2
z^{-1}
-1



$y_{v_0}[n]$
↑2
$1+z^{-1}$
$y_{w_0}[n]$
↑2
$1-z^{-1}$
$y[n]$