WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

# Lecture 45: Frequency-Domain Analysis of Two-Band Filter Bank

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## 1 Introduction

Previous session we have placed before you a tutorial on two-band filter bank from the point of view of the time domain. We want to understand in depth how a signal progress through a different stages of a two-band filter bank and emerges with perfect reconstruction subjected to a delay and some constant multiplication factor.

Now, we will see the frequency domain analysis of two-band filter bank. *How do we see the sinusoidal being treated by different stages of the two-band filter bank? What does the two-band filter bank do in frequency domain?* Now can we illustrate it with a use of a domain for looking a two-band filter bank. Each one of them has advantages and limitations.

Advantage of time domain precisely is to understand what filter bank does to the signal in the natural domain. When we have long term signal in mind and we wish to look at the sinusoidal content, both at the input and output, it is the frequency characteristics of a two-band filter bank.

## 2 Two-Band Filter Bank

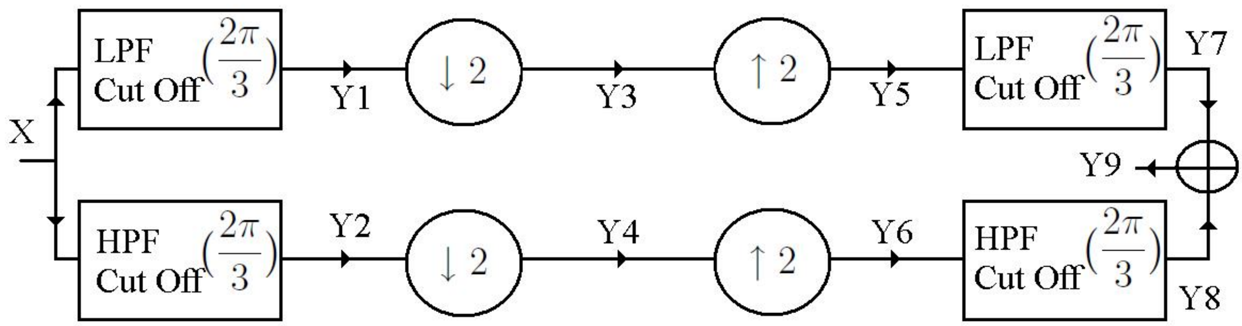


Figure 1: Two-Band filter Bank

The deviation from the ideal two-band filter bank is that LPF and HPF are with cutoff $\left(\frac{2\pi}{3}\right)$.

We will consider the 'Prototype' input as shown in the figure.2. We consider this input because it's amplitude is linear with the frequency scale.

