Module 3: Recording and play back theories Lecture 11: The Writing process

Objectives:

We have so far discussed various types of magnetic materials and the electronic structure of the magnetic materials.In magnetic recording, the understanding of the writing process of information on the magnetic medium and the methods to retrieve the recorded information from the medium are one of the core parts of the magnetic recording [1]. Hence, in this module, we shall first cover

- (1) Writing process, which includes
	- a. Introduction
	- b. Definition of a transition length
	- c. The Demagnetization field
	- d. Nature of the magnetic transitions
	- e. Williams-Comstock model for writing
	- f. Effect of imaging from the head and transition relaxation
	- g. Different types of writing process

and then discuss

- (2) Play-back theories
	- a. Read back voltage,
	- b. Wallace solution,
	- c. Reciprocity principle,
	- d. Readback from single transition,
	- e. Pulse width and current optimization,
	- f. Magnetoresistive readback

to retrieve the information from the recorded medium.

Introduction:

The magnetic recording process converts electric current signal into an equivalent magnetization in the magnetic materials placed on a medium (tape or disk). This process is done with the help of a transducer which transforms the electrical signal into a magnetic field through which the medium to write the information passes.

The criterion for magnetic recording on to a media coating is quite simple: if the applied field magnitude $H(x, y)$ from the recording head at the element Δvol is greater than the switching field (H_C) , then the particles within the Δvol will switch its magnetization in accordance with the applied field. On the other hand, when Δvol has moved away from the strong magnetic field and if the applied field $H(x, y)$ is less than H_{Ci} , then the particles will no longer follow the field and their magnetization have been frozen at the point where $H(x, y)$ became just less than H_{Ci} .

Definition of a transition length:

Let us consider a disk medium moving with a constant speed under the head magnetic field. Now, the magnetization of the medium would also act as a source of an additional field, i.e., the demagnetization field, along with the head field. Hence, one needs to understand response of the medium for the write field and then incorporate the demagnetization field in the writing process.

Figure 11.1: Head fields from the ring type head.

Figure 11.2: Schematic of initiation of writing process.

First we shall assume that the medium only responds to the longitudinal component of the head field and the contour of the head field lines for the *x*-direction (H_x) being constant are circles with the centers located along the *y*-axis as shown in Figure 11.1. Now, we shall consider that the medium is initially magnetized in the positive direction, i.e., in its remnant state (state 1) as shown in Figure 11.2. With increasing the head field in the negative direction, the magnetization in the medium takes down any value along its hysteresis loop. Now, if we decrease the head field to zero, then the magnetization of the medium follow the arrow marks, i.e., along its minor loop. The contour that ends up in a completely demagnetized state had to have been driven slightly beyond coercivity value (H_C) , which is often referred as remnant coercivity (H_{Cr}) . A careful view along the medium as shown in Figure 11.2(b) reveals that at the location 2, the magnetization is reduced from the remnant state, but still in the positive direction, whereas the magnetization at location 3 has been reversed partially. This clearly indicates that there is a length over which the transition from the negative to positive direction occurs. This can be determined as follows,

$$
\frac{dM}{dx} = \frac{dM}{dH} \times \frac{dH}{dx}
$$
\n(11.1)

Assuming that the terms on the right hand side of the eqn. (11.1) are constant, the transition length can be estimated to be

$$
a = \frac{M_R}{\left[\frac{dM}{dH} \times \frac{dH}{dx}\right]}
$$
(11.2)

This clearly indicates the requirement of a square hysteresis loop and high field gradient in writing sharp transitions.Since the magnetization varies spatially through the transition region, a demagnetization field will be generated. Therefore, the field at the locations 2 and 3 will not be zero, except at the centres of the transition. This suggests that the estimation of demagnetization field is important, which is discussed below.

The Demagnetization field:

When a specimen is magnetized, a self-field is developed within the specimen, which opposes the magnetizing field. This is typically called as demagnetization field. We shall briefly cover as it plays an important role in the magnetization process. Consider a uniformly magnetized specimen with a volume *V* and surface *S*. Its magnetization \vec{M} gives rise to surface poles, which in turn gives rise to a demagnetization field $\overrightarrow{H_d}$ within the specimen. The field $\overrightarrow{H_d}$ is proportional to \overrightarrow{M} , but in the opposite direction. For example,

$$
\overrightarrow{H_d} = -\widehat{N}\overrightarrow{M} \tag{11.3}
$$

$$
\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = - \begin{pmatrix} N_{xx} & N_{yx} & N_{zx} \\ N_{yx} & N_{yy} & N_{zy} \\ N_{zx} & N_{zy} & N_{zz} \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix}
$$
 (11.4)

Where \hat{N} is the demagnetization factor tensor that relates the demagnetization field with a specimen magnetization.The tensor function of position is given by [2]

$$
\widehat{N}(r) = -\frac{1}{4\pi} \iiint d^3r' \nabla' \left(\nabla' \left(\frac{1}{r - r'} \right) \right) \tag{11.5}
$$

This tensor is given by an integral over the object volume and can be evaluated either inside or exterior to the body. The value of tensor \hat{N} significantly depends on the specimen shape, which are difficult to obtain in closed-form. It may be calculated exactly for an ellipsoidal shape only. In many symmetrical materials such as any ellipsoids of revolution, the demagnetization factor tensor only has three principal components, i.e.,

$$
\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = -\begin{pmatrix} N_1 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix}
$$
 (11.5)

Where $N_1 + N_2 + N_3 = 1$ (in SI) and $N_1 + N_2 + N_3 = 4\pi$ (Gaussian). The demagnetization factors for the selected shapes are summarized in Table 11.1.

| Shape | N_I | N_2 | N_3 |
|-------------------------------------------------------------------|------------------|----------|----------|
| Sphere | $4\pi/3$ | $4\pi/3$ | $4\pi/3$ |
| Long Cylinder along z-axis | 2π | 2π | θ |
| Infinite plate normal to z -axis | $\boldsymbol{0}$ | θ | 4π |
| Strip film normal to z -axis | 0 | 8t/W | 4π |
| (with t – thickness, W – Width, L – Length; $t \le W \le L$) | | | |

Table 11.1: Demagnetization factors (in Gaussian units) of selected shapes:

A detailed calculation of demagnetization factor for various objects can be found in Ref.[2]. Note that the infinite plate has no demagnetization within its *x*-*y* plane but suffers a 4π demagnetization factor (Gaussian unit) on magnetization components out of the plane. A sphere suffers a $4\pi/3$ factor in all directions. A long cylinder has no demagnetization along its axis, but suffers 2π in the *x* and *y* directions of its cross sections.

The demagnetization field obtained from the Maxwell equation is given as

$$
\nabla. \vec{H} = -4\pi \nabla. \vec{M} \tag{11.6}
$$

In the absence of the currents, $\nabla \times \vec{H} = 0$ and the field can be derived from a scalar potential, $\vec{H} = -\nabla \varphi$. Combining this field with the Maxwell equation given in eqn.(11.6) results the Poisson equation,

$$
\nabla^2 \varphi = -4\pi \nabla \cdot \vec{M} \tag{11.7}
$$

which has the solution,

$$
\varphi(\vec{r}) = -\int \frac{\nabla' \cdot \overline{M(r')} }{|\vec{r} - \vec{r'}|} d^3 \vec{r'}
$$
\n(11.8)

Therefore, the demagnetization field from eqn.(11.6) becomes

$$
H_d(\vec{r}) = -\int \frac{\nabla' \cdot \overline{M(\vec{r})}(\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3} d^3 \vec{r'}
$$
\n(11.9)

It is well known that the demagnetizing field associated with a uniform magnetized ellipsoidal sample is *−*4π*NM* inside the sample. For a solid sphere, the demagnetizing factor *N* is 1/3. In spherical coordinates (r, θ, φ) , the divergence of magnetization is described as [3],

$$
\nabla. \vec{M} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 M_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \, M_\theta) + \frac{1}{r \sin \theta} \frac{\partial M_\varphi}{\partial \varphi}
$$
(11.10)

