

Department of Physics  
 Indian Institute of technology Madras  
 Select/Special Topics in Classical Mechanics  
 Self-Assessment-1 (Questions)

Answer all the 16 (**SIXTEEN**) questions **STRICTLY** in the space provided for the same. Extra pages are added for your rough work. *However, do not forget to insert your final answer in the specific space designated for each question. Answers written elsewhere will **NOT** be evaluated.*

p1

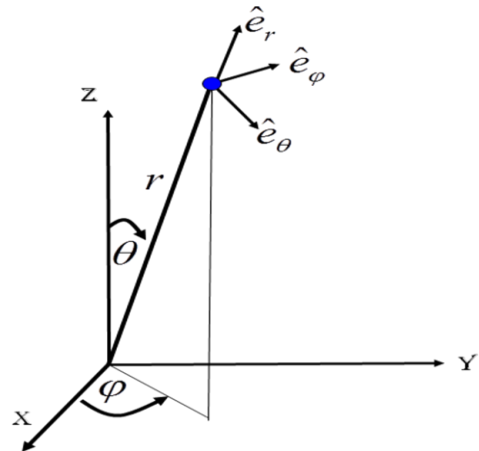
**NOTE:** Notations/symbols used have their usual meanings. Page numbers are given in 'circles' in left-side margin.

1. Express the quantities on the left hand sides of the equations below in terms of the coordinates and unit vectors of the spherical polar coordinate system; give your answer in the blank space on the right hand side of the equation:

$$\left[ \vec{r} \right]_{\text{sph.polar coord.}} = \underline{\hspace{10cm}}$$

$$\left[ d\vec{r} \right]_{\text{sph.polar coord.}} = \underline{\hspace{10cm}}$$

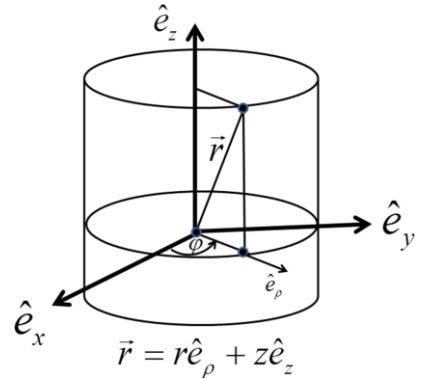
$$\left[ \frac{\partial \hat{e}_\varphi}{\partial \varphi} \right]_{\text{sph.polar coord.}} = \underline{\hspace{10cm}}$$



2. Fill in the blank spaces in the equations below by appropriate coefficients that are functions of the cylindrical polar coordinates  $(\rho, \varphi, z)$ :

$$-e_\rho = e_x(\underline{\hspace{2cm}}) + e_y(\underline{\hspace{2cm}})$$

$$-e_\phi = e_x(\underline{\hspace{2cm}}) + e_y(\underline{\hspace{2cm}})$$



p2

3. A vector point function is defined by  $\vec{A}(\vec{r}) = 3y\hat{e}_x + 4z\hat{e}_y - 6x\hat{e}_z$ .  
Fill in the blanks by writing this vector in cylindrical polar coordinates.

$$\vec{A}(\vec{r}) = \hat{e}_\rho \underline{\hspace{2cm}}$$

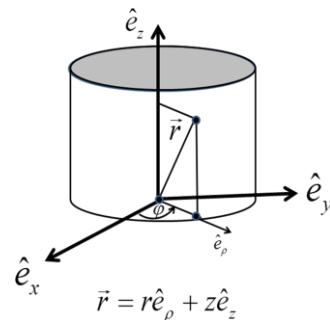
$$+ \hat{e}_\phi \underline{\hspace{2cm}}$$

$$+ \hat{e}_z \underline{\hspace{2cm}}$$

4. A vector point function is defined by

$$\vec{G}(\vec{r}) = \hat{e}_\rho \{ \sin \phi + \cos \phi \} + \hat{e}_\phi 10 \cos \phi - 10 \hat{e}_z$$

in the cylindrical polar coordinates. Find the flux of  $\vec{G}$  crossing the surface of the cylinder shown in the figure from its inside to outside. The cylinder is closed at its top face, at  $z=2$ , and it is open at the bottom, which is in the  $XY$  plane at  $z=0$ .



**Write your answer to Q4 here:**

The required FLUX is = \_\_\_\_\_

5. We use the usual notation:  $\vec{r}$  : position vector;  $\vec{p}$  : linear momentum;  $\vec{L}$  : angular momentum.

5(a) Express the triple product  $\{\vec{r} \cdot (\vec{p} \times \vec{L})\}$  in terms of an appropriate function of the angular momentum  $\vec{L}$  alone.

