

1. Consider  $^{197}\text{Au}$  nucleus. Up to what radius are 50 nucleons situated? You can use Woods Saxon function for nucleon density with  $\rho_0 = 0.17$  nucleon/ $\text{fm}^3$ ,  $R_0 = 1.1$  fm,  $a = 0.55$  fm.
2. Electrons of kinetic energy 620 MeV are made to scatter from a metal sheet. The differential cross section shows the first minimum at an angle of 10 deg. Estimate the radius of the nucle in the metal.
3. Estimate the radius of  $^{209}_{83}\text{Bi}$ .
4. Suppose I write the electric charge density inside a nucleus as  $\rho(r) = \frac{\rho_0}{1 + \exp(r-R)/a}$ . As the mass number  $A$  changes from  $A = 12$  to 96, approximately by what fraction does  $R$  change? Approximately by what fraction does  $a$  change?
5. The radius of a nucleus is  $R = R_0 A^{1/3}$  and its Coulomb energy is  $\frac{3}{5} \frac{(Ze)^2}{4\pi\epsilon_0 R}$ . A nucleus  $^A_Z X$  decays to its mirror nucleus  $^A_{z-1} Y$  in its ground state. Derive an expression for the  $Q$  value in terms of  $A$ ,  $R_0$  and universal constants.
6. Alpha particles are scattered from gold foil ( $A=197$ ). At what minimum kinetic energy of alpha particle, you expect the results to deviate significantly from those predicted by Rutherford Scattering formula?
7. K-alpha X-ray energies of a number of isotopes of an element are measured. This energy is plotted as a function of  $A^x$  where  $A$  is the mass number. What should be the value of  $x$  so that the graph is a straight line (at least approximately).
8. Why muonic X-ray isotopic shift in energy is more sensitive to estimate finite size of nucleus than the electronic X-ray isotopic shift?
9. Using a one electron model, calculate the energies of muonic K X-rays in Fe nucleus using a point nucleus. Now, treat this nucleus as a spherical nucleus of radius  $R = R_0 A^{1/3}$  and calculate the correction in the X-ray energy for  $^{56}\text{Fe}$ .
10. How many bound states does a deuteron have?
11. What is the total spin  $I$  of a deuteron?
12. Is the value of orbital angular momentum quantum number definite in a deuteron? If not, what values are possible?
13. The magnetic moment of a deuteron due to the spins of the neutron and proton is about 2% higher than the observed value. What could be the reason for this discrepancy?
14. Find the energy of magnetic dipole-dipole interaction in the deuteron and compare it with the deuteron binding energy.
15. Assume n-p potential to be a square well potential of depth 27 MeV and width 2.4 fm. Find the probability of finding the separation between the nucleons in deuteron to be more than the width of the potential.
16. Consider the deuteron wave function outside the nuclear potential well. Find the difference  $r_2 - r_1$  such that  $\psi(r_2)$  is half of  $\psi(r_1)$ .
17. Assume that the nuclear potential for the n-p system for its relative motion is given by a square well of depth 36 MeV and width 2 fm. A deuteron is formed because of this nuclear potential. Find the separation between the neutron and proton at which the deuteron wave function (for the relative motion) is maximum. Binding Energy of deuteron = 2.25 MeV,  $\hbar^2/m_p = 41.3\text{MeV}\cdot\text{fm}^2$ .

18. The  ${}^3D_1(l=1)$  state of a deuteron is given by

$$|\psi\rangle = \frac{u(r)}{r} \left[ \sqrt{\frac{3}{5}} Y_2^2 \chi_1^{-1} - \sqrt{\frac{3}{10}} Y_2^1 \chi_1^0 + \sqrt{\frac{1}{10}} Y_2^0 \chi_1^{-1} \right]$$

where Y represents the spherical harmonics and  $\chi_s^{ms}$  the spin wave functions. Calculate the magnetic moment  $\langle \psi | \mu_z | \psi \rangle$  in pure  ${}^3D_1$  state.

