Lecture 1: Complexity in physical systems: various forms

The complexity manifest itself in diversified forms in nature which leads to its many definitions, based on the type of a system and its interaction with its environment [1, 2, 3, 4, 5, 6, 7, 8]. In general, it can be defined as a measure of the relationships among various parts of the system, their variations due to randomness, or emergence of coherent patterns out of randomness due to various constraints and also to the ability of frequent switching among such patterns. In particular, "complexity" has a definition specific to the scientific area. For example, In physical systems, complexity is defined based on nature and strength of the interactions, scattering of particles due to interactions and stochasticity, and, effect of system parameters e.g dimensionality, system size, temperature etc. In computational complexity theory, it is a measure of the resources (time involved, memory-space as well hardware) required for the execution of an algorithm and is therefore a relative property. In information processing, it measures the amount of information transmitted by an object and detected by an observer. In mathematical sciences, complexity is a measure associated with finite semi-groups, automata, graph and network theory etc. The complexity in economy describes the variants and their impacts in various fields such as product portfolio, technologies, markets and market segments, locations, manufacturing network, customer portfolio, IT systems, organization, processes etc. In this course, we restrict ourselves to the study of complexity in physical systems.

The complexity can also be described as an emerging phenomenon as a system goes from microscopic dimensions to macroscopic dimensions, adding more and more sub-units and the interactions among them. The emergence phenomenon leads to division of complexity into two broad types [1, 2, 3]:

Organized Complexity

The non-random, or correlated, local interactions within a sub-unit lead to creation of local structures which can then interact with other local structures. The collective expression of the rules governing the individual parts can be very different from their individual expression. As a consequence, the combination of all these structures can behave in a way very different from the individual parts, it can also manifest properties not dictated by them. The complexity in the combination therefore emerges as a consequence of the organization of varios sub-units and is referred as organized complexity. The world around us is full of the examples of organized complexity e.g emergence of regularity or order through structural pattern formation in biological systems in which many subunits conspire with each other in an intricate yet cohesive way to create such an order. Another example is the emergence of order in dynamical systems etc.

Disorganized Complexity

Disorganized Complexity refers to a system consisting of many components, often with largely random or approximately known interactions. The lack of detailed information about the interactions among components leads to unpredictability of the system-behavior. As a consequence, the properties of the system can be best described only by statistical tools. One important characteristic of the disorganized complexity is that the properties often show scale independence and universal features. In this course, our main focus is the study of *disorganized complexity in physical systems.*

Examples

Some examples of natural complex physical systems from widely diverse areas are strongly interacting quantum many body systems e.g nano-systems, disordered systems e.g industrial Glass, chaotic systems, the human brain and the financial markets. This can further be explained by a brief discussion of four cases with different origins of complexity:

(i) Noise due to particle-particle interactions

The exact analysis of the physical properties of a typical clean many body system e.g. nuclei, atoms, molecules etc is often not possible and one has to apply various approximation techniques, linear response theories and perturbation schemes. The complexity in these systems originates in the complicated interactions among various system sub-units. Even if the nature of their interactions is known and the equations of motion can be described exactly, the large number of subunits and their dynamics often renders their solution technically impossible.

Consider the Hamiltonian of a system of $N > 2$ quantum particles subjected to pairwise inverse-square interactions e.g N electrons subjected to coulomb interactions.

$$H = \sum_k \frac{\mathbf{p_k^2}}{2m} + \frac{1}{4\pi\epsilon_0} \sum_{k,l;k\neq l} \frac{e^2}{|\mathbf{r_k} - \mathbf{r_l}|} \tag{1}$$