**Write your answer to 5(a) here:**  $\{\vec{r} \cdot (\vec{p} \times \vec{L})\} = f(\vec{L}) =$  \_\_\_\_\_

5(b) Given:  $\vec{V} = \vec{L} \times [\vec{p} \times \vec{L}]$ . Tick mark  in the box that corresponds to the correct statement(s) amongst the following:

- (i)  The vector  $\vec{V}$  is parallel to the linear momentum vector  $\vec{p}$ .
- (ii)  The vector  $\vec{V}$  is anti-parallel to the linear momentum vector  $\vec{p}$ .
- (iii)  The vector  $\vec{V}$  is orthogonal to the linear momentum vector  $\vec{p}$ .
- (iv)  The vector  $\vec{V}$  makes an angle between  $0^\circ$  and  $90^\circ$  with respect to the linear momentum vector  $\vec{p}$ .
- (v)  The vector  $\vec{V}$  makes an angle between  $90^\circ$  and  $270^\circ$  with respect to the linear momentum vector  $\vec{p}$ .

5(c) Given: The Laplace-Runge-Lenz vector is  $\vec{A} = (\vec{p} \times \vec{L} - mk\hat{e}_r)$ , and we define a new vector

$\vec{F} = \vec{L} \times \vec{A}$ . Tick mark  in the box that corresponds to the correct statement(s) amongst the following:

- (i)  The vector  $\vec{F}$  is parallel to the linear momentum vector  $\vec{p}$ .
- (ii)  The vector  $\vec{F}$  is parallel to the major axis of the ellipse.
- (iii)  The vector  $\vec{F}$  orthogonal to the plane of the Kepler orbit.
- (iv)  The vector  $\vec{F}$  is parallel to the minor axis of the ellipse.
- (v)  None of the above.

p4

Q.6 A point mass 'm' moves along a curved trajectory in a plane at  $z=0$  such that it is at a radial distance  $\rho$  from a centre of force, and  $\phi$  is the azimuthal angle made by the position vector with the ZX plane. NOTE: Use cylindrical polar coordinate system to answer Q6.

Given: The force acting on the particle is  $\vec{F} = -\frac{k}{\rho^2}\hat{e}_\rho$ ; *k is a positive constant.*

**Now, fill in the following blanks:**

- (a) The work done on the particle by the force in bringing it from infinity, where the potential energy of the particle is zero, to a distance  $\rho$  from the center of the force

is : \_\_\_\_\_.

- (b) The Lagrangian of the particle is: \_\_\_\_\_.

- (c) The generalized momentum for the particle is/are:

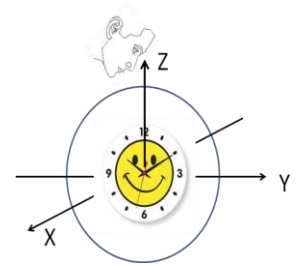
\_\_\_\_\_.

- (d) The Hamilton's principal function for the particle is: \_\_\_\_\_.

7. Find the work done by a force given by

$$\vec{F}(\vec{r}) = (3R \sin \varphi \cos \varphi + 4z \sin \varphi) \hat{e}_\rho + (4z \cos \varphi - 3R \sin^2 \varphi) \hat{e}_\varphi - 6\rho \cos \varphi \hat{e}_z$$

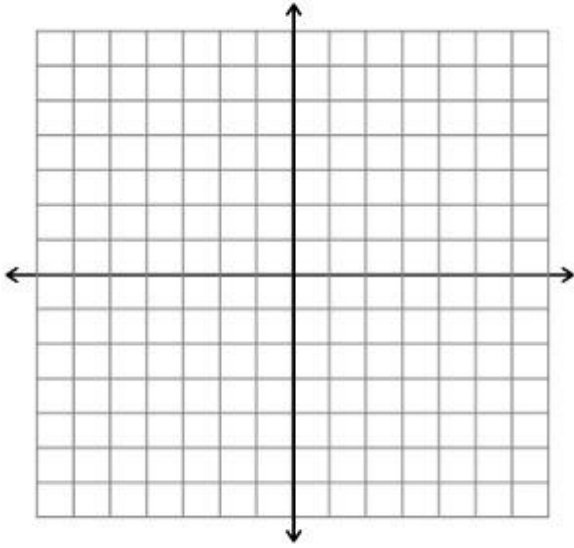
over a circular path in the XY plane with center at the origin and radius R. The sense in which the path is traversed is clockwise as viewed from above the XY plane.



**Write your answer to Q7 here:**

**The required WORK DONE is =** \_\_\_\_\_

8. Sketch the vector field of the gradient of the function  $\phi = 100 + xy$  :



9. Determine if the following statements are ‘**TRUE**’ or ‘**FALSE**’, and tick  the right choice. Also, provide the reason:

(a) If the fluid velocity flow is in a ‘steady state’ and it is also ‘irrotational’, then

$\left\{ \frac{p(\vec{r})}{\rho} + \phi + \frac{|\vec{v}|^2}{2} \right\}$  is a constant only for a particular streamline in the fluid but not for the whole velocity field.