19. Consider a helium nucleus to be a bound state of two deuterons interacting via attractive finite square well potential of width 2.0 fm. The binding energy of helium nucleus is 28 MeV and that of deuteron 2.3 MeV. Find the depth of this potential which can give correct ground state energy of helium nucleus.

20. The spin dependent nuclear potential between neutron and proton is given as  $V = V_c(r) + V_s(r)P_\sigma$  where  $V_c(r)$  and  $V_s(r)$  are attractive central potentials and  $P_\sigma$  is spin exchange potential given by

$$P_\sigma = \frac{1}{2} [1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2].$$

Here  $\sigma$  represents Pauli spin operator. Show that when  $P_\sigma$  operates on the  $S=0$  state of n-p system, the individual spins are exchanged.

21. Using a square well potential for n-p interaction that can give deuteron binding energy, the calculated low energy n-p scattering cross-section comes out to be 4.2 barns. The observed value is however 20 barns. Explain this discrepancy and obtain the scattering cross-section for  $S=0$  state.

22. Show that p-p scattering cannot occur in  ${}^3S$ ,  ${}^1P$  and  ${}^3D$  states.

23. The scattering length for low energy singlet and triplet n-p scattering are reported to be  $a_s = -2.3 \times 10^{-12} \text{ cm}$  and  $a_t = 0.52 \times 10^{-12} \text{ cm}$ . Calculate the total cross-section in unpolarized n-p scattering.

24. Suppose the n-p triplet potential is given by an attractive square well of 35 MeV depth and 2.0 fm width. Calculate the phase shift  $\delta_0$  and scattering cross-section  $\sigma_1$  for n-p scattering at  $E_{lab} = 20 \text{ keV}$ .

25. Consider scattering of 100 eV neutrons from a proton target. The s-wave phase shift is found to be 30 degrees. Plot the s-wave components  $|\Psi_{s,inc}|$  (use dotted line) and  $|\Psi_{s,scatt}|$  (use solid line) of the incident and scattered waves against r on the same diagram. Give the values of r where the two plots cross the r-axis for the first time.

26. The total scattering cross-section of elastic scattering of 1 MeV neutrons (lab energy) from a proton target is 5.0 barns in triplet state scattering and 65 barns in singlet state scattering. Find the phase shifts in the two cases.

27. The maximum spin polarization observed in p-p scattering for energies  $E_1$ ,  $E_2$  and  $E_3$  are 0.00, 0.05 and 0.40 respectively. (a) Give an estimate of the upper limit on  $E_1$  (b) Arrange  $E_1, E_2, E_3$  in ascending order. Justify your answer.

28. Compute the binding energy per nucleon for (a)  ${}^7\text{Li}$ , (b)  ${}^{56}\text{Fe}$  and (c)  ${}^{235}\text{U}$  using semi empirical mass formula.

29. Evaluate the neutron separation energy for  ${}^7\text{Li}$ .

30. Compute the approximate mass of Carbon, Germanium and Bismuth nuclei.

31. It is energetically possible for a nucleus to decay by both  $\beta^+$  and  $\beta^-$  emission. What can you say about the Mass number A (even, odd) of this nucleus?

32. There are only four odd-Z, odd-N nuclei which are stable. Name as many of them as you remember.

33. Using semi empirical mass formula derive an expression for the number of protons Z for which the isobar with a fixed A is stable (does not beta-decay). Find this Z for A = 125. Neglect the mass difference between neutron and proton.

34. A neutron star consists of only neutrons. Gravitational potential energy  $\left(-\frac{3}{5} \frac{Gm^2}{r}\right)$  plays a major role in giving the star stability by making the binding energy positive. Find the minimum number of neutrons expected in a neutron star based on properly modified semi empirical mass formula.
35. Why does a neutron have magnetic moment when it is electrically neutral particle?
36. Suppose the spin-orbit part of the N-N potential is given by.

$$V_{ls} = -\frac{g}{4\pi} \mathbf{L} \cdot \mathbf{S} \frac{e^{-\mu r}}{r}$$

Write whether this potential is attractive, repulsive or zero in the following states.