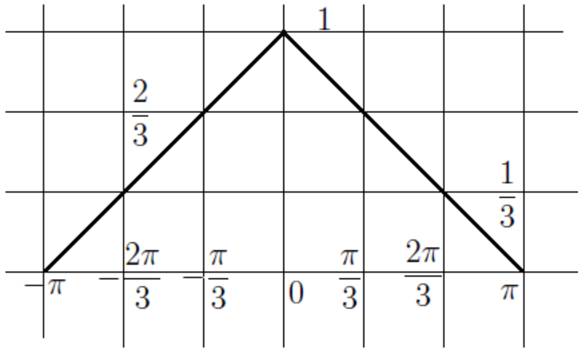


Figure 2: Prototype Input

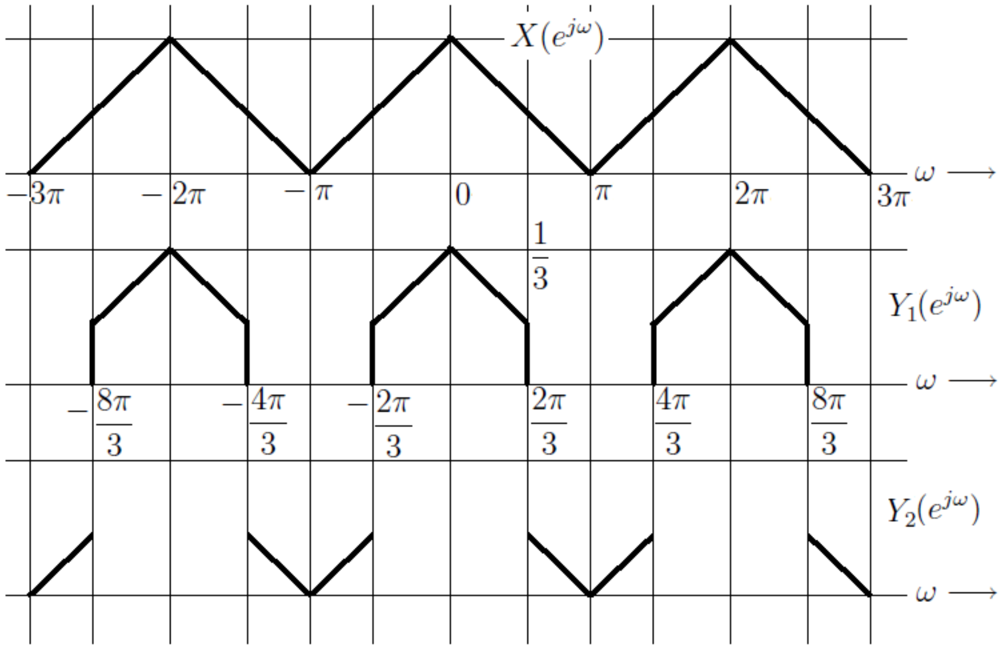


Figure 3: Output at $Y_1$ and $Y_2$ in Two-Band filter Bank

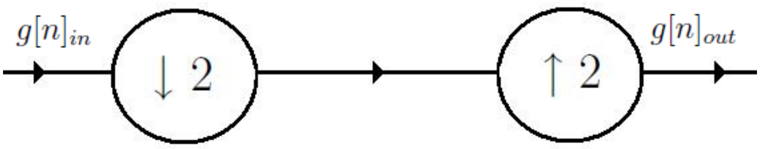


Figure 4: Intermediate Branch of Two-Band Filter Bank

$$G_{out}(Z) = \frac{1}{2}\{Gin(Z) + Gin(-Z)\}$$

In $Z$ Domain $\quad Z \longleftarrow e^{j\omega}$

$$G_{out}(Z) = \frac{1}{2}\left\{Gin(e^{j\omega}) + Gin(e^{j(\omega\pm\pi)})\right\}$$

$$G_{out}(Z) = \frac{1}{2}(\text{Original DTFT}) + \frac{1}{2}(\text{Aliased DTFT})$$

$$Y_6(e^{j\omega}) = \frac{1}{2}\left\{Y_2(e^{j\omega}) + Y_2(e^{j(\omega\pm\pi)})\right\}$$

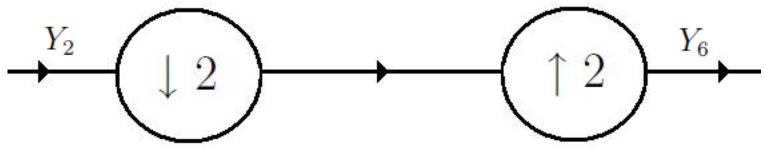


Figure 5: Intermediate Branch of Two-Band Filter Bank

Essentially multiplication by $-1$ of variable $Z$ accounts of phase shift by $\pi$.
Aliased is obtained by replacing $Z$ by $-Z$,$\Rightarrow e^{j\omega} \leftarrow -e^{j\omega}$.

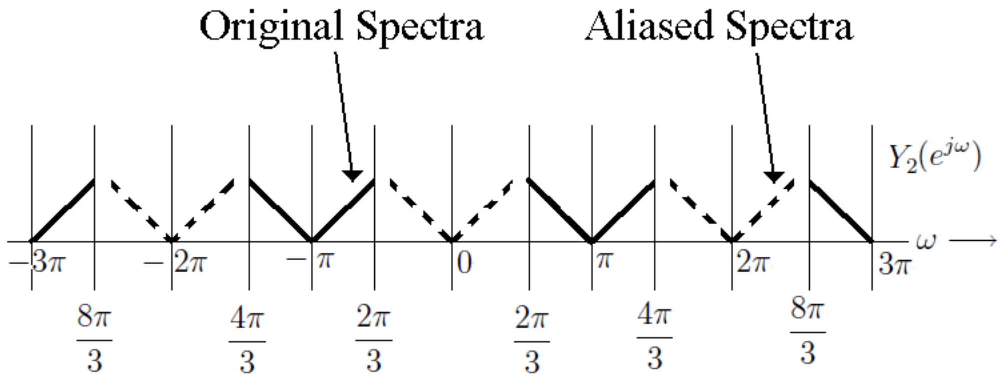


Figure 6: DTFT Spectra

Now subjecting to the action of high pass filters of cutoff $(\frac{2\pi}{3})$. Retains the original spectra. HPF with cutoff $(\frac{\pi}{3})$ retains $\frac{1}{2}Y_2(e^{j\omega})$ and destroy $\frac{1}{2}Y_2(e^{j(\omega\pm\pi)})$ ( aliased).

$$Y_5(e^{j\omega}) = \frac{1}{2}Y_1(e^{j\omega}) + \frac{1}{2}Y_1(e^{j(\omega\pm\pi)})$$

Now it illustrate very clearly in the frequency domain what the consequence of non ideal cutoff is although the two filters is looked two be complementary because one of the filter did not obey the requirement of aliasing or a rather had a passband beyond $\frac{\pi}{2}$ we observed the aliasing taking place.

As expected the aliasing takes place to the extend that we exceeded the $\frac{\pi}{2}$ band. The excess was from $\frac{\pi}{2}$ to $\frac{2\pi}{3}$ and therefore we have aliasing between $\frac{\pi}{2}$ + $\frac{\pi}{3}$ and $\frac{\pi}{2}$ - $\frac{\pi}{3}$.

Aliasing has occurred in a band of extent $\frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$ on either side of $\frac{\pi}{2}$.

This is an example of frequency domain behavior where we are not adhering to the requirement of cutoff.

As we can see in the Figure.7 the overlap between the original spectra and the aliased spectra resulted into a straight line.