Although the Hamiltonian is exactly described by eq.(2), an exact calculation of its matrix elements in a generic basis is still not possible. The complexity here arises due to each particle being subjected to multiple interactions due to other particles which is further complicated if the particles are moving. The local variation in particle density or nature of the local interactions also adds up to complexity of the system: If the local interactions are complicated in a specific part of the system, the evaluation of the corresponding matrix elements of the Hamiltonian becomes technically difficult. As a consequence, these elements can be determined only within a certain degree of accuracy and can best be described by a probability density. However the system may also contain parts where interactions are simple and the related matrix elements can exactly be calculated. The Hamiltonian then turns out to be a matrix with both random and non-random elements

(i) External disorder potential

Recent advances in nanotechnology and quantum information theory have motivated an extensive research on the topic of wave transport through disordered regions. The random distribution of impurities in such systems give rise to fluctuations in transport properties from sample to sample. The fluctuations can also be observed in a single disordered sample under an external perturbation or a slight variation of a system parameter.

The simplest model of a non-interacting disordered system is a particle of mass m moving in a random, white-noise potential $V(\mathbf{r})$ (also known as the standard Anderson model) :

$$H = \frac{\mathbf{p^2}}{2m} + V(\mathbf{r}) \tag{2}$$

where $V(\mathbf{r})$ is described only in a statistical way, that is, by its disordered averaged -value and the variance:

$$\begin{aligned} \langle V(\mathbf{r}) \rangle &= 0, \\ \langle V(\mathbf{r}).\mathbf{V}(\mathbf{r}') \rangle &= \delta(\mathbf{r} - \mathbf{r}') \end{aligned} \tag{3}$$

where $\delta(x)$ refers to the Dirac-delta function. This can describe, for example, the dynamics of electrons in a disordered nano-device e.g a quantum dot, an essential component of many modern day technologies. As the model describes a single-particle dynamics, it is valid under independent electrons approximation. However an analysis of the physical properties of the electrons within this simplified model still requires statistical tools. The origin of complexity in this case can be explained as follows: The motion of an electron in a periodic lattice with no disorder can be described by plane waves, extended throughout the system and the energy levels can be modeled by the standard band theory of solid. However, the waves associated with the electrons moving in a crystal with random impurities get randomly scattered which results in their partial or full localization or extended behavior based on the degree of disorder. In a real-space basis, the wave-function of the electron therefore has varying strengths of its components. The random real-space scattering must also affect the Hamiltonian, the latter being the generator of the wave-function dynamics. As a consequence, the Hamiltonian matrix can be either sparse (for two or more dimensional lattice), banded (for one-dimension) or full (that is, same order of magnitude for all elements) with its non-zero elements best described by a probability distribution.

(iii) Chaotic dynamics

The classical Hamiltonian dynamics of a typical system can reveal two different types of motion : the regular motion of integrable systems and the chaotic motion of nonintegrable systems [9]. The harmonic oscillator and the Kepler problem show regular motion, while a periodically driven pendulum or an autonomous conservative double pendulum can display chaotic dynamics. To identify the type of motion of a given system, one may look at a bundle of trajectories originated from a narrow cloud of points in phase space. For regular motion, the distance between any two trajectories may increase like a power of time. For chaotic case on the other hand, the distance between any two such trajectories grows exponentially with time, the growth rate is the so called Lyapunov exponent.

Chaotic systems are deterministic, dynamical systems, with their future behavior fully determined by their initial conditions, and without any random elements. However small differences in initial conditions lead to widely diverging trajectories for chaotic systems. This extreme sensitivity to initial conditions renders long-term prediction of their dynamics impossible. Thus although the chaotic phenomena obeys a deterministic law but its future is probabilistic and not predictable. The unpredictability of a given clean system is caused not by the many degrees of freedom but its non-linearity.

Consider a particle moving inside a clean stadium billiard shape geometry. Unlike previous examples, this system had no many body interactions or impurity scattering. The dynamics of the particle is also governed by a simple Hamiltonian $H = \frac{\mathbf{p}^2}{2m}$; the complexity however arises due to nature of the confining geometry. The multiple scattering of the particle from the boundaries leads to chaotic dynamics and introduces unpredictability.