- (a)  $^1S_0$  (b)  $^3S_0$  (c)  $^1P_1$  (d)  $^3P_0$  (e)  $^3P_1$  (f)  $^3S_2$

In which of the cases the potential strongest?

37. Assume a single particle central potential to describe the shell structure of nuclei. An additional of  $V_{so} \mathbf{L} \cdot \mathbf{S}$  term gives the correct shell closing. Let  $V_{so} = -1.0 \text{ MeV} / h^2$ . Find the change in the energy of 1f state as the state becomes (a)  $1f_{7/2}$  (b)  $1f_{5/2}$ . Do the same calculation for  $1d_{5/2}$  and  $1d_{3/2}$ .
38. When the orbital and intrinsic spin angular momenta of the last nucleon couple in shell model, the wave function  $|j, m_j\rangle$  can be written as a linear combination of the products.  $Y_1^{m_1} \chi_s^{m_s}$ . Write the expression for (a)  $|j = 1 + 1/2, m_j = j\rangle$  (b)  $|j = 1 - 1/2, m_j = j\rangle$  in terms of such products. You can leave the coefficients of these products terms as a, b, c, etc.
39. Consider harmonic oscillator potential for constructing shell model energy levels, Addition of spin-orbit potential  $V(\mathbf{r}) \mathbf{L} \cdot \mathbf{S}$  gives rise to magic numbers 2, 8, 20, 28, 50, 82, 126. Suppose one uses  $[-V(r) \text{ l.s}]$  for the spin-orbit potential. Find the magic numbers coming out from such a scheme.
40. What is the magnetic moment of  $^{16}\text{O}$  in ground state?
41. Give the expected shell model spin and parity assignments for the ground states of (a)  $^7\text{Li}$  (b)  $^{15}\text{C}$  (c)  $^{31}\text{P}$  (d)  $^{141}\text{Pr}$ .
42. The ground state of  $^{203}\text{Tl}_{122}$  has spin-parity  $\frac{1}{2}^+$ . Give the shell model configuration of this nucleus that is consistent with the observed states.
43. Find the expectation values  $\langle j, j | g_1 l_z | j, j \rangle$  and  $\langle j, j | g_s s_z | j, j \rangle$  in the ground state of (a)  $^{41}\text{Ca}$  (b)  $^{35}\text{Cl}$  using extreme single particle shell model. Hence find their magnetic dipole moments.
44. Compute the magnetic dipole moment expected from extreme single particle shell model for  $^{75}\text{Ge}(\frac{1}{2}^-)$ .
45. Find the magnetic dipole moment of  $^{41}\text{Ca}$  and  $^{35}\text{Cl}$  in their ground states using extreme single particle shell model.
46. The unpaired neutron in an odd-A nucleus is in the state  $d_{5/2}$ . Using extreme single particle shell model calculate the magnetic moment expected in units of nuclear magneton.  $g_{sn} = -3.826$ .
47. The 1<sup>st</sup> two excited states of  $^{17}_8\text{O}$  nucleus are found to have spin parity as  $\frac{1}{2}^+$  and  $\frac{1}{2}^-$ . Write the proton and neutron configurations for each of these states.

