WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 18: The Time-Bandwidth Product

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Self Evaluation Quizzes

Q 1. How you can decide by just looking at the function only whether its time and frequency center are zero or not?
Ans. If $|x(t)|^2$ is symmetric around $t = 0$ in time domain then time-center is zero. In case of real signals frequency center is zero because in that case Fourier Transform is symmetric around zero frequency point.

Q 2. What is the difference between taking the $\mathbb{L}_2$ norm and modulus square of a function? Why we considered $\mathbb{L}_2$ norm in calculating variance and not any other norm?
Ans. Norm is a scalar quantity and is given by

$$\|x(t)\|^2 = \{2\}^{1/2}$$

while $|x(t)|^2$ is just the modulus square of a function. We considered $\mathbb{L}_2$ norm because our assumed function belongs to $\mathbb{L}_2$.

Q 3. Does a function need to be compactly supported in space to have finite variance?
Ans. No, to have finite variance a function need not be compactly supported however it should not diverge as t or f tends to infinity.

Q 4. What do you think, when you can achieve minimum time-bandwidth product in among the following cases (just qualitative answers). Support your answer with some examples.
(a)Signal is band-limited in time-domain
(b)Signal is band-limited in frequency domain
(c)No restrictions of being band-limited in both domains
Ans. As we know if a function is band-limited in one domain we cant make it band-limited in other domain too. So if we are getting some very small value of variance in one domain then it is most probable that we will get large value of variance in other domain. So to have optimal value of time-bandwidth product we need to look at functions not just band-limited for example in case of Gaussian function where we get minimum time-bandwidth product irrespective of the fact that it is not band-limited in any of the domains.

Q 5. Are all orthogonal transformations like Fourier transform are invariant to time-bandwidth product?(Yes/No)
Ans. No.