(iv) Biological Systems

A typical biological system is a combination of various units, with complicated, often not well-known, interactions among them. Variation of these interaction over time, an important part of the evolutionary process, leads to emergence of complexity in the living beings [6, 7, 8]. Here we consider three examples:

The human brain and those phenomena that result from its activity form the most complex systems known to exist in the universe. The brain of a living being is the tool which controls the manner in which neurons are connected to each other and has a strong influence on the dynamics that emerge from it. Since begining of life, the brain has evolved from simple networks of neurons to complex networks, with most of the rules governing this evolutionary process still unknown. Systematic investigations of neuronal connectivity and of large-scale inter-regional pathways in animals reveal that the topology of these networks is neither entirely random nor entirely regular. The details analysis show that these networks show small-world properties (similar to many other complex networks), revealing a tendency of a high degree of clustering, with short path lengths linking individual components. These structural characteristics in turn govern the functions of the brain.

A cell, the basic unit of all organs of living beings, is another important example of biological complexity. The latter originates through the interactions and inter-dependence of many functionally diverse units or modules e.g. proteins, DNA, RNA and small molecules. An important characteristic of such modular systems is collectivity: the properties of elements in the same module are more similar than the elements from different modules (which in fact is the basis of a separate function of each module). The properties of a cell therefore can be modeled by complex networks of modules, with each module acting as a a sub-network that structurally has more links within itself than links with other sub-nets.

The functionally of a cell-module is governed by the expression of genes in a DNA which leads to formation of amino acid sequences that are the basic building blocks of proteins. The message contained in a DNA then manifests itself through a specific structure of protein which in turn determines its functionality. In fact, the protein after its birth, acts as a feedback and leads to creation of new copies of the parent DNA. The structure of a protein is determined purely by the amino acid sequences, and its function depends on the ability of the protein to fold rapidly to its native structure. The folding process is governed by many parameters, e.g. the sequence of amino acids, intermolecular interactions, the solvent, temperature and chaperon molecules which make it a complex process, still not fully understood. However the complexity results in a well-known pre-determined configuration, despite availability of an infinite number of possibilities of the protein.

(iv) Complex adaptive systems

A complex adaptive system (CAS) is a combination of individual agents, free to act in ways that are not always totally predictable, with their actions inter-connected such that one agent's action can influence that of others [5]. Each agent can also operate according to its own strategy, i.e set of rules for the response to environment and can also have its own interpretation of events. The rules and interpretations of one agent need not be explicit, may or may not be shared by others, and could change over time. to change. As the agents can change themselves, share their strategies, learn and adapt to each other as well as to their environment, their combination is referred as a complex adaptive system. Examples of complex adaptive systems are wide-ranging e.g the stock market, a business industry, a social organization, a colony of ants, the immune system etc.

A CAS is an inherently self-organizing system, with order as its inherent property, and governed by the interactions among agents. Its behavior however is an emergent, non-linear phenomenon, with its origin in the time-varying nature of the interactions among agents. The non-linearity manifests itself through drastic changes in system behavior even for seemingly small changes in their interactions. The possibility of changes in the agents themselves further adds up to complexity of a CAS and it can exhibit novel behaviors. Although novelty and non-linearity makes the detailed behavior of a CAS fundamentally unpredictable, the useful information about its average behavior can still be derived through statistical tools.

The stock market is a good example of a CAS with a wide range of agents e.g buyers, sellers, companies and regulators who can act independently as well as act in a shared, common way. Each such agent is governed by another complex system i.e human brain which acts based on its capacity of the interpretation of the signals (i.e information) received; the actions of agents are therefore sensitive to the complexity of their brain. The unpredictability of the specific actions of each stock-agent over time as well as their impact on the other's actions makes the stock market a complex, non-linear system. The average behavior of the stock market however is known to show universal, predictable trends.

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