48. The first excited state of  ${}^{120}_{52}\text{Ti}$  appears at about 600 keV and has  $I^\pi = 2^+$ . The next three excited states occur at around 1.2 MeV and have  $I^\pi = 0^+, 2^+, 4^+$ . Explain the origin of these excited states.
49. In many of the even-even nuclei with  $A < 150$ , the first excited state occurs at about 0.5–1 MeV with spin-parity  $2^+$ . What kind of motion of the nucleus gives this state and why it is  $2^+$ ?
50. A deformed nucleus ( $150 < A < 190$ ) has the first excited state at 90 keV with parity  $2^+$ . What is the expected energy, spin and parity for the next excited state?
51. What spin parity are expected for a vibrating nucleus with two quadrupole phonon excitation?
52. What is the expected ratio of the energy of the first  $4^+$  state to the energy of the first  $2^+$  state for (i)  ${}^{130}_{50}\text{Sn}$  and for (ii)  ${}^{160}_{66}\text{Dy}$ .
53. The lowest state of a rotational band is found to have  $J^\pi = 2^+$ . What will be  $J^\pi$  in the next higher state in the band.
54. The excited state sequence in  ${}^{27}\text{Al}$  are 0.842 MeV( $1/2^+$ ), 1.013 MeV( $3/2^+$ ) and 2.729 MeV( $5/2^+$ ). Assuming it to be a  $K=1/2$  rotational band, find the value of  $\hbar^2 / 2\mathcal{I}$ .
55. The ground state rotational band of  ${}^{238}\text{Pu}$  is as follows;  $0^+ : 0$ ,  $2^+ : 44.11\text{keV}$ ,  $4^+ : 146\text{keV}$ ,  $6^+ : 303.7\text{keV}$ ,  $8^+ : 514\text{keV}$ . Calculate the moment of inertia of the nucleus from this data. What is the moment of inertia of this nucleus is assumed to be a rigid sphere? Find the ratio of these two values.
56.  ${}^{251}\text{Fm}$  decay to  ${}^{247}\text{Cf}$  by emitting alpha particles of several energies. The highest three energies are 7.305 MeV, 7.251 MeV and 7.184 MeV. Assuming that these alpha decays populate the rotational band of  ${}^{247}\text{Cf}$  built on ground state find the spins of these states and the moment of inertia of the daughter.
57. Show that when three quadrupole phonons are coupled together, only states with  $J^\pi = 0^+, 2^+, 3^+, 4^+$  and  $6^+$  are allowed.
58. What are the spin-parity values of the vibrational state of an even-even nucleus corresponding to one octupole phonon excitation?
59. Consider two phonon excitation of octupole vibration of an even-even nucleus originally in ground state. What will be the spin-parity of different states resulting from this excitation.
60. What is Geiger-Nuttall law? Give typical range of Q-value and of life time in  $\alpha$ -decay reactions.
61. The half life of  ${}^{198}\text{Au}$  is 2.7 days. (a) What is the decay constant? (b) What is the probability that a given Au nucleus will decay in 1 second (c) What is the activity of 1.00  $\mu\text{g}$  sample of  ${}^{198}\text{Au}$ ? (d) How many decays per second occur when the sample is one week old?
62. A radiation detector is in the form of a circular disc of diameter 3.0 cm. It is held at a distance of 25 cm from a source of radiation where it records 1250 counts per second. Assuming that the detector records every radiation incident upon it, find the activity of the source in Curies.
63. The Q values and half lives of decay for certain isotopes of Th are given below.
- |               |                      |                      |      |      |                   |                      |                      |
|---------------|----------------------|----------------------|------|------|-------------------|----------------------|----------------------|
| (a) A         | 220                  | 222                  | 224  | 226  | 228               | 230                  | 232                  |
| (b) Q(MeV)    | 8.95                 | 8.13                 | 7.31 | 6.45 | 5.52              | 4.77                 | 4.08                 |
| $T_{1/2}$ (s) | $1.0 \times 10^{-5}$ | $2.8 \times 10^{-3}$ | 1.04 | 1854 | $6.0 \times 10^7$ | $2.5 \times 10^{12}$ | $4.4 \times 10^{20}$ |
64. In the decay of  ${}^{242}\text{Cm}$  to  ${}^{238}\text{Pu}$  the maximum  $\alpha$  energy is  $6112.9 \pm 0.1$  keV (mass of  ${}^{238}\text{Pu}$  is 238.049555 u). Find the mass of  ${}^{242}\text{Cm}$ .
65. Estimate  $\alpha$  decay energy of  ${}^{218}_{90}\text{Th}$  from the semi-empirical mass formula.

