WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 1: Introduction

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Self Evaluation Quizzes

Q 1. Why Sine waves are popular in signal analysis ?
Ans. There are several reasons for popularity of sine waves, Following are few:

1. Sine waves occur naturally in different circumstances.
2. They are smoothest possible periodic functions
3. They inherently have power to express other signals.
4. Linear combination/ differentiation/ integration of sine waves gives the output in the same domain of sinusoids.

Q 2. What is the difference between waves and wavelets ?
Ans. *Wave* is a repeated periodic disturbance that moves through a medium from one location to another. *Wavelet* means "small wave" *i.e.* wave that doesn't last forever. For example, a sinusoid ranging from $-\infty$ to ∞ can be treated as wave whereas Morlet wavelet is a type of wavelet given by $x(t) = e^{-t^2/2}\cos(5t)$. Figure 1 shows the plot of a wave and a Morlet wavelet. The corresponding code is as follows.

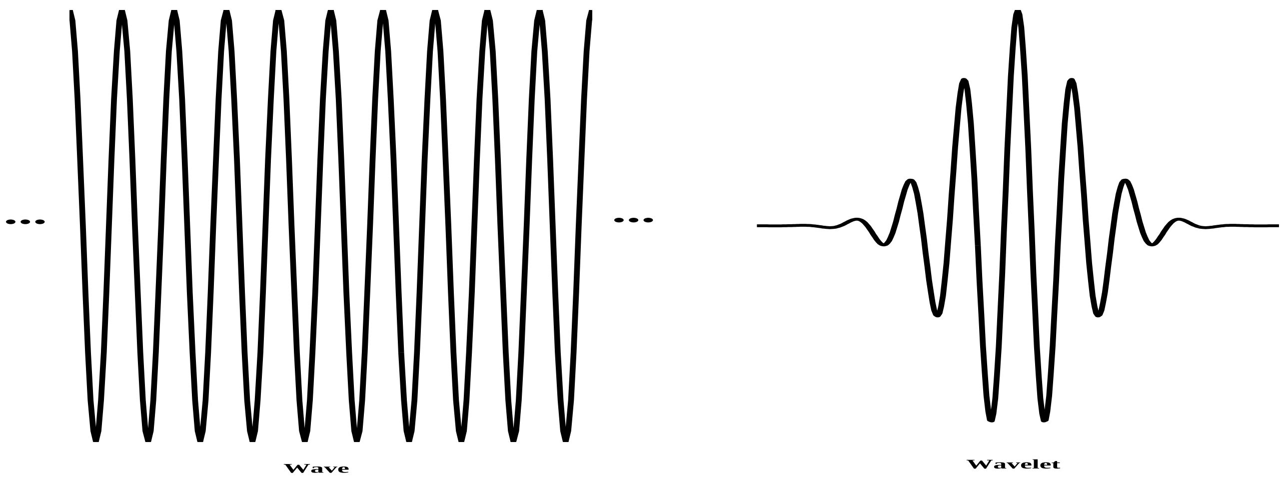

Figure 1: Wave versus Wavelet.

```
% Title: Wave versus Wavelet.
% Author: Ankit Bhurane (ankit.bhurane@gmail.com)
% Last Modified: 28-Apr-2012.

clc; clear all; close all;

Fs = 100; % Sampling frequency
```

```
ts = -100:100; % Time axis for sinusoid
s = cos(2*pi*5/Fs*ts); % Sinusoid
tw = -3:0.02:3; % Time axis for Morlet wavelet
w = exp(-tw.^2).*cos(10.*tw); % Morlet wavelet equation
subplot(121); plot(ts/Fs,s); title('Wave');
subplot(122); plot(tw/Fs,w); title('Wavelet');
```

Q 3. Compare Fourier transform with Wavelet transform.
Ans. Fourier analysis of a signal gives all the frequency components present in the signal. So, if we take the Fourier transform of the signal then we know about the frequency content of that signal. But cannot comment on the instance or the time at which those frequencies exits. So the time information is lost. On the other hand, Wavelet transform deals with both time and frequency information simultaneously.

E.g., consider an impulse function. Its Fourier transform contains all the frequency components from $-\infty$ to ∞. This doesn't specify at what time these frequencies are present. Whereas, wavelet provides us both time and frequency information at multiple scales as shown in the Figure 2. The corresponding code is given below.

