WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 7: Frequency domain behaviour of Haar filter banks

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Self Evaluation Quizzes

Q 1. Why is the Haar MRA considered not adequate even though it can give perfect reconstruction?
Ans. The frequency response of the Haar low pass and high pass filter is $cos(\frac{\omega}{2})$ and $sin(\frac{\omega}{2})$ respectively. This leads to poor localization in frequency domain as the filter cutoff is not sharp. Hence the Haar MRA is not considered adequate.

Q 2. Why is a filter with a zero phase response necessarily causal?
Ans. Consider a filter with transfer function $h(t)$ and fourier transform $H(\Omega)$.

$$\begin{aligned} H(\Omega) &= \int_{-\infty}^{\infty} h(t)e^{-j\Omega t}dt \\ &= \int_{-\infty}^{\infty} h(t)cos(\Omega t)dt - j\int_{-\infty}^{\infty} h(t)sin(\Omega t)dt \end{aligned}$$

For a zero phase response, imaginary part of $H(\Omega)$ is zero. Hence,

$$\int_{-\infty}^{\infty} h(t)sin(\Omega t)dt = 0$$

Thus $h(t)$ must be an even function of t, *i.e.* symmetric about zero. Thus it will have values for $t < 0$ and hence will be non-causal.

Q 3. Why is linear phase important? What is pseudo linear phase?
Ans. A transfer function is said to have linear phase if the phase angle is directly proportional to frequency and the line of angle v/s frequency passes through the origin. Linear phase is important because it ensures that the entire waveform is shifted by the same time. In non-linear phase system, since the different frequency components are delayed by different amounts, the waveform is distorted.
A transfer function is said to have Pseudo-linear phase if the phase angle ϕ is related to frequency Ω by $\phi = m\Omega + c$, where $c \neq 0$(**NOTE**: c can be different for different parts of Ω axis). Refer to figures of magnitude and phase plots in lecture notes for further clarification.

Q 4. Why does 2-D processing NOT require causality as a condition for filter?
Ans. For applications such as image processing, the entire data is already present. Hence, the past, present and future samples are already known. Thus a non-causal filter can be employed for 2-D applications. However, if in 2-D processing, we have 2 different 1-D data streams which have to be operated in real time, then we cannot employ non-causal systems.