Figure 11.3: A sphere of radius *a* **with an uniform permanent magnetization of magnitude** *M***.**

For a uniformly magnetized sphere of radius *a*, as shown in Figure 11.3,

$$
M_r = M \cos \Theta \theta (a - r)
$$

\n
$$
M_{\Theta} = M \sin \Theta \theta (a - r)
$$
\n
$$
M_{\varphi} = 0
$$
\n(11.11)

where θ (*a* – *r*) is the theta function,

$$
\theta(a-r) = \begin{cases} 1 \, r < a \\ 0 \, r > a \end{cases} \tag{11.12}
$$

Therefore,

$$
\nabla. \vec{M} = -M\cos\Theta \delta(r - a) \tag{11.13}
$$

Combining eqn.(11.13) with the spherical harmonic expansion [3] results,

$$
\frac{1}{|\vec{r} - \vec{r'}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(\frac{4\pi}{2l+1} \right) \frac{r_<^l}{r_<^{l+1}} Y_l^{m^*}(\theta', \varphi') Y_l^m(\theta, \varphi)
$$
(11.14)

Only the $l=1$, $m=0$ term survives the integral for $\varphi(r)$, which results

$$
\varphi(r) = \frac{4\pi M}{3} a^2 \left(\frac{r}{r_s^2}\right) \cos \theta \tag{11.15}
$$

Where $r₅$ is smaller than *r*, i.e., inside the sphere and $r₅$ is larger than *a*, i.e., outside the sphere.This potential gives the constant demagnetizing field inside the sphere and the familiar dipole field outside.

References:

- [1]. R. M. White, Introduction to Magnetic Recording, IEEE, 1985.
- [2]. H. Neal Bertram, Theory of magnetic recording, Cambridge University Press, 1994.
- [3]. J.D. Jackson, Classical Electrodynamics, 3rd Edition, Wiley-India, 2007, Chap5.

Module 3: Recording and play back theories Lecture 12: Nature of the transitions in the writing process – Part 1

Infinitely sharp transition in the horizontal magnetization:

The nature of the transition between the bits in a recorded medium plays a major role on the nature of the magnetic interaction between the bits and the stray field from the written information. Let us assume that the transition between the bits is infinitely sharp in the horizontal magnetization as shown in Figure 12.1:

Figure 12.1: Infinitely sharp transition in the horizontal magnetization.

Then

$$
\nabla. \vec{M} = \nabla. \{ M[\Theta(-x)] - M[\Theta(x)] \} \hat{x} = -2M\delta(x) \tag{12.1}
$$

and hence the demagnetization field from eqn.(11.6) is

$$
H_d(x,y)_x = -2M \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \left[\frac{x \, dy' \, dz'}{[x^2 + (y - y')^2 + z'^2]^{\frac{3}{2}}} \right]
$$
(12.2)

Here, *w* is the trackwidth and δ is the medium thickness. If we assume that the value of *w*is the longest dimension, and taking *w*→∞; the eqn.(12.2) turns out to be

$$
H_d(x, y)_x = -4M \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \left(\frac{x \, dy'}{x^2 + (y - y')^2} \right) \tag{12.3}
$$

The above equation is two-dimensional problem. Taking $y = 0$ in the mid-plane of the medium, the eqn.(12.3) changes to

$$
H_d(x,0)_x = -8M \tan^{-1} \left(\frac{\delta}{2x}\right)
$$
 (12.4)

which has its maximum value, $4\pi M$, at the discontinuity as shown in Figure 12.2.

Figure 12.2: Variation of demagnetization field around the transition.

The lowering of the energy associated with this discontinuity is possible, if one adoptsdifferenttypes of magnetization configuration at the transition. Figure 12.3 shows one such configuration, called as zig-zagtransition wall.

Figure 12.3: Zig-Zagtransition in the horizontal magnetization.

The Arctangent transition:

The zig-zag wall shown in the figure 12.3 reduces the magnetic pole density, as the poles are now spread over a finite distance. However, this arrangement increases the wall energy, which eventually determines the shape of the wall. In order to simply the understanding, we shall assume that the amplitude of magnetization has the arctangent form rather than considering any complex wall configuration. In such case, the magnetization along the *x*-direction is

$$
M_x(x) = -\frac{2M}{\pi} \tan^{-1} \left(\frac{x}{a}\right) \tag{12.5}
$$

Where *a* is the transition length, which is an adjustable parameter. The bigger it is, the wider the transition. Taking $a = 0$, the transition becomes an ideal sharp step transition. Figure 12.4 depicts both the types of transition.

Figure 12.4:Variation of magnetization in Step and Zig-Zag transition.

The longitudinal gradient of the arctangent transition is

$$
\frac{\partial M(x)}{\partial x} = \frac{2M}{\pi} \frac{a}{x^2 + a^2} \tag{12.6}
$$

Substituting eqn.(12.5) in eqn.(11.6) results [1,2]

$$
H_d(x, y)_x = 4M \left\{ \tan^{-1} \left[\frac{\left(\frac{\delta}{2} + y \right) x}{x^2 + a^2 + \left| \frac{\delta}{2} + y \right| a} \right] + \tan^{-1} \left[\frac{\left(\frac{\delta}{2} - y \right) x}{x^2 + a^2 + \left| \frac{\delta}{2} - y \right| a} \right] \right\}
$$
(12.7)

and

$$
H_d(x, y)_y = 2M \left[ln \left(\frac{x^2 + \left[\left(\frac{\delta}{2} + y \right) + a \right]^2}{x^2 + \left[\left(\frac{\delta}{2} - y \right) + a \right]^2} \right) \right]
$$
(12.8)

For calculational convenience, H_{dx} may also be written [1],

$$
H_d(x,y)_x = -4M \left\{ \tan^{-1} \left[\frac{\left(a + \frac{\delta}{2} + y \right)}{x} \right] + \tan^{-1} \left[\frac{\left(a + \frac{\delta}{2} - y \right)}{x} \right] - 2 \tan^{-1} \left(\frac{a}{x} \right) \right\}
$$
(12.9)

Figure12.2 shows the variation of $H_{dx}(x,0)$, where the maximum occurs at

$$
x_{max} = \pm a \sqrt{\left(1 + \frac{\delta}{2a}\right)}\tag{12.10}
$$

It may be noted there is also a vertical demagnetization field, $H_d(x, y)$ ^y, which is largest at the surface of the medium. Nevertheless, this large field is generally ignored as we considered that the medium responds only to the longitudinal field.

References:

- [1]. R.I. Potter, J. Appl. Phys. 41 (1970) 1647.
- [2]. IBM San Jose Technical Report TR 02 (1969) 465.

Module 3: Recording and play back theories

Lecture 13: Nature of the transitions in the writing process – Part 2

Now, it is important to highlight the amplitude of the demagnetization field compared to the coercivity of the medium. If the demagnetization field exceeds the coercivity, then the magnetization will readjust itself to lower this field. Therefore, the transition length can be determined by equating the maximum value of $H_d(x,0)$ _xto the coercivity of the medium. For the current transition type, the minimum transition length can be found by substituting the maximum value of x [Eqn.(12.10)] in the eqn.(12.7),

$$
H_d(x,0)_x = 4M \left\{ \tan^{-1} \left[\frac{\left(\frac{\delta}{2}\right)x}{x^2 + a^2 + \frac{\delta}{2}a} \right] + \tan^{-1} \left[\frac{\left(\frac{\delta}{2}\right)x}{x^2 + a^2 + \frac{\delta}{2}a} \right] \right\}
$$
(13.1)

$$
H_d(x,0)_x = 8M \left\{ \tan^{-1} \left[\frac{\left(\frac{\delta}{2}\right)x}{x^2 + a^2 + \frac{\delta}{2}a} \right] \right\}
$$

$$
H_d(x,0)_x = 8M \left\{ \tan^{-1} \left[\frac{\left(\frac{\delta}{2}\right)a\sqrt{\left(1 + \frac{\delta}{2a}\right)}}{a^2\left(1 + \frac{\delta}{2a}\right) + a^2 + \frac{\delta}{2}a} \right] \right\} \implies
$$

$$
H_d(x,0)_x = H_c = 8M \left\{ \tan^{-1} \left[\frac{\delta}{4a\sqrt{\left(1 + \frac{\delta}{2a}\right)}} \right] \right\}
$$
(13.2)

