WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 6: The Haar Filter Bank

Prof. V.M.Gadre, EE, IIT Bombay

Self Evaluation Quizzes

Q 1. Comment on the linearity, time-invariant and invertibility property of Up-sampler and Down-sampler.

Ans. Up-sampler: General equation for up-sampling by M is given as,

$$\begin{aligned} x_{out}[n] &= x_{in}\left[\frac{n}{M}\right], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned}$$

Since zeros are added in between successive samples of a signal, there is no loss of information during up-sampling and we can retrieve the original sequence by passing through the down-sampler by M. Hence, it is an **Invertible** operation.
Now consider,

$$\begin{aligned} x_{1out}[n] &= x_{1in}\left[\frac{n}{M}\right], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned}$$

$$\begin{aligned} x_{2out}[n] &= x_{2in}\left[\frac{n}{M}\right], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned}$$

Now, if we apply the combine input x_{1in} and x_{2in}, we get,

$$\begin{aligned} x_{out}[n] &= x_{1in}\left[\frac{n}{M}\right] + x_{2in}\left[\frac{n}{M}\right], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned}$$

which is same as,

$$\begin{aligned} x_{out}[n] &= x_{1out}[n] + x_{2out}[n], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned}$$

Hence, upsampling is a **Linear** operation.
Now, again consider our general equation of up-sampler by M. Now, delay the input sequence by k samples.

$$\begin{aligned} x_{out}[n] &= x_{in}\left[\frac{n}{M} - k\right], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned} \tag{1}$$

Now, replace n by $n-k$ in equation, we get

$$\begin{aligned} x_{out}[n] &= x_{in}\left[\frac{n-k}{M}\right], \qquad where\, n\, is\, multiple\, of\, M \\ &= 0, \qquad otherwise \end{aligned} \tag{2}$$

Now, equation (1) and (2) are not equal. Hence, upsampling by M is **Time-variant**.
Down-sampler: General equation for down-sampling by M is given as,

$$x_{out}[n] \quad = \quad x_{in}[Mn]$$

Here, the samples which are at non-multiple of M are discarded. Hence, it is not possible to retrieve the discarded samples by passing through up-sampler. Hence, it is **Non-invertible**. Similar to up-sampler, we can show that downsampling is **Linear** and **Time-variant** operation.