NPTEL COURSE

TOPICS IN NONLINEAR DYNAMICS

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Quiz 1

1. Are the statements in quotation marks true or false?

   (a) "A conservative dynamical system, given by an equation of motion of the form $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, cannot have any attractors."

   (b) "The harmonic oscillator is the only system whose time period of oscillation is independent of the amplitude of oscillation."

   (c) Let $(q\,,\,p) \longrightarrow (Q\,,\,P)$ be a canonical transformation of an autonomous Hamiltonian system with Hamiltonian $H(q\,,\,p)$.

   "Under such a transformation, the form of Hamilton's equations is preserved, although the functional form of the Hamiltonian in the new variables need not remain the same as the original one."

   (d) "Every dynamical system given by an equation of motion of the form $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ can be transformed into a gradient system by a suitable choice of dynamical variables."

   (e) "Homoclinic orbits can occur in both conservative and dissipative systems."

   (f) "Linear stability analysis need not reveal the correct nature of the flow in the vicinity of a critical point that has a centre manifold."

   (g) "The Liouville-Arnold criterion for integrability is applicable to *any* even-dimensional dynamical system."

   (h) "A bifurcation occurs at some value of a parameter in a dynamical system if the nature of the flow changes qualitatively as the parameter crosses that value."

   (i) "The critical points corresponding to an undamped simple pendulum can only be centres or saddle points."

   (j) Consider the two-dimensional dynamical system given by
   $\dot{x} = x^2 - y^2 \quad \text{and} \quad \dot{y} = 2xy.$

   "The critical point at the origin is a saddle point."

   (k) "A Hopf bifurcation can only occur in a dissipative system."

(l) " If the Poisson bracket of A with B vanishes, and that of B with C vanishes, then the Poisson bracket of A with C *necessarily* vanishes."

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2. Select the correct alternative(s).

   (a) Consider a general Hamiltonian system.

   (A) The Hamiltonian is always a sum of a kinetic energy term and a potential energy term which depends only on the generalized coordinates.

   (B) Saddle-node bifurcations cannot occur in this system.

   (C) The dynamical symmetry group of transformations need not necessarily be identical to the group of canonical transformations.

   (D) Action-angle variables necessarily exist for this system.

   (b) Consider a general autonomous dynamical system described by a set of n coupled, nonlinear, first-order, ordinary differential equations.

   (A) The phase space can be either even or odd dimensional.

   (B) There is always at least one attractor in the system.

   (C) The dynamics is necessarily measure-preserving.

   (D) There must exist at least n functionally independent constants of the motion that do not have any explicit time dependence.

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3. Consider the two-dimensional system given in plane polar coordinates by the equations $\dot{r} = \sin(\pi/r),\ \dot{\theta} = r$.

   (a) Find the limit cycles of the system and determine whether they are stable or unstable.

   (b) Schematically sketch the phase portrait of the system.