Upon inverting the eqn.(13.2), the minimum transition length is found to be [1],

$$
\tan\left[\frac{H_C}{8M}\right] = \frac{\delta}{4a\sqrt{\left(1 + \frac{\delta}{2a}\right)}}
$$

$$
a\left(1 + \frac{\delta}{2a}\right)^{\frac{1}{2}} = \frac{\delta}{4}\frac{\cos\left[\frac{H_C}{8M}\right]}{\sin\left[\frac{H_C}{8M}\right]}
$$

Expanding the left hand side as power series gives,

$$
\Rightarrow a + \frac{\delta}{4} = \frac{\delta}{4} \frac{\cos\left[\frac{H_C}{8M}\right]}{\sin\left[\frac{H_C}{8M}\right]}
$$

$$
a_{min} = \frac{\delta}{4} \left(\frac{\cos\left[\frac{H_C}{8M}\right]}{\sin\left[\frac{H_C}{8M}\right]} - 1\right)
$$

$$
a_{min} = \begin{cases} \frac{\delta}{4} \left(\csc\left[\frac{H_C}{8M}\right] - 1\right), & \frac{H_C}{M} < 4\pi \\ 0, & \frac{H_C}{M} \ge 4\pi \end{cases}
$$
(13.4)

 \overline{M}

 $\geq 4\pi$

For the thin media with $a \gg \delta$, the eqn.(13.2) turns out to be

$$
H_C = \frac{2\delta M}{a_{min}};
$$

$$
\therefore \tan^{-1} \left[\frac{\delta}{4a\sqrt{1 + \frac{\delta}{2a}}} \right] = \frac{\delta}{4a}
$$

$$
a_{min} = \frac{2\delta M}{H_C}
$$
 (13.5)

Since *amin* in eqn.(13.5) represents the closest approach of any two magnetic reversals, the high density magnetic recording requires a small value of *amin*, which means that one needs a thin media (δ) with high coercivity(H_C) and a moderate remanent magnetization for generating the readback signal. Therefore, it is clear that a medium with higher coercivityand a moderate magnetization would be more appropriate for the high density recording.

Sinusoidal variation in magnetization:

The demagnetization field also can be evaluated for a sinusoidal variation of magnetization, which is given as

$$
M_x(x) = M \sin kx \tag{13.6}
$$

where $k = 2\pi/\lambda$ and the demagnetization field from eqn.(11.6) is defined as

$$
H_d(x, y, z = 0)_x = -kM \iiint \frac{\cos kx^{'} (x - x^{'}) dx^{'} dy^{'} dz^{'} }{((x - x^{'})^2 + (y - y^{'})^2 + z^{'}^2)^{\frac{3}{2}}}
$$
(13.7)

As considered earlier in lecture 12 that the trackwidth (*W*) is much larger than the thickness of the medium(δ) gives,

$$
H_d(x,y)_x = 2kM \int_{-\infty}^{\infty} \cos kx' \left(\tan^{-1} \left[\frac{\left(y - \frac{\delta}{2} \right)}{x - x'} \right] - \tan^{-1} \left[\frac{\left(y + \frac{\delta}{2} \right)}{x - x'} \right] \right) dx'
$$
(13.8)

$$
H_d(x,y)_x = -2\pi M \sin(kx) \left(2 - e^{-k\left(y + \frac{\delta}{2} \right)} - e^{k\left(y - \frac{\delta}{2} \right)} \right)
$$
(13.9)

The demagnetization field can be calculated both at the mid plane and at either surface of the medium as a function of wavelength of the medium.

Taking $y = 0$ in the mid plane, the eqn.(13.9) becomes,

$$
H_d(x,0)_x = -2\pi M \sin(kx) \left(2 - e^{-\frac{k\delta}{2}} - e^{-\frac{k\delta}{2}}\right)
$$

= -12.56 M sin(kx) $\left(1 - e^{-\frac{k\delta}{2}}\right)$ (13.10)

On the other hand, the demagnetization field at either surface $(y = \pm \delta/2)$ turns out to be,

$$
H_d\left(x, \pm \frac{\delta}{2}\right)_x = -2\pi M \sin(kx) \left(2 - e^{-k\delta} - e^{-k\delta}\right)
$$

= -6.28 M sin kx $\left(1 - e^{-k\delta}\right)$ (13.11)

Figure 13.1: Variation of demagnetization field associated with the horizontal sinusoidal magnetization distribution.

The variations of demagnetization field at the mid plane and at the surface are shown in Figure 13.1. It is clearly seen from the figure that with increasing $k\delta$, the demagnetization field reaches to 12.56*M*at the mid plane, while on the surface a maximum of 6.28*M* was observed.

References:

[1]. R.I. Potter, J. Appl. Phys. 41 (1970) 1647.

Module 3: Recording and play back theories

Lecture 14: Model for the writing process

William–Comstock (WC) Model

In order to understand the role of various parameters in different recording configurations, one needs to obtain an analytic form for the transition length that incorporates both the head field and the demagnetization field. An analytical model was first proposed by Williams and Comstock [1] with few assumptions, which provides an insightful and relatively simple analysis of write process. In this lecture, we shall discuss the WC model for the writing process.

Figure 14.1: Variation of magnetization, head field, and the demagnetization field along the center plan of the magnetic medium. The dashed line indicates the transition center.

Figure 14.1 shows the schematic of the write process in the longitudinal recording along with the variation of field along the *x* direction. The first assumption considered in the model is that the zig-zag nature of the magnetic transition is ignored and the shape of the transition is assumed to be Arctangent function. The second assumption is that the written magnetization pattern is represented by $M_x(x)$ which produces the demagnetization field $H_d(x)$. At a given point *x* in the magnetic medium, the demagnetization field and the head field together determine $M_x(x)$ through the intrinsic M-H loop $M_x(H)$ of the recording medium. Therefore,

$$
H(x) = H_{tot}(x) = H_x(x) + H_d(x)
$$

$$
M_x(x) = M_x[H_{tot}(x)] = M_x[H_x(x) + H_d(x)]
$$
 (14.1)

The second assumption rests on the first assumption that the shape of the magnetization is already known to calculate the demagnetization field based on the following longitudinal gradient of the magnetization given as,

$$
\frac{dM_x}{dx} = \frac{dM_x}{dH_{tot}} \frac{dH_{tot}}{dx} = \frac{dM_x(H)}{dH} \left[\frac{dH_x(x)}{dx} + \frac{dH_d(x)}{dx} \right]
$$
(14.2)

The eqn.(14.2) suggests that a sharp transition means a larger magnetization gradient. Since the demagnetization field opposes the head field gradient, the demagnetization field tends to broaden the transition in the write process. Hence, it should be considerably small.

Figure 14.2: Definition of various parameters characterizing the hysteresis loop.