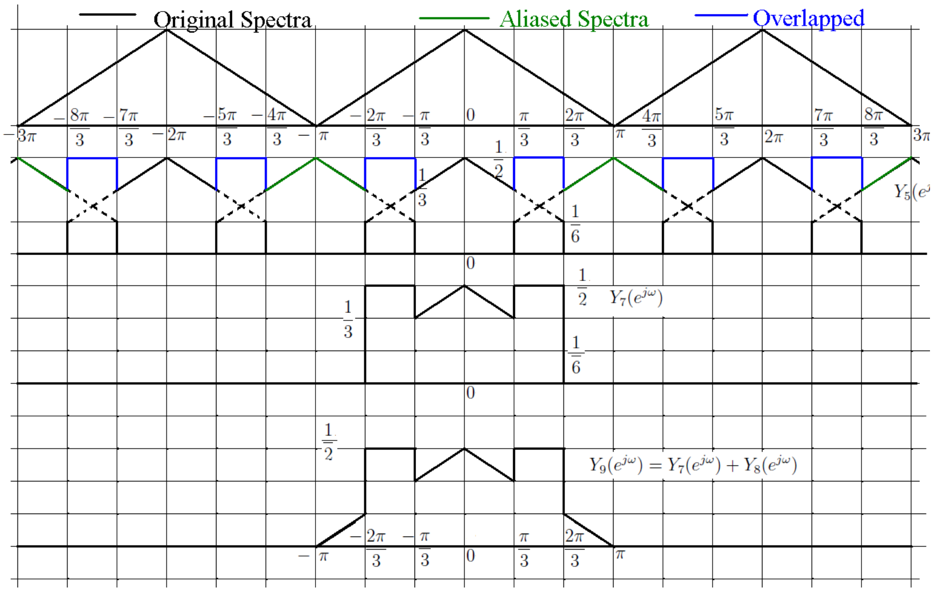


Figure 7: DTFT Spectra

# 3 Hybrid Filter Bank

Now lets look at some of the variants of the two-band filter bank. This time not just the two-band filter bank but also the hybrid filter bank where we have a three-band and two-band combination.

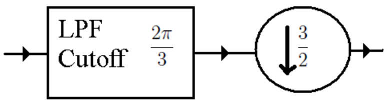


Figure 8: Hybrid Filter bank

We can interpret the operation $\downarrow \frac{3}{2}$ as two separate operations down-sampling by 3 and up-sampling by 2. The problem is down sampling operation is not reversible whereas up-sampling operation is reversible.

Therefore we first do the up-sampling by 2 and then down-sampling by 3. Thus the loss is incurred at the end only.

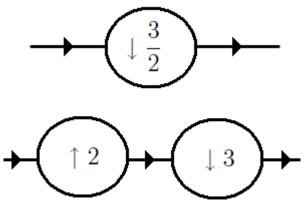


Figure 9: Down Sampler

Now we again put the ‘Prototype’ input (shown in Figure.2) to the LPF with cutoff $\frac{2\pi}{3}$

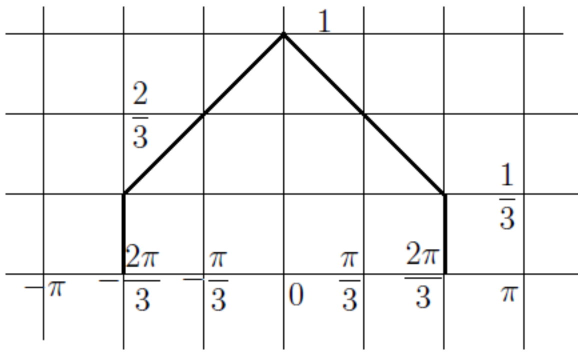


Figure 10: Output after passing the prototype input through LPF with cutoff $\frac{2\pi}{3}$

We will subject the input $X(e^{j\omega})$ to first up-sampling by 2. Then some filtering $F_1$ followed by down-sampling by 3 to get output as $Y_0(e^{j\omega})$


Figure 11: Equivalent structure for down-sampling by $\frac{3}{2}$

Output of $Y_1(e^{j\omega})$ after subjecting it to the down-sampling by 2 will shrink to half due to up-sampling. The filter $F_1$ needs to be an ideal low pass filter with cutoff $\frac{\pi}{2}$. After passing through the lowpass filter $F_1$ we will get the output at $Y_2(e^{j\omega})$

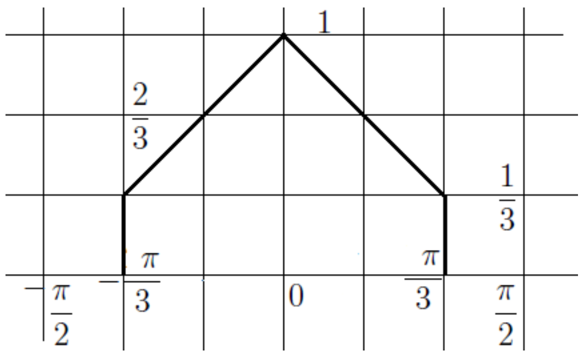


Figure 12: Spectrum of $Y_2(e^{j\omega})$