NPTEL COURSE
TOPICS IN NONLINEAR DYNAMICS

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Quiz 2

1. Are the statements in quotation marks true or false?

   (a) "Any map of the unit interval that is non-invertible leads to dynamics that is chaotic."

   (b) "The Lyapunov exponent of the logistic map $x_{n+1} = \mu\, x_n(1 - x_n)$ at $\mu = \mu_\infty \simeq 3.566\ldots$, where μ_∞ is the limit point of the period-doubling cascade of bifurcations, is equal to ln 2."

   (c) "For any chaotic attractor, the generalized dimension D_0 is equal to the dimensionality of the phase space itself."

   (d) "If the Lyapunov exponent of a one-dimensional map is positive, we may conclude that the dynamics is chaotic for all initial conditions."

   (e) "Stability analysis using a Lyapunov function enables us to decide on the stability of a critical point even in cases where linearization in the vicinity of the critical point is invalid."

   (f) "The logistic map $x_{n+1} = \mu\, x_n(1 - x_n)$ undergoes a Hopf bifurcation at $\mu = 3$."

   (g) "A Hopf bifurcation cannot occur in a Hamiltonian system."

   (h) "The origin $x = 0$, $y = 0$ is a global attractor for the dynamical system $\dot{x} = y\,, \quad \dot{y} = x - x^3 - y$."

   (i) "The winding number of the singularity at the origin of the planar vector field
   $$\mathbf{f}(x, y) = \left(\frac{x^2 - y^2}{(x^2 + y^2)^2}\,, \frac{-2xy}{(x^2 + y^2)^2} \right)$$
   is equal to -2."

   (j) "The damped, unforced Duffing oscillator cannot have any limit cycles."

   (k) Consider the map $x_{n+1} = x_n(3 - 4x_n^2)$, where $x_0 \in [-1\,, 1]$.

   "This map has a stable period-3 cycle."

(1) Let $x(t)$ be a dichotomous Markov process, in which x jumps randomly between two values x_1 and x_2, with mean residence times τ_1 and τ_2 in the two states. Further, let the mean value of $x(t)$ be zero.

"The autocorrelation function $\langle x(0)\, x(t) \rangle$ of the process is a decaying exponential function of t."

2. Let $S \in [0, 1]$ be the set of numbers such that the *decimal* expansion of any $x \in S$ is of the form $x = 0.a_1\, a_2\, a_3\, \ldots$, where each digit is even. (In other words, the digits can only have the values $0, 2, 4, 6$ and 8.) Find the box-counting (or fractal) dimension D_0 of the set S.

3. Three identical tall glasses A, B and C contain water to respective heights x_0, y_0 and z_0. The levels in A and B are first equalised by pouring water from the glass containing more water to the one containing less water. The levels in B and C are then similarly equalised. Finally, the levels in C and A are equalised by the same procedure.

   The entire procedure is iterated over and over again. What are the levels in the three glasses after n iterations of this process?