Answer here: True

False

Reason:

(b) If the velocity field of a fluid flow is in 'steady state' and it is also 'solenoidal', then the fluid is compressible.

Answer here: True

False

Reason:

p7

Q10. In the Lagrangian description of fluid flow, the velocity field of a given field is:

$\vec{u} = 2z\hat{e}_x + xt\hat{e}_y + xy^2\hat{e}_z$ . For this case, find the acceleration of a fluid particle in this flow.

Write your answer to Q10 by filling in the blanks here:

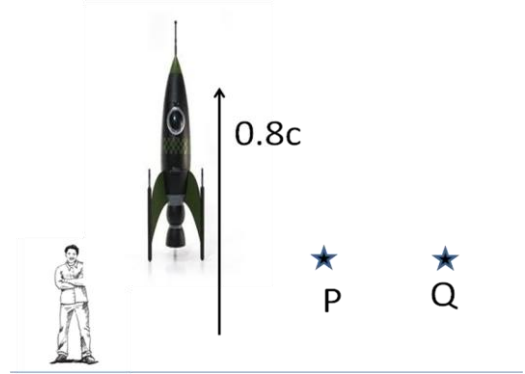
$$\vec{a} = \text{_____} \hat{e}_x + \text{_____} \hat{e}_y + \text{_____} \hat{e}_z$$

Q11. A rocket flying horizontally above Canada flying at speed of  $0.8c$  crosses a certain region below on ground in 1.0 second as per the clock on the rocket. Find the corresponding time interval as would be clocked by an observer on the ground.

Write your answer to Q11 here: The required time interval is = \_\_\_\_\_ seconds.



12. An observer in a rocket flying as shown in the figure at a speed of  $0.8c$  measures a distance between two objects P and Q which are stationary with respect to the observer on ground to be  $2ly$ . Find the corresponding distance as would be estimated by an observer on the ground.



**Write your answer to Q12 here: The required distance is = \_\_\_\_\_ ly.**

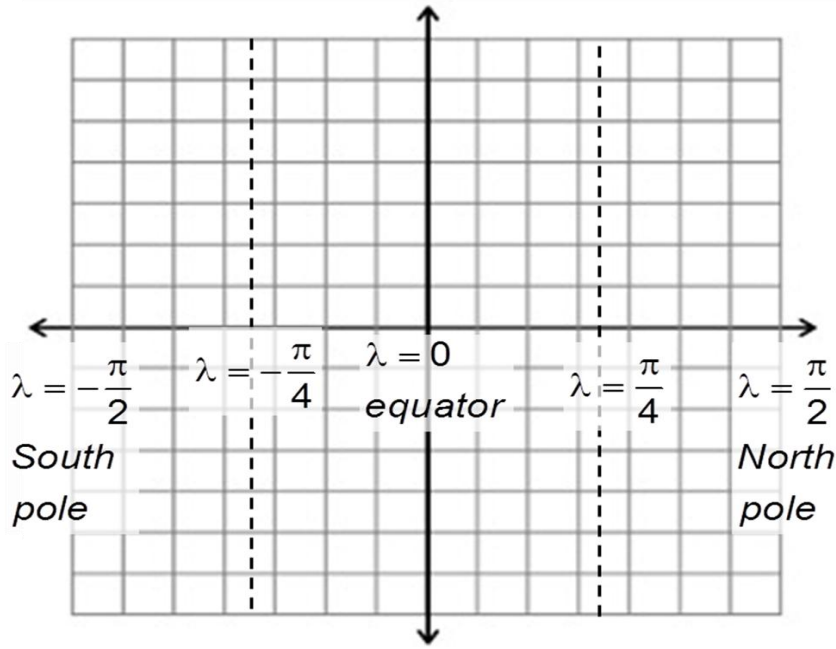
13. (a) Suppose you are standing at the equator. You throw an object directly toward the east. In which direction (if any) will it get deflected?

**State your answer here:** \_\_\_\_\_

- (b) Suppose you are standing at the equator. You throw an object directly toward the west. In which direction (if any) will it get deflected?

**State your answer here:** \_\_\_\_\_

14. Sketch the centrifugal acceleration of an object on earth as a function of the earth's latitude  $\lambda$  on the graph below:



15. Fill in the blanks:

(a) The 'attractor' of simple harmonic oscillator is \_\_\_\_\_.

(b) The 'attractor' of a damped linear harmonic oscillator is \_\_\_\_\_.

(c) The approximate numerical value of the 'golden ratio' (upto two places of decimal) is \_\_\_\_\_.

16. The fractal dimension  $d$  of the Sierpinski triangle is (fill in the blank space below):

$d =$  \_\_\_\_\_