66.  ${}_{96}^{242}\text{Cm}$  decay by alpha emission to  ${}^{238}\text{Pu}$ . The spin and parities of the initial nucleus is  $0^+$  and the ground state and some of the excited states of the daughter nucleus are  $0^+, 2^+, 4^+, 1^-, 2^-, 4^-$ . Which of these states are accessible to the daughter nucleus by alpha decay?
67. A nucleus having spin parity  $1^-$  decays by emitting  $\alpha$  particles of energies 4.687 and 4.650 MeV to a nucleus having ground state spin parity of  $0^+$ . Emission of  $\gamma$  rays of energies 266 and 305 keV is also observed. From this information construct the decay scheme.
68. In the certain decay process a nucleus in the vicinity of mass 230 emits  $\alpha$  particles with the following energies in MeV, 5.545, 5.513, 5.469, 5.417, and 5.389. The following  $\gamma$  rays from the daughter nucleus are seen (energies in keV) 26, 33, 43, 56, 60, 99, 103 and 125. From this formation construct the decay scheme assuming the highest energy particles coming from decay to the ground state.
69. Suppose the angular momentum of an alpha particle coming out of a heavy nucleus ( $mass = 200 u$ ) corresponds to  $l = 2$ . Find the raise in Coulomb potential barrier at  $r = r_0 (= 8 \text{ fm})$  from that with  $l = 0$ .
70. Take the well depth to be 35 MeV and  $Q$  to be 5 MeV for an alpha particle in the nucleus. Estimate the speed of the particle inside the nucleus. Assuming  $A=216$ , estimate the frequency at which the alpha particle is presenting itself to the barrier.
71. Why  $\alpha$ -particle emission is much more common than emission of other lighter particles such as proton, deuteron etc?
72. Compute the  $Q$  values for the following decay (a)  ${}^{11}\text{Be} \rightarrow {}^{11}\text{B}(\beta^-)$  (b)  ${}^{10}\text{C} \rightarrow {}^{10}\text{B}(\beta^+)$  (c)  ${}^{19}\text{C} \rightarrow {}^{19}\text{B}(EC)$
73.  ${}^{156}\text{Au}$  can decay by  $\beta^+$ ,  $\beta^-$  and electron capture. Find the  $Q$  values for these decays.
74. The decay of  ${}^{191}\text{Os}$  leads to an excited state of  ${}^{191}\text{Ir}$  at 171keV. Compute the maximum kinetic energy of the beta spectrum.
75. Find the recoil energy of the proton in free neutron beta decay if (a) the neutrino takes away negligible energy in. (b) the electron takes away negligible energy.
76.  ${}^{23}\text{Ne}$  decay to  ${}^{23}\text{Na}$  by negative beta emission. What is the maximum kinetic energy of the emitted beta particles.
77.  ${}^{12}\text{N}$  decays to an excited state of  ${}^{12}\text{C}$  which subsequently decays to the ground state with the emission of 4.43 MeV gamma ray. What is the maximum kinetic energy of the emitted beta particles?
78.  ${}_{53}^{124}\text{I}$  decays by beta emission to  ${}_{52}^{124}\text{Te}$ . Find the maximum energy of the beta particles possible in this decay process. Atomic masses;  ${}_{53}^{124}\text{I}:123.906207 \text{ u}$ ,  $m_e:0.511003 \text{ MeV}/c^2$ .
79.  ${}^{75}\text{Se}$  decays by electron capture to  ${}^{75}\text{As}$ . Find the energy of the emitted neutrino.
80. The weak force is thought to originate from the exchange of particles of mass 75 GeV. What is the range of this interaction?
81. Find the  $Q$  value of the reaction  ${}^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}$ .
82. The reactions  ${}_{90}^{220}\text{Th} \rightarrow {}_6^{12}\text{C} + {}_{84}^{208}\text{Po}$  has  $Q$  value much larger than that for  ${}_{90}^{220}\text{Th} \rightarrow {}_2^4\text{He} + {}_{88}^{216}\text{Ra}$ . Still  ${}^{220}\text{Th}$  is observed to decay by  $\alpha$ -emission and not by carbon emission. Why?
83. A tree is planted few hundred years ago and people still use its leaves for religious rituals. Can nuclear radioactivity experiments tell the age of this tree?
84. Classify the following decays according the degree of forbiddenness.

