WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 11: Two Channel Filter Bank

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Self Evaluation Quizzes

Q 1. Obtain the Z-Transform of a general down-sampler (down-sampling by a factor of M).
Ans. Down-sampler can be broken down into two steps as modulation by a sequence and then inverse up-sampling. General sequence for modulation in case of down-sampling by a factor of M can be written as:

$$\frac{1}{M}\sum_{l=0}^{M-1} e^{\frac{-j2\pi l}{M}} \tag{1}$$

Using this equation, we can write the Z-Transform of down sampler by factor M as:

$$X_{out,d}(Z) = \frac{1}{M}\sum_{l=0}^{M-1} X_{in}(Z^{\frac{1}{M}} * e^{-j2\pi\frac{l}{M}}) \tag{2}$$

Q 2. Obtain the output of M-channel filter bank.
Ans. Z-Transform of M-channel filter bank which has M analysis filters denoted by $H_0(Z)$, $H_1(Z), \cdots H_{M-1}(Z)$ and corresponding M synthesis filter denoted by $F_0(Z)$, $F_1(Z) \cdots F_{M-1}(Z)$ can be written as follows:

$$X_{out,d}(Z) = \frac{1}{M}\sum_{l=0}^{M-1}(X_{in}(Z * e^{\frac{-j2\pi l}{M}})\sum_{k=0} M-1(H_k(Z * e^{\frac{-j2\pi l}{M}})) * F_k(Z)) \tag{3}$$

(For more reading on filter banks, you can refer "Multi-rate Systems and Filter Banks" by P.P.Vaidyanathan).