The factor *dM/dH* brings in the role of the hysteresis loop. As the head approaches the point x_0 , the magnetization at that point moves away from its remanent value as illustrated on the hysteresis loop. To have zero remanence at this point after the removal of the field, one needs to overshoot to some point *I*so that it returns to zero along a minor loop path characterized by a slope *χ*. The slope of the major loop at the point *I* is

$$
\left. \frac{dM}{dH} \right|_{I} = \frac{M_r}{H_c (1 - S^*)} \tag{14.3}
$$

Where *S** measures the squareness of the loop as defined in Figure 14.2 and the value of the field at *I* is

$$
H_{cr} = \frac{-H_c}{1 - \chi(1 - S^*)\frac{H_c}{M}} = -\frac{H_c}{r}
$$
(14.4)

We have already discussed that to obtain a sharp transition the head field gradient should be at the maximum. Also, in general, the write field follows the write current almost instantaneously. Letus assume that the current into the head has been adjusted such that the maximum gradient also occurs at the transition, i.e, $H_h = H_l$. In other wods, the optimum write current is given by the maximum field gradient requirement:

$$
\left. \frac{d^2 H_x \left(x, d + \frac{\delta}{2} \right)}{dx^2} \right|_{x = x_0} = 0 \tag{14.5}
$$

Now we shall use the WM model to derive the transition parameter written with a Karlqvist head with the head field given as

$$
H_x = \frac{H_g}{\pi} \left[\tan^{-1} \left(\frac{x + \frac{g}{2}}{y} \right) - \tan^{-1} \left(\frac{x - \frac{g}{2}}{y} \right) \right]
$$
 (14.6)

The derivative of the eqn.(14.6) is

$$
\frac{dH_x}{dx}\bigg|_{x_0} = \frac{H_g}{\pi y} \left[\frac{1}{1 + \left(\frac{x_0 + \frac{g}{2}}{y}\right)^2} - \frac{1}{1 + \left(\frac{x_0 - \frac{g}{2}}{y}\right)^2} \right] = \left(\frac{H_c Q}{y}\right)
$$
(14.7)

where *Q* is a function of *y* and x_0 with typical values of 0.65 – 0.85.

$$
Q(y) = \frac{H_g}{\pi H_c} \left[\frac{y^2}{y^2 + (x_0 + g/2)^2} - \frac{y^2}{y^2 + (x_0 - g/2)^2} \right]
$$
(14.8)

The magnetization gradient at the transition center at the instant of transition formation is

$$
\left. \frac{dM_x}{dx} \right|_{x_0} = \frac{2M_r}{\pi a} \tag{14.9}
$$

The demagnetization field along the center of the medium is given as [2]

$$
H_d(x) = \frac{2M}{\pi} \left(\tan^{-1} \left(\frac{x - x_0}{a} \right) - \tan^{-1} \left(\frac{x - x_0}{a + \delta/2} \right) \right)
$$
(14.10)

Taking derivative of eqn.(14.10) at the transition center gives \rightarrow

$$
\frac{dH_d(x)}{dx}\Big|_{x=0,y=0} = -\frac{2M}{\pi} \left(\frac{1}{a} - \frac{1}{a + \delta/2}\right)
$$
\n
$$
\frac{dH_d(x)}{dx}\Big|_{x=0,y=0} = -\frac{M}{\pi} \left(\frac{\delta}{a(a + \delta/2)}\right)
$$
\n
$$
\frac{dH_d(x)}{dx}\Big|_{x=0,y=0} = -4M_r \left(\frac{\delta}{a_I \left(a_I + \frac{\delta}{2}\right)}\right) \tag{14.11}
$$

The gradient of the demagnetizing field for small medium thickness is

$$
\left. \frac{dH_d(x)}{dx} \right|_{x_0} = -\left. \frac{4M_r \delta}{a^2} \right| \tag{14.12}
$$

Combining all these derivatives (eqns.(14.3), (14.7), (14.9), and (14.12) and substituting in eqn.(14.2) leads to the transition length at *I*,

$$
\frac{2M_r}{\pi a} = \frac{M_r}{H_c(1 - S^*)} \left[\frac{H_c Q}{y} - \frac{4M_r \delta}{a^2} \right]
$$
(14.13)

Multiplying by a^2 on both sides of the eqn.(14.13) and re-arranging the terms gives

$$
\frac{2aM_r}{\pi} = \frac{M_r}{H_c(1-S^*)} \frac{H_c Q a^2}{y} - \frac{M_r}{H_c(1-S^*)} 4M_r \delta
$$

Re-arranging gives \rightarrow

$$
a^{2} - \frac{2(1 - S^{*})y}{\pi Q}a - \left(\frac{2M_{r}\delta}{H_{C}}\right)\left(\frac{2y}{Q}\right) = 0
$$
\n(14.14)

The above equation is a quadratic equation, which has a solution, as given below,

$$
a = \frac{(1 - S^*)y}{\pi Q} + \sqrt{\left[\frac{(1 - S^*)y}{\pi Q}\right]^2 + \left(\frac{2M_r \delta}{H_c}\right)\left(\frac{2y}{Q}\right)}
$$
(14.15)

where *y* is taken as the distance from the head pole tip to the center of the medium, $y = d$ $+ \delta/2$ and *d* is the magnetic spacing between the head and medium. In order to achieve a higher linear density, we need to reduce the value of *a* as much as possible, which leads to the following requirements:

- (1) Large medium coercivity,
- (2) Small $M_r \delta$ product (but sufficient enough to maintain readback signal level).
- (3) Small magnetic spacing
- (4) Large coercivity squareness S*. A range of 0.7 to 0.9 is required typically for recording with significantly less media noise. It should be noted that the large value of S^* indicate the good exchange interaction between the grains, which induces additional noise called media noise.

Hence, for a square $M - H$ loop, $S^* = 1$, eqn.(14.15) becomes,

$$
a = \sqrt{\left(\frac{2M_r\delta}{H_C}\right)\left(\frac{2y}{Q}\right)}
$$
(14.16)

This is an equation widely used for calculating the transition length in the magnetic recording.

References:

[1]. M.L. Williams, and R.L. Comstock, AIP Conf. Proc. 5 (1971) 738.

[2]. S.X. Wang, A. M. Taratorin, Magnetic information storage technology, Academic press, New York, 1999, Chap 2.

Module 3: Recording and play back theories Lecture 15: Effect of imaging from the head and the relaxation of transition parameter

When a transition is under a recording head, the transition shape can be modified due to the imaging effect of the recording head, which is made of high permeability materials. The imaging effect is schematically shown in Figure 15.1.

Figure 15.1: Imaged transition of a high permeability recording head. Negative charges are imaged into positive charges in the head.

A negative magnetic charge at $y = 0$ will have an positive magnetic charge at $y = 2d + \delta$ and the field from the image charge modifies the transition shape significantly. Therefore, the demagnetization field in the medium is reduced to

$$
H_d^{net} = H_d(x)|_{y=d+\frac{\delta}{2}} - H_d(x)|_{y=-d-\frac{\delta}{2}}
$$
\n(15.1)

The second terms comes from the imaged transition. Considering the demagnetization field along the center of the medium,

$$
H_d(x) = \frac{2M}{\pi} \left(\tan^{-1} \left(\frac{x - x_0}{a} \right) - \tan^{-1} \left(\frac{x - x_0}{a + \frac{\delta}{2}} \right) \right)
$$
(15.2)

and taking derivative of eqn.(15.2) at the transition center gives \rightarrow

$$
\frac{dH_d(x)}{dx}\Big|_{x=0} = -\frac{2M}{\pi} \left(\frac{1}{a} - \frac{1}{a + \delta/2}\right)
$$
\n
$$
\left.\frac{dH_d(x)}{dx}\right|_{x=0} = -\frac{M}{\pi} \left(\frac{\delta}{a(a + \delta/2)}\right)
$$
\n(15.3)

Recalculating the demagnetization field for the net demagnetization field due to the imaging effects results us,

$$
\frac{dH_d^{net}}{dx}\bigg|_{x=0} = -\frac{M}{\pi} \left(\frac{\delta}{a_{im} \left(a_{im} + \frac{\delta}{2} \right)} \right) + \frac{M}{\pi} \left(\frac{\delta}{(a_{im} + 2d + \delta)^2 - \left(\frac{\delta}{2} \right)^2} \right) \tag{15.4}
$$

where, a_{im} represents the transition parameter after taking the image effect into consideration. For thin medium, the eqn.(15.4) can be approximated to

$$
\left. \frac{dH_d^{net}}{dx} \right|_{x=0} = -\frac{M\delta}{\pi} \left(\frac{1}{a_{im}^2} - \frac{1}{(a_{im} + 2d + \delta)^2} \right) \tag{15.5}
$$

We know that if the magnetic medium has a nearly square loop, then the following relation must hold:

$$
\frac{dH_x}{dx} + \frac{dH_d}{dx} = 0\tag{15.6}
$$

This means that from eqn.(14.7),

$$
-\frac{H_{c}Q}{y} = \begin{cases}\n-\frac{M\delta}{\pi a^{2}} & \text{without imaging effect} \\
-\frac{M\delta}{\pi} \left(\frac{1}{a_{im}^{2}} - \frac{1}{(a_{im} + 2d + \delta)^{2}}\right) & \text{with imaging effect}\n\end{cases}
$$
\n(15.7)