- (a)  $^{89}\text{Sr}(5/2) \rightarrow ^{89}\text{Y}(1/2)$   
 (b)  $^{35}\text{Cl}(2^+) \rightarrow ^{35}\text{Ar}(0)$   
 (c)  $^{26}\text{Si}(0^+) \rightarrow ^{26}\text{Al}(0^+) \rightarrow ^{26}\text{Mg}(0^+)$

85. Can  $^{242}\text{Cm} \rightarrow ^{238}\text{Pu}^* + ^4\text{He}$  reaction take place?  $J^\pi$  of  $^{242}\text{Cm}$  is  $0^+$  and that of  $^{238}\text{Pu}^*$  is  $2^-$ . Justify your answer by giving proper reason.
86. Assuming that the nuclear force starts acting at 2 fm of separation between two protons, find the height of the coulomb Barrier in  $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu$  reaction.
87. Find the height of the Coulomb barrier for an alpha decay from  $^{220}\text{Fr}$ .
88. Consider two nuclei of mass numbers  $Z_1$  and  $Z_2$ , and radii  $r_1$  and  $r_2$ , moving towards each other. The reduced mass of the nuclei is  $M$  and the kinetic energy in the center of mass frame is  $E_0$ . Find the height and the width of the Coulomb barrier.
89. A beam of 2 MeV alpha particles hits a carbon target and the reaction  $^4\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O}^*$  is studied. The number of reactions per unit time is  $N_0$ . Keeping the density of  $\alpha$ -particles (number/volume) in the beam same, their energy is increased to 4 MeV. What will be the new number of reactions per unit time?
90. Consider a nuclear reaction  $X + Y \rightarrow Z^*$ . As the kinetic energy (CM frame) of the reactant particles is increased, the nuclear reaction cross-section often increases sharply at certain energies and again drops. Write the condition in which this resonance occurs.
91. Fusion reactions take place in plasma at temperature  $T$ . If  $\sigma(v)$  be the reaction cross-section for two nuclei moving at relative speed  $v$ , the average value of  $\langle \sigma(v)v \rangle$  can be written as  $\int_0^\infty f(E) dE$  where  $E = \frac{1}{2}mv^2$ . Sketch  $f(E)$  as a function of  $E$  for two temperature  $T_0$  and  $2T_0$ , both in keV range.
92. The pp-I chain going on in a star having largely hydrogen is given by
- $$^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu \quad (10^9 \text{ Years})$$
- $$^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma \quad (1 \text{ second})$$
- $$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2^1\text{H} \quad (10^6 \text{ Years})$$
- Explain the different between the time scales of these reaction.
93. Write the three most abundant nuclei in universe.
94. The core of a star consists of  $^{12}\text{C}$  only and the temperature is sufficient for fusion. How can  $^{28}\text{Si}$  form in this core?
95. Given the availability of  $^{12}\text{C}$ ,  $^1\text{H}$  and  $^4\text{He}$  in a star with sufficiently high temperature, write reactions that can produce a neutron.
96.  $^8\text{Be}$  nucleus is unstable with a half life of  $10^{-17}$  s. Then how is  $^{12}\text{C}$  formed in the core of stars?
97. The atomic mass of  $^4\text{He}$  is 4.002603 u. Find the total loss in rest mass energy if 3  $\alpha$ -particles join to form a  $^{12}\text{C}$  nucleus. Neglect electronic binding energies of atoms.
98. Give the reactions that produce  $^{59}\text{Co}$  in stars.