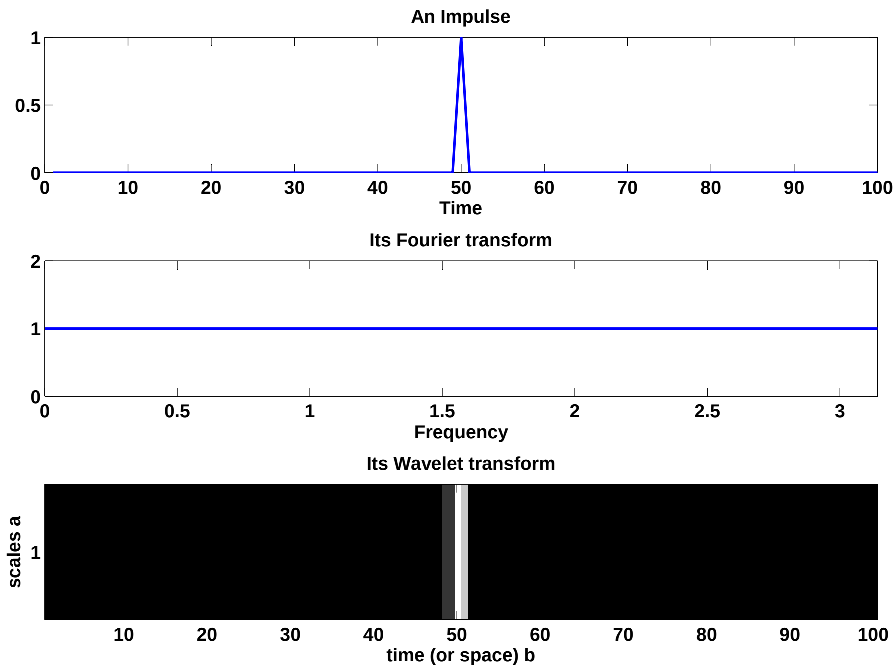

Figure 2: Fourier Vs Wavelet transform

```
% Title: Fourier Transform versus Wavelet Transform.
% Author: Saket Porwal and Newton
% Last Modified: 29-Apr-2012.

clc; clear all; close all;

x = zeros(1,100);
x(50) = 1; % Impulse
f= linspace(0,pi,100);
```

```
subplot(311); plot(x); axis([0 100 0 1]);
xlabel('Time'); title('An Impulse')
F = abs(fftshift(fft(x))); % Fourier transform
subplot(312);
plot(f,F); axis([0 pi 0 2]);
xlabel('Frequency'); title('Its Fourier transform')
subplot(313);
w = cwt(x,1,'sym2','plot'); % Wavelet tranform
colormap gray; title('Its Wavelet transform')
```

Q 4. What is STFT(Short Time Fourier Transform) and its drawbacks compare to wavelet transform?

Ans. In STFT, signal is first windowed using different type of window functions like Triangular window, Rectangular window, Gaussian window etc. Now Fourier Transform of resulting windowed signal is taken. This gives the STFT of signal for particular time. As window slides along time axis, so basically STFT maps input signal $x(t)$ into two dimensional function in a time-frequency plane or tiling.

So, drawback of STFT is that once a window has been chosen for STFT then time-frequency resolution is fixed over the entire time-frequency plane or say STFT moves a tile of constant shape in the time-frequency plane. But in case of wavelet transform, the time resolution becomes arbitrarily good at high frequencies, while the frequency resolution becomes arbitrarily good at low frequencies. In other words we can say that CWT moves a tile of variable shape but constant area which is governed by time-bandwidth product.

Q 5. What is uncertainty principle? Explain it for different domains.

Ans. The basic Uncertainty principle also called Heisenberg's Uncertainty Principle is that, the position and velocity of an object cannot both be measured exactly, at the same time. Uncertainty principle derives from the measurement problem, the connection between the wave and particle nature of quantum objects. The uncertainty principle is alternatively expressed in terms of particle's momentum and position. The momentum of a particle is equal to the product of its mass and its velocity. The principle applies to other related (conjugate) pairs of observable, such as energy and time. Finally uncertainty principle says that exact knowledge of complementarity pairs (position, energy/velocity, time) is impossible.

In time-frequency perspective, the Uncertainty Principle puts a restriction on localizing/ resolving/ focusing both time and frequency simultaneously. This trade-off of time-frequency resolution will be studied in detail further in this course.

Q 6. What are basis functions? What type of basis function is used in Fourier Transform? How is it different from the basis functions used in Haar multiresolution analysis?

Ans. A functional object can be represented as a linear combination of elementary functional building blocks. These basic functional blocks are known as *basis functions*. For example, complex exponential functions form the basis for Fourier analysis. These basis are extremely smooth functions. On the other hand, Haar analysis uses piecewise constant function(ex. Haar scaling and Wavelet function) as basis *i.e.* every signal is represented as a linear combination of piecewise constant functions.