Department of Physics
IIT-Madras

PCD_STiAP_Self_Assesment_5
20 marks

1. What are the parameters that can be measured in atomic photoionization process? Explain them briefly. – 3 marks

a. Cross sections: The probability measure of a transition to take place during the process
b. Angular distribution of photoelectrons: The direction in which the photoelectrons are ejected out with reference to the incident beam and polarization direction.
c. Spin-polarization parameters of the photoelectrons: Gives the orientation of the angular momentum of the photoelectrons.

2. Consider a system of 20 electrons. Write down the relativistic configuration of the system and give the dipole transition from the valence subshell. – 2marks

The relativistic Configuration:

$$1s_{1/2}^{2}2s_{1/2}^{2}2p_{1/2}^{2}2p_{3/2}^{4}3s_{1/2}^{2}3p_{1/2}^{2}3p_{3/2}^{4}4s_{1/2}^{2}$$

The allowed dipole transition from 4s subshell is:

$$4s_{1/2} \rightarrow \varepsilon p_{1/2}$$

$$4s_{1/2} \rightarrow \varepsilon p_{3/2}$$

3. Determine the number of states within the solid angle $d\Omega$ in a cubical box of volume V and length L, with energy between E and E+dE. - 4 marks

The number of states in volume element is

$$n^2 dn d\Omega = n^2 \frac{dn}{dE} dE d\Omega \qquad n = \frac{L}{2\pi}k \Rightarrow dn = \frac{L}{2\pi}dk$$

$$= n^2 \frac{dk}{dE}\frac{L}{2\pi} dE d\Omega$$

[mul and div by L/2∏]

$$= n^2 \frac{dk}{dE}\left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{L} dE d\Omega$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\frac{dk}{dE} = \frac{m}{\hbar^2 k}$$

$$= \left(\frac{L}{2\pi}k\right)^2 \left(\frac{m}{\hbar^2}\frac{1}{k}\right)\left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{L} dE d\Omega$$

$$= \left(\frac{L}{2\pi}\right)^3 \frac{mk}{\hbar^2} dE d\Omega$$

Number of states within the solid angle $d\Omega$ in a cubical box with energy between E and E+dE is given by:

$$\rho(E)d\Omega = \left(\frac{1}{2\pi}\right)^3 \frac{mk}{\hbar^2}V \text{ ; Where } L^3 = V$$

4. a. What is Born approximation. Show that $\frac{v_f}{2c} <<< 1$. - 2+1 marks

Born approximation is a high energy approximation. The incident photon energy is very high compared to the $|E_{1s}|$ i.e $\hbar\omega >> |E_{1s}|$. So we have

$$\hbar\omega = \frac{\hbar^2 k_f^2}{2m} + I.P \approx \frac{\hbar^2 k_f^2}{2m} \qquad E = \hbar\omega = \hbar kc$$

$$kc = \frac{\hbar k_f^2}{2m} \Rightarrow \frac{k}{k_f} = \frac{\hbar k_f}{2mc} = \frac{p_f}{2mc} = \frac{v_f}{2c}$$

$$\frac{v_f}{2c} <<< 1$$

b. Under Born approximation in what powers of energy and atomic number do the photoionization cross section depends on?

$$\sigma \rightarrow E^{-7/2}, Z^5$$

5. Given the matrix element $\langle f \,|\, e^{i\vec{k}\bullet\vec{r}}\hat{\varepsilon}\bullet\vec{\nabla} \,|\, i\rangle$ where $|i\rangle$ represents initial bound state and $|f\rangle$ represents final continuum state, under dipole approximation, determine the momentum and length form of the matrix elements.

– 5 marks

$$\langle f | e^{i\vec{k}\bullet\vec{r}}\hat{\varepsilon}\bullet\vec{\nabla}|i\rangle = \frac{1}{(-i\hbar)}\langle f | e^{i\vec{k}\bullet\vec{r}}\hat{\varepsilon}\bullet(-i\hbar\vec{\nabla})|i\rangle$$

$$= \frac{i}{\hbar}\langle f \,|\, e^{i\vec{k}\bullet\vec{r}}\hat{\varepsilon}\bullet\vec{p} \,|\, i\rangle$$; In dipole approximation $e^{+i(\vec{k}\bullet\vec{r})} \approx 1$

$$\langle f | e^{i\vec{k}\bullet\vec{r}}\hat{\varepsilon}\bullet\vec{\nabla}|i\rangle = \frac{i}{\hbar}\langle f | p_x | i\rangle$$

$$[r_k, p_k^2] = r_k r_k p_k - p_k p_k r_k$$

$$= r_k p_k p_k - p_k r_k p_k \underbrace{+ p_k r_k p_k - p_k p_k r_k}$$

$$= [r_k, p_k]p_k + p_k[r_k, p_k]$$

$$= 2i\hbar p_k \qquad [r_k, H_0] = \left[r_k, \frac{p^2}{2m}\right] = \frac{i\hbar}{m}p_k$$

$$\langle f | e^{i\vec{k}\cdot\vec{r}}\hat{\varepsilon}\bullet\vec{\nabla}|i\rangle = \frac{i}{\hbar}\langle f|\frac{m}{i\hbar}[x,H_0]|i\rangle$$
$$\langle f | e^{i\vec{k}\cdot\vec{r}}\hat{\varepsilon}\bullet\vec{\nabla}|i\rangle = \frac{i}{\hbar}\langle f|\frac{m}{i\hbar}[xH_0 - H_0x]|i\rangle$$
$$\langle f | e^{i\vec{k}\cdot\vec{r}}\hat{\varepsilon}\bullet\vec{\nabla}|i\rangle = \frac{m}{\hbar^2}\left(E_i - E_f\right)\langle f|x|i\rangle$$
$$\langle f|p_x|i\rangle = \frac{im}{\hbar}\left(E_i - E_f\right)\langle f|x|i\rangle$$
$$\langle f|p_x|i\rangle = im\omega_{fi}\langle f|x|i\rangle \qquad \Rightarrow \vec{\mathrm{p}}_{\mathrm{fi}} = \mathrm{i}\mathrm{m}\omega_{\mathrm{fi}}\vec{\mathrm{r}}_{\mathrm{fi}}$$

6. Given the differential cross section equation $\left[\frac{d\sigma}{d\Omega}\right]_{\hat{k}_f}^{\hat{\varepsilon}} = \frac{\sigma_{Total}}{4\pi}\left[1+\beta P_2\cos\Theta\right]$ where $P_2\cos\Theta = \frac{1}{2}\left(3\cos^2\Theta - 1\right)$. Find the range of β. - 3 marks

Since cross section cannot go negative $1+\frac{\beta}{2}(3\cos^2\Theta-1)\geq 0$

$$\frac{\beta}{2}(3\cos^2\Theta-1)\geq -1 \qquad 0\leq\Theta\leq\pi \qquad 0\leq\cos^2\Theta\leq 1$$
$$\left[\cos^2\Theta\right]_{\max} = 1 \Rightarrow \beta \geq -1$$
$$\left[\cos^2\Theta\right]_{\min} = 0 \Rightarrow \beta \leq 2$$
$$\Rightarrow -1\leq\beta\leq 2$$