Therefore, the transition parameter without imaging effect is related to that considering imaging effect by the following relations:

$$
\frac{1}{a^2} = \frac{1}{a_{im}^2} - \frac{1}{(a_{im} + 2d + \delta)^2}
$$
 (15.8)

Therefore, the net imaging effect reduces the transition parameter written by a recording head because a_{im} <*a*. This imaging effect is quite significant when *d* is small $(d \sim \delta)$, but negligible, when $d > 3\delta$. In this scenario, what would be the final transition parameter in the recording after the relaxation? This can be calculated by analysing the magnetization nature after relaxation. The relaxed magnetization M_f is related to the initial magnetization *M* through a minor loop susceptibility of χ , as shown in Figure14.2:

$$
M_f - M = \chi \left(H_{df} - H \right) \tag{15.9}
$$

where H_{df} and H are the final demagnetization field after relaxation and the total field at the instant of formation of transition, respectively. Taking derivative both sides with respect to *x* gives,

$$
\frac{dM_f}{dx} - \frac{dM}{dx} = \chi \left(\frac{dH_{df}}{dx} - \frac{dH}{dx} \right)
$$
\n
$$
\frac{dM_f}{dx} - \frac{dM}{dx} = \chi \left(\frac{dH_{df}}{dx} - \frac{dH}{dM} \frac{dM}{dx} \right)
$$
\n(15.10)

Assuming that the initial and final transition centres are close (where χ becomes small), and the eqn.(15.10) simplifies to (following the eqns.(14.9) and (14.3))

$$
\frac{2M}{\pi a_f} - \frac{2M}{\pi a} = \chi \left(\frac{M\delta}{\pi a_f^2} - \frac{H_c (1 - S^*)}{M} \frac{2M}{\pi a} \right) \implies
$$

$$
\frac{2M}{\pi a_f} - \frac{2M}{\pi a} = \frac{M \delta \chi}{\pi a_f^2} - \frac{2\chi H_c (1 - S^*)}{\pi a}
$$

Multiplying by $[\pi/(2M)]$ on both side of the equation provides

$$
\frac{1}{a_f} - \frac{1}{a} = \frac{\delta \chi}{2a_f^2} - \frac{\chi H_c (1 - S^*)}{Ma}
$$

Rearranging the terms gives

$$
0 = \frac{\delta \chi}{2a_f^2} - \frac{1}{a_f} + \frac{1}{a} \left[1 - \frac{\chi H_c (1 - S^*)}{M} \right]
$$

From eqn.(14.4), the parameters within the square bracket of the above equation can be correlated to *r* and the equation turns out to be

$$
0 = \frac{\delta \chi}{2a_f^2} - \frac{1}{a_f} + \frac{r}{a}
$$
 (15.11)

Rearranging the eqn.(15.11) results,

$$
a_f^2 - \frac{a}{r}a_f - \frac{\delta \chi a}{2 r} = 0 \tag{15.12}
$$

This is aquadratic equation on final transition parameter after relaxation. Solving the eqn.(15.11) provides the positive solution for the final transition parameter.

$$
a_f = \frac{a}{2r} + \sqrt{\left(\frac{a}{2r}\right)^2 + \frac{\delta \chi a}{2r}} \tag{15.12}
$$

This provides $a_i \approx a$, only when $x \ll 1$ and $r \approx 1$. Otherwise, the final relaxed transition parameter is always greater than the initial value written. This analytical result shows that it is important to have a sharp head field gradient (large *Q*), a very square M-H loop (*S** \rightarrow 1), a small demagnetization field (4 $\pi M \ll H_C$), and considerably thin medium in obtaining high densities.

Module 3: Recording and play back theories Lecture 16: Different types of writing process

Perpendicular versus In-plane Recording:

There are two types of magnetic recording processes: one is longitudinal recording, where the information is written on the magnetic medium using longitudinal field of the recording head and written bits are oriented along the film plan direction, another process is perpendicular recording, where the medium is written by the perpendicular field emerging out of the head and the written bits are aligned perpendicular to the disk plane. WC model was used to compare the transition length of the perpendicular and in-plane magnetic recording [1]. This constitutes a nice example of the usefulness of this model.

In-plane recording Case*:*

A small gap of the Karlqvist head field was taken by Middleton and Wright [1] for the head field, which is defined as $[from eqn. (14.6)],$

$$
H_x = \frac{H_g}{\pi} \left(\tan^{-1} \left(\frac{\left(x + \frac{g}{2}\right)}{y} \right) - \tan^{-1} \left(\frac{\left(x - \frac{g}{2}\right)}{y} \right) \right) \tag{16.1}
$$

Expanding the arctangent function for the limit $g \rightarrow 0$, we get

$$
\tan^{-1}\left(\frac{\left(x+\frac{g}{2}\right)}{y}\right) = \tan^{-1}\left(\frac{x}{y}\right) + \frac{\frac{yg}{2}}{x^2 + y^2} + \cdots
$$

$$
H_x = \frac{H_g}{\pi} \left(\tan^{-1}\left(\frac{x}{y}\right) + \frac{\frac{yg}{2}}{x^2 + y^2} - \left(\tan^{-1}\left(\frac{x}{y}\right) - \frac{\frac{yg}{2}}{x^2 + y^2}\right)\right)
$$

$$
H_x = g\frac{H_g}{\pi}\frac{y}{x^2 + y^2}
$$
(16.2)

Taking gradient of the head field gives,

$$
\frac{dH_x}{dx} = -\frac{gH_g}{\pi} \frac{2xy}{(x^2 + y^2)^2}
$$
\n(16.3)

The above equation has its maximum value at $x = y\sqrt{3}$. If H_g is adjusted such that H_x has the value $-H_I \approx H_C$ at a point where the gradient has its maximum, then eqn.(16.2) turns out to be

$$
-H_C = g \frac{H_g}{\pi} \frac{y}{3y^2 + y^2}
$$

$$
H_g = -\frac{4\pi}{g} H_C y
$$
(16.4)

Substituting eqn.(16.4) in eqn.(16.3) results,

$$
\frac{dH_x}{dx}\Big|_{max} = -\frac{g}{\pi} \left(-\frac{4\pi}{g} H_C y \right) \frac{2y\sqrt{3}y}{(3y^2 + y^2)^2}
$$
\n
$$
\frac{dH_x}{dx}\Big|_{max} = \frac{\sqrt{3}}{2} \left(\frac{H_C}{y} \right)
$$
\n(16.5)

If the hysteresis loop is very square so that *dM/dH* is very large, then if *dM/dx* is to have a reasonable value, then we must have

$$
\frac{dH_x}{dx} = -\frac{dH_d(x)}{dx} \tag{16.6}
$$

The derivative of the demagnetization field along the center of the medium is given as (following eqn.(14.10) and (14.11))

$$
\left. \frac{dH_d(x)}{dx} \right|_{x=0,y=0} = -4M_r \left(\frac{\delta}{a_I(a_I + \delta/2)} \right) \tag{16.7}
$$

where,*a_I*denotes the "longitudinal" transition length.Substituting eqns.(16.5) and (16.7) in eqn.(16.6) gives \rightarrow

$$
\frac{\sqrt{3}}{2}\left(\frac{H_C}{y}\right) = 4M_r \left(\frac{\delta}{a_I(a_I + \delta/2)}\right)
$$

Re-arranging the above equation results,

$$
a_l^2 + \frac{\delta}{2}a_l - \frac{4M_r\delta y}{\sqrt{3}H_c} = 0
$$
\n(16.8)

This is a quadratic equation for the longitudinal transition length and gives a general solution as

$$
a_{I} = -\frac{\delta}{4} + \sqrt{\frac{\delta^2}{16} + \frac{8M_r\left(d + \frac{\delta}{2}\right)\delta}{\sqrt{3}H_c}}
$$
(16.9)

In the limit of a thin medium, this above eqn. (16.9) is in consistent with the WC model result, described in eqn.(14.15).

