WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 5: Equivalance between Function and Sequence

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Self Evaluation Quizzes

Q 1. What are basis functions? Write down the bases for spaces V_1, V_{-1} and W_{-1} of Haar MRA.
Ans. Basis functions are the basic building blocks of a function space. Any function in the function space, for example $\mathbb{L}_2(\mathbb{R})$ space, can be represented as linear combination of the basis functions or bases of that space. Bases for spaces V_1, V_{-1} and W_{-1} of Haar MRA are $span\{\Phi(2t-n)\}_{n\in\mathbb{Z}}$,$span\{\Phi(\frac{t}{2}-n)\}_{n\in\mathbb{Z}}$ and $span\{\Psi(\frac{t}{2}-n)\}_{n\in\mathbb{Z}}$ respectively.

Q 2. Consider a triangular pulse $x(t)$ as described by

$$\begin{aligned} x(t) &= t, & 0 \le t \le 1 \\ &= 2-t, & 1 \le t \le 2 \\ &= 0, & otherwise \end{aligned}$$

Write down a sequence $x[n]$ using dilates and translates of $x(t)$.
Ans. $x(t)$ can be sketched in terms of its own dilates and translates as shown in figure 1. So

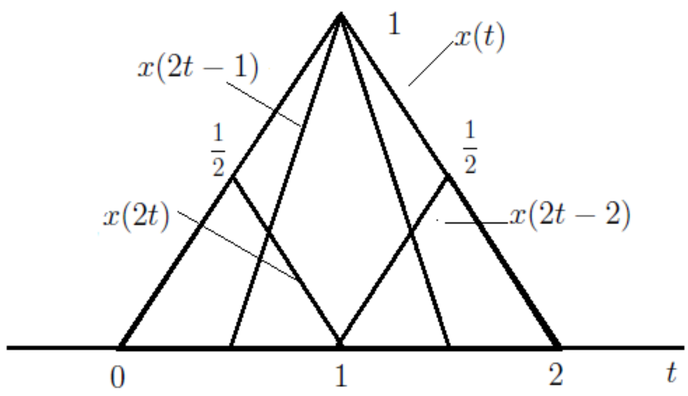

Figure 1: $x(t) = \frac{1}{2}x(2t) + x(2t-1) + \frac{1}{2}x(2t-2)$

corresponding sequence is $x[n] = [\frac{1}{2}, 1, \frac{1}{2}]$

Q 3. In the case of orthogonality and independence of vectors, which is necessary and sufficient condition for the other?
Ans. When two vectors are orthogonal then they are also independent. However, if two vectors are independent then they are not necessarily orthogonal. In other words, orthogonality of vectors is sufficient condition for the vectors to be independent but independence of vectors is necessary for vectors to be an orthogonal.
Moreover, for two subspaces $V1$ and $V2$ of vector space V, $V1$ and $V2$ are said to be orthogonal if every vector in $V1$ is orthogonal to every vector in $V2$ which implies that bases of subspace

$V1$ and $V2$ are orthogonal. In case of independence, bases of subspace $V1$ and $V2$ are independent and need not not be orthogonal.

Q 4. Is orthogonal decomposition unique?
Ans. Orthogonal decomposition is decomposing or splitting the vector space V in two the orthogonal subspaces. Consider a function from V_1 space. Then,

$$V_1 = V_0 \oplus W_0$$

Also we can write

$$V_0 = V_{-1} \oplus W_{-1}$$

In general,

$$V_{m+1} = V_m \oplus W_m$$

where $m\epsilon\mathbb{Z}$. Hence orthogonal decomposition is not unique but once the subspaces are fixed then the decomposition is unique.
For example, a function from V_1 space can be represented as linear combination of scaling function $\phi(t)$ and wavelet function $\psi(t)$ if space is decomposed into subspaces V_0 and W_0 i.e.

$$f(t) = \sum_{n\epsilon\mathbb{Z}} \phi(t-n) + \sum_{n\epsilon\mathbb{Z}} \psi(t-n)$$

Similarly, the same function can be represented as linear combination of dilates of scaling and wavelet function from subspaces V_{-1}, W_{-1} respectively and wavelet function from space W_0 as

$$f(t) = \sum_{n\epsilon\mathbb{Z}} \phi\Big(\frac{t}{2} - n\Big) + \sum_{n\epsilon\mathbb{Z}} \psi\Big(\frac{t}{2} - n\Big) + \sum_{n\epsilon\mathbb{Z}} \psi(t-n)$$