WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 2: Haar Multiresolution analysis

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Self Evaluation Quizzes

Q 1. Define L_p norm.
Ans. L_p norm of a function $x(t)$ is defined as,

$$\left\{\int_{-\infty}^{\infty} |x(t)|^p dt\right\}^{1/p}$$

Q 2. What is $L_2(\mathbb{R})$ space.
Ans. It is a space of functions whose L_2 norm is finite.

Q 3. Why in general, only functions in $L_2(\mathbb{R})$ are considered for piecewise constant approximations?
Ans. The L_2 norm signifies the energy of a signal. In general most of the real time signals have finite energy. So we are interested in the signals which have finite energy. So functions in $L_2(\mathbb{R})$ are considered for piecewise constant approximations.

Q 4. Calculate the L_2 norm of the following function $x(t)$, where $x(t) = 1- \mid t \mid$ in the interval [-1,1] and zero elsewhere.
Ans. Since the function is symmetric between -1 to 0 and 0 to 1, the norm will be twice that of the value calculated in any one of the above two intervals.
Hence L_2 norm of $x(t) = [2\int_0^1 \mid (1-t) \mid^2 \ dt]^{\frac{1}{2}} = [2\int_{-1}^0 \mid (1+t) \mid^2 \ dt]^{\frac{1}{2}} = \sqrt{2/3}$

Q 5. Give an example of function which does not exist in $L_2(\mathbb{R})$.
Ans. All the exponential functions having geometric ratio greater than 1 does not exist in $L_2(\mathbb{R})$. For example consider function 2^t for $t > 0$. Its L_2 norm does not exist because $\left[\int_0^\infty \mid 2^t \mid^2 dt\right]$ does not converge.

Q 6. Give some examples of functions for the following cases:
(a) Function that exists in $L_1(\mathbb{R})$ but does not exist in $L_2(\mathbb{R})$.
(b) Function that exists in $L_2(\mathbb{R})$ but does not exist in $L_1(\mathbb{R})$.
(c) Functions which does not exist in both the spaces.
Ans.
(a) Consider the function $\frac{1}{\sqrt{t}}$ in the interval (0,1] and 0 else where. Its L_1 norm converges but L_2 norm does not converge.
(b) Consider the function $\frac{1}{t}$ in the interval [1,∞). Its L_2 norm converges, but L_1 norm diverges.
(c) All the periodic functions such as $sin(t)$ and $cos(t)$ does not exist in both $L_1(\mathbb{R})$ and $L_2(\mathbb{R})$.

Q 7. Give the axioms that are to be satisfied by a vector space.
Ans. A real vector space is a set X with a special element 0 called as a zero vector, and three operations:

Addition: Given two elements x, y in X, one can form the sum $x+y$, which is also an element of X.
Inverse: Given an element x in X, one can form the inverse $-x$, which is also an element of X.
Scalar multiplication: Given an element x in X and a real number c, one can form the product cx, which is also an element of X.

These operations must satisfy the following axioms:
Additive axioms: For every x, y, z in X, we have
a) $x + y = y + x$.
b) $(x + y) + z = x + (y + z)$.
c) $0 + x = x + 0 = x$.
d) $(-x) + x = x + (-x) = 0$.
Multiplicative axioms: For every x in X and real numbers c,d, we have
a) $0x = 0$.
b) $1x = x$.
c) $(cd)x = c(dx)$.

Distributive axioms: For every x, y in X and real numbers c, d, we have
a) $c(x + y) = cx + cy$.
b) $(c + d)x = cx + dx$. Scalar multiplication: Given an element x in X and a real number c, one can form the product cx, which is also an element of X.