Perpendicular recording case*:*

For perpendicular recording, one can write the medium using either a ring head or a single pole head. In such scenario, the head field gradients due to the ring type and single pole heads are

$$
\left. \frac{dH_{y}^{ring}}{dx} \right|_{max} = \frac{1}{2\sqrt{3}} \frac{H_{c}}{y}
$$
\n(16.10)

$$
\left. \frac{dH_{y}^{pole}}{dx} \right|_{max} = \frac{\sqrt{3}}{2} \frac{H_c}{y}
$$
\n(16.11)

It is clear from the above equations that the single pole head has a larger field gradient at the point where the transition occurs.

The demagnetization field associated with a vertical arctangent transition is

$$
H_{dy} = -4M_0 \left[\tan^{-1} \left(\frac{x_0}{a_v + \left| y_0 - \frac{\delta}{2} \right|} \right) + \tan^{-1} \left(\frac{x_0}{a_v + \left| y_0 + \frac{\delta}{2} \right|} \right) \right]
$$
(16.12)

where a_v is the transition length, $M_0 \leq M_r$ depending on the coercivity, i.e., if $H_c > 4\pi M_r$ then $M_0 = M_r$, but if $H_C \leq 4\pi M_r$, then $M_0 = H_C / 4\pi$. That is, far from the transition, i.e., as $|x_0|$ becomes large, H_d goes to $-4\pi M_0$. Hence, the demagnetization field gradient is

$$
\frac{dH_{dy}}{dx} = -\frac{8M_0}{\left(a_v + \frac{\delta}{2}\right)}\tag{16.13}
$$

For Ring Heads:

Equating the eqns.(16.10) and (16.13) for the field gradients gives the transition length for the ring heads.

$$
\frac{1}{2\sqrt{3}} \left(\frac{H_c}{d + \frac{\delta}{2}} \right) = \frac{8M_0}{\left(a_v^{ring} + \frac{\delta}{2} \right)} \implies
$$
\n
$$
\left(a_v^{ring} + \frac{\delta}{2} \right) = 16\sqrt{3} \frac{M_0}{H_c} \left(d + \frac{\delta}{2} \right) \implies
$$
\n
$$
a_v^{ring} = -\frac{\delta}{2} + 16\sqrt{3} \frac{M_0}{H_c} \left(d + \frac{\delta}{2} \right) \implies
$$
\n(16.14)

$$
a_v^{ring} = \frac{\delta}{2} \left(16\sqrt{3} \frac{M_0}{H_c} - 1 \right) + 16\sqrt{3} \frac{M_0 d}{H_c}
$$
 (16.15)

For Pole heads:

Equating the eqns.(16.11) and (16.13) for the field gradients gives the transition length for the pole heads.

$$
\frac{\sqrt{3}}{2} \left(\frac{H_c}{d + \frac{\delta}{2}} \right) = \frac{8M_0}{(a_v^{pole} + \frac{\delta}{2})} \implies
$$
\n
$$
\left(a_v^{pole} + \frac{\delta}{2} \right) = \frac{16 M_0}{\sqrt{3} H_c} \left(d + \frac{\delta}{2} \right) \implies
$$
\n
$$
a_v^{pole} = -\frac{\delta}{2} + \frac{16 M_0}{\sqrt{3} H_c} \left(d + \frac{\delta}{2} \right) \implies
$$
\n
$$
a_v^{pole} = \frac{\delta}{2} \left(\frac{16 M_0}{\sqrt{3} H_c} - 1 \right) + \frac{16 M_0 d}{\sqrt{3} H_c} \implies
$$
\n
$$
a_v^{pole} = \frac{\delta}{2} \left(\frac{4}{\sqrt{3} \pi} \frac{4 \pi M_0}{H_c} - 1 \right) + \frac{4}{\sqrt{3} \pi} \frac{4 \pi M_0}{H_c} d
$$
\n(16.17)

Eqn.(16.15) reveals that the longitudinal transition length increases with increasing the thickness of the medium and also for the increase in (M_0/H_C) . On the other hand, eqn.(16.17) indicates that the transition length gets shorted as the medium gets thicker. This arises from the fact that $4\pi M_0$ is always less than H_C and hence, the term $\left(\frac{4}{\sqrt{2}}\right)$ $\frac{4}{\sqrt{3}\pi}$ $\bigg(\frac{4\pi M_0}{H_c}\bigg)$ $\frac{d^{1}m_0}{dt_c}$ < 1. The above results suggest that it is not the demagnetization fields themselves which are important, but rather their gradients. Therefore, the larger gradient makes the pole heads preferable for writing on a vertical medium.

Fields due to a finite gap of the head:

It is generally assumed that the track widths are considerably larger as compared to the gap length or flying height so that the fields are virtually constant over the width of the track so that two-dimensional can be applied. The constant potential pole pieces are taken to be infinitely long in the recording direction. In order to study in detail the approximations were taken into far field, near field and medium field approximations. We shall briefly look at these approximations and analyse the variation of lone of constant potential with these approximations.

Far field approximation:

If the gap is assumed to be infinitesimally small [2] and the poles are infinitely long as shown in Fig. 16.1, the lines of constant potential (LCP) in the regime above the head are radially directed emanating from the gap and increase linearly with angle. Therefore, the field lines are circular coincident with the field magnitude contour. Utilizing the path integral and taking the integration path to be a field line at a fixed radial distance r from one potential face to the other provides,

$$
\left|\overrightarrow{H(r)}\right| = \frac{NIE}{\pi r} \tag{16.18}
$$

Where *NI* is the magmetomotive force and *E* is head efficiency.

Figure 16.1: Potential lines and field contour for far field approximation [2].

Medium range approximation (Karlqvist field):

The far field approximation is not accurate in the vicinity of the gap. Hence, it is assumed that the head is infinitely long and wide with an infinitely deep gap. The only length parameter is the gap length as given in Figure 16.2. The field above the head can be calculate using eqns.(12.7)and (12.8). Unfortunately, the potential variation across the gap is not known in advance.

Figure 16.2: Gap region fields and potentials. H_g is the deep gap field, and *g* is the gap length so that $H_g = NIE/g$.

Near Field approximation:

Ruigrok – Westmijze [3] has introduced a simple analytic form for the near field of a finite gap head utilizing a conformal map solution for a finite gap head. They introduced a complex function W defined as

$$
W = U + iV \tag{16.19}
$$

Where $V(x,y)$ is considered as a scalar potential and $U(x,y)$ is the field stream function (flux lines). The transformation to the *W* plane, in normalized form (*z* by *g*/2 and *W* by *NIE*) is given by

$$
z = i\sinh(\pi W) \tag{16.20}
$$

By symmetry, the field across the gap $(-g/2 \langle x \langle g/2 \rangle)$ is longitudinal and is given simply as

Figure 16.3: Surface longitudinal field component across the hap for the head.

The exact form of field variation at the head surface with respect to the eqn.(16.20) is given in Fig.16.3. Ruigrok demonstrated that the exact surface field is given by half the Karlqvist field and half of the above thin gap field.

$$
H_x(x) = \frac{H_g}{2} \left(1 + \frac{2}{\pi \sqrt{1 - \left(\frac{2x}{g}\right)^2}} \right)
$$
 (16.21)

The above equation yields $H_x(0) = 0.82 H_g$ as does the exact field and becomes infinite at the corners. The above equation reveals that the form of approach to infinity varying as the square root of the distance from the corner is not exact. This is because for the 90 corner of a real head the variation should be as the 3/2 power [4]. Nevertheless, the eqn.(16.21) follows the surface fields quite accurately and yields the values of $H_x(0) \cong H_g$ at $2x/g \sim 0.77$, as indicated in Fig.16.3. This approximation clarifies the surface charge dilemma of the Karlqvist field.

Figure 16.4: LCP for far-field, medium field (Karlqvist) and near-field (Westmijze) approximations for a two-dimensional head with infinitely long poles and infinitely deep gap.

Fig.16.4 shows the summary of the various approximations for the LCP with two dimensional head with infinitely long poles and infinitely deep gap. Half the head is shown where the right core potential is $V/(NIE) = -0.5$ and the gap center line potential is $V = 0.0$. Deep inside the gap, far from the surface, the potential varies linearly: the potential lines are parallel to the gap faces and yield a deep gap field given by $H_g = NIE/g$. On the other hand, the potential at the surface does not vary linearly and the potential lines spread leaving the gap resulting in a slightly reduced field at the gap center and much increased field near the gap corners.

In the magnetic recording process, the information is written in the form of magnetic bits oriented either in the film plane or perpendicular to the film plane. In general, there exists a considerable magnetic interaction between the neighbouring bits depending on its nature, i.e., 0 or 1. In the micromagnetic analysis it has been noticed that the fluctuations of the interbit magnetostatic interactions are a dominant aspect. Interbit interactions have been studied for single transition noise models involving non-linear bit shift by Middleton and Miles [5] and Semenov et al [6]. A more detailed discussion on the interbit interactions involving non-linear bit shift and overwrite process has been discussed by Neal Bertram [2] in chapter 9.

References:

[1]. B.K. Middleton and C.D. Wright, IERE Conf. Proc. 54 (1982) 181.

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Quiz:

(1) What are the materials' properties required to write the information with narrow transition length?

(2) What are the different types of transitions considered for demonstrating the writing process?

(3) How does the transition parameter relax by the effect of imaging from the write head made of high permeability material?

(4) What are the physical limitations faced by the in-plane magnetic recording?

(5) How does the transition length vary with the imaging effect?

Module 3: Recording and play back theories Lecture 17: The Read back Voltage

In the earlier lectures, we have discussed the writing process in a magnetic medium. The written magnetic medium consists of magnetic bits either on the plane of the medium or perpendicular to the medium. The presence of such magnetization distribution in the medium generates "demagnetization" fields, which extend beyond the medium, as shown in Figure 17.1. If these field lines made to pass through a coil, which is moving at a speed relative to the medium, a voltage will be induced in the coil. Wallace has investigated the read back voltage for the first time in 1957 [1].

Figure 17.1: The demagnetization field emanating from the magnetization bits written in the magnetic medium.

Figure 17.2: The variation of magnetization inside the magnetic medium.

The Wallace Solution:

Wallace had assumed that the magnetization in the medium had the sinusoidal form longitudinally. The general interest is to consider the magnetic field above the medium, which can be expressed as discussed in the writing process. Hence, the fields at a point (*x*,*y*) above the medium are

$$
H_x(x, y) = -2\pi M \sin(kx) e^{-ky} \left(e^{k\frac{\delta}{2}} - e^{-k\frac{\delta}{2}} \right)
$$

\n
$$
H_y(x, y) = -2\pi M \cos(kx) e^{-ky} \left(e^{k\frac{\delta}{2}} - e^{-k\frac{\delta}{2}} \right)
$$
\n(17.1)

Notice the exponential dependence on *y*is a general feature of Poisson's equation in two dimensions.Wallace assumed that the reproduce head consists of a semi-infinite block of high permeability material with a flat face spaced at a distance *d*from the recording medium. Also, the block was assumed to be infinitely thick so that it collected all the field lines, and then thread through a coil wound around the block.The value of the induction *B* inside the block is easily obtained by the method of images [2] as,

$$
B_x(x,y) = -\left(\frac{2\mu}{\mu+1}\right) 2\pi M \sin(kx) e^{-ky} \left(e^{k\frac{\delta}{2}} - e^{-k\frac{\delta}{2}}\right)
$$
 (17.2)

Then, the flux per unit width can be calculated as,

$$
\Phi_x = \int_{d+\frac{\delta}{2}}^{\infty} B_x dy = -\left(\frac{2\mu}{\mu+1}\right) 2\pi M \sin(kx) \left(e^{k\frac{\delta}{2}} - e^{-k\frac{\delta}{2}}\right) \int_{d+\frac{\delta}{2}}^{\infty} e^{-ky} dy
$$
\n
$$
\Phi_x = \int_{d+\frac{\delta}{2}}^{\infty} B_x dy = -\left(\frac{2\mu}{\mu+1}\right) 2\pi \delta M \sin(kx) \left(\frac{e^{-kd} - e^{-k(\delta+d)}}{k\delta}\right)
$$
\n(17.3)

If $x = vt$, and the width of the head is given as w ,

$$
\Phi_x = -\left(\frac{2\mu}{\mu+1}\right) 2\pi M \sin(kvt) \left(\frac{e^{-kd} - e^{-k(\delta+d)}}{k}\right)
$$
\n
$$
\frac{d\Phi}{dt} = -\left(\frac{2\mu}{\mu+1}\right) 2\pi M v \left(e^{-kd} - e^{-k(\delta+d)}\right) \cos \omega t
$$
\n(17.4)

The voltage across the coils for the head with N turns and an efficiency of η turns out to be,

$$
V(t) = N\eta \frac{d\Phi_x}{dt} = -\left(\frac{2\mu}{\mu + 1}\right) 2\pi N\eta M v \left(e^{-kd} - e^{-k(\delta + d)}\right) \cos \omega t \tag{17.5}
$$

The eqn.(17.5) reveals a number of features that are characteristics of magnetic recording. First issue is that there is a spacing loss:

$$
20\log_{10}e^{-kd} = -54.6\left(\frac{d}{\lambda}\right)dB\tag{17.6}
$$

Another feature to note is the thickness dependence of the output voltage:

$$
(1 - e^{-k\delta}) \approx k\delta = \frac{2\pi\delta}{\lambda} = \frac{\delta\omega}{\nu}
$$
 (17.7)

Figure 17.3: Reciprocity between recording head and magnetic medium.

Reciprocity principle:

This principle is based on the fact that the mutual inductance between any two objects is one quantity and the same, i.e., $M_{12} = M_{21}$. Let us consider two objects: one is a recording head and another is a magnetic element of recording medium located at (*x*,*y*,*z*) with a volume of *dxdydz*, as shown in Figure 17.3. The distance between the pole tip and the top of the magnetic recording layer can be taken as d and δ is the thickness of the medium. If we assume that the magnetization in the medium is in horizontal direction, i.e., along the *x*-axis, then we need only a horizontal component of the magnetic field. If the coil has an imaginary write current i_l , which produces a head field H_x , then the magnetic flux through the medium element due to the current i_l , is

$$
d\Phi_{21} = \mu_0 H_x(x, y, z) dy dz \qquad (17.8)
$$

The equivalent current at the four surfaces (parallel to *x*-axis as shown in Figure 17.3) of the magnetic element is

$$
i_2 = M_x(x - \bar{x}, y, z)dx\tag{17.9}
$$

where $\bar{x} = vt$ is the typical moving distance of the medium relative to the head. Therefore, the magnetic flux through the head coil due to the magnetic element, $d\Phi_{12}$, is related to $d\Phi_{21}$ by,

$$
\frac{d\Phi_{12}}{i_2} = \frac{d\Phi_{21}}{i_1} \tag{17.10}
$$

By combining eqns.(17.8)-(17.10), the magnetic flux through the head coil due to the magnetic element *dxdydz*can be given as

$$
d\Phi_{12} = \frac{\mu_0 H_x(x, y, z)}{i_1} M_x(x - \bar{x}, y, z) dx dy dz
$$
 (17.11)

Thus, the total magnetic flux through the head coil is expressed by head field per unit head current and the medium magnetization:

$$
\Phi = \mu_0 \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \int_{-\infty}^{\infty} dz \frac{H_x(x, y, z)}{i} M_x(x - \bar{x}, y, z) \tag{17.12}
$$

The eqn.(17.12) is called as reciprocity formula or reciprocity principle.This equation can further be simplified by assuming that both the magnetization and head field are uniform over the data track (along the track width *W*) along the *z*-axis and the magnetization is uniform through the medium thickness along the *y*-axis,

$$
\Phi = \mu_0 W \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \frac{H_x(x, y)}{i} M_x(x - \bar{x})
$$
 (17.13)

Subsequently, the readback voltage is given as (by the Faraday's law):

$$
V_x(\bar{x}) = -\frac{d\Phi}{dt} = -\frac{d\Phi}{d\bar{x}}\frac{d\bar{x}}{dt} = -\mu_0 vW \int_{-\infty}^{\infty} dx \int_{d}^{d+\delta} dy \frac{H_x(x, y)}{i} \frac{dM_x(x - \bar{x})}{d\bar{x}} \tag{17.14}
$$

As the derivative of magnetization represents the magnetic charge density, the eqn.(17.14) confirms the fact that the head field senses the moving magnetic charge. If the derivative of magnetization is zero or the velocity is zero, then there is no readback signal.

References:

[1]. R.L. Wallace, Bell Syst Tech J, 30 (1957) 1145.

[2]. A. Pramanik, Electromagnetism: Theory and Applications, PHI learning Pvt Ltd, 2004, page 272.

Module 3: Recording and play back theories Lecture 18: Readback from a single transition

The reciprocity formula obtained in the earlier lecture would readily help to derive the readback voltage from various types of magnetic recording patterns. To begin with, let us consider a single magnetic transition made of either an infinitely sharp step transition or an arctangent transition.

Infinitely sharp step transition:

A single infinitely sharp step transition as discussed in Figure 12.4 with a moving center $\bar{x} = vt$ is expressed as

$$
M_x(x - \bar{x}) = \begin{cases} -M_r \text{ for } x < \bar{x} \\ M_r \text{ for } x > \bar{x} \end{cases} \tag{18.1}
$$

where M_r is the remanence magnetization of the medium. Then,

$$
\frac{dM_x(x-\bar{x})}{d\bar{x}} = -\frac{dM_x(x-\bar{x})}{dx} = -2M_r \delta(x-\bar{x})
$$
\n(18.2)

where $\delta(x)$ is the δ -function (not the thickness of the medium). From the reciprocity formula, we then obtain,

$$
V_x(\bar{x}) = 2\mu_0 v W M_r \int_d^{d+\delta} dy \frac{H_x(\bar{x}, y)}{i}
$$
 (18.3)

For thin medium with $\delta \ll d$, we get

$$
V_x(\bar{x}) = 2\mu_0 v W M_r \delta \frac{H_x(\bar{x}, d)}{i}
$$
\n(18.4)

The eqn.(18.4) reveals that the readback signal of a thin magnetic medium with an infinitely sharp magnetic transition is in the shape of the head field profile at the medium. Hence, the head field expression is often called head sensitivity function.

 (10.4)

Arctangent transition:

In reality, the magnetic transition is not a sharp one (as discussed in the writing process), but rather a transition with a finite width. This could be approximated to an arctangent function (see Fig.12.4):

$$
M_x(x - \bar{x}) = \frac{2M_r}{\pi} \tan^{-1} \left(\frac{x - \bar{x}}{a}\right)
$$
 (18.5)

where*a* is the transition parameter. When $a = 0$, the arctangent function transfers into a step function.Following the reciprocity formula, Karlqvist equation, and the following identities:

$$
\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2};
$$
\n
$$
\int_{-\infty}^{\infty} \frac{1}{a^2 + (x - \bar{x})^2} \left[\tan^{-1}\left(\frac{x+c}{y}\right) - \tan^{-1}\left(\frac{x-c}{y}\right) \right]
$$
\n
$$
= \frac{\pi}{a} \left[\tan^{-1}\left(\frac{\bar{x}+c}{y+a}\right) - \tan^{-1}\left(\frac{\bar{x}-c}{y+a}\right) \right]
$$
\n(18.6)

The readback voltage can readily be obtained from an arctangent transition as

$$
V_x(\bar{x}) = -\mu_0 vW \int_{-\infty}^{\infty} dx \int_d^{d+\delta} dy \frac{H_x(x, y)}{i} \frac{dM_x(x - \bar{x})}{d\bar{x}}
$$

\n
$$
= \mu_0 vW \int_{-\infty}^{\infty} dx \int_d^{d+\delta} dy \left\{ \frac{H_g}{\pi i} \left[\tan^{-1} \left(\frac{x + \frac{g}{2}}{y} \right) \right] - \tan^{-1} \left(\frac{x - \frac{g}{2}}{y} \right) \right\} \left(\frac{2M_r}{\pi} \frac{a}{a^2 + (x - \bar{x})^2} \right) \implies
$$

\n
$$
= \frac{2\mu_0 M_r vW aH_g}{\pi^2 i} \int_d^{d+\delta} dy \int_{-\infty}^{\infty} dx \left\{ \left(\frac{1}{a^2 + (x - \bar{x})^2} \right) \left[\tan^{-1} \left(\frac{x + \frac{g}{2}}{y} \right) \right] - \tan^{-1} \left(\frac{x - \frac{g}{2}}{y} \right) \right\} \implies
$$

$$
= \frac{2\mu_0 M_r vW aH_g}{\pi^2 i} \frac{\pi}{a} \int_d^{d+\delta} dy \left[\tan^{-1} \left(\frac{x + \frac{g}{2}}{y + a} \right) - \tan^{-1} \left(\frac{x - \frac{g}{2}}{y + a} \right) \right] \implies
$$

$$
V_x(\bar{x}) = \frac{2\mu_0 M_r vW H_g}{\pi i} \int_d^{d+\delta} dy \left[\tan^{-1} \left(\frac{\bar{x} + \frac{g}{2}}{y + a} \right) - \tan^{-1} \left(\frac{\bar{x} - \frac{g}{2}}{y + a} \right) \right]
$$
(18.7)

Or, we can write it as a general equation as

$$
V_x(\bar{x}) = \frac{2\mu_0 M_r v}{i} \int_d^{d+\delta} dy \, H_x(\bar{x}, y + a) = \frac{2\mu_0 M_r v}{i} \int_{d+a}^{d+a+\delta} dy \, H_x(\bar{x}, y) \tag{18.8}
$$

This suggest that an arctangent transition at a magnetic spacing of *d* is equivalent to a step transition at an effective magnetic spacing of *d*+*a*. If the magnetic medium is thin enough (for $<< d$), then,

$$
V_x(\bar{x}) = 2\mu_0 M_r v W \delta \frac{H_x(\bar{x}, d+a)}{i}
$$
 (18.9)

Hence, its peak voltage is

$$
V^{peak} = V_x(\bar{x} = 0) \approx 4\mu_0 M_r v W \delta \frac{H_g}{\pi i} \tan^{-1} \left(\frac{\frac{g}{2}}{d+a}\right)
$$
 (18.10)

The readback voltage pulse from an arctangent transition is in the shape of head field profile with an effective magnetic spacing of *d*+ *a*.

Now we shall consider a similar thin medium read by a small gap Karlqvist head. Then the readback voltage pulse is

$$
V_x(\bar{x}) = 2\mu_0 M_r v W \delta \frac{H_g}{\pi i} \tan^{-1} \left(\frac{g(d+a)}{\bar{x}^2 + (d+a)^2 - \left(\frac{g}{2}\right)^2} \right)
$$

$$
V_x(\bar{x}) \approx 2\mu_0 v W M_r \delta \frac{g H_g}{\pi i} \frac{(d+a)}{\bar{x}^2 + (d+a)^2}
$$
 (18.11)

This is a Lorentzian pulse, which can be written in the form of

$$
V_{sp}(x) = \frac{V^{peak} \left(\frac{PW_{50}}{2}\right)^2}{x^2 + \left(\frac{PW_{50}}{2}\right)^2}
$$
 (18.12)

where, $V^{peak} = 2\mu_0 vW M_r \delta \frac{gH_g}{\pi i (d+r)}$ $\frac{g \mu g}{\pi i (d+a)}$ is the pulse amplitude and $PW_{50} = 2(d+a)$ is the pulse width at half-amplitude. This is often used in magnetic recording channel modelling.

Module 3: Recording and play back theories Lecture 19: Pulse width and Current Optimization

The readback pulse width at half maximum is generally expressed as

$$
PW_{50} = \begin{cases} \frac{1}{2} + \frac{\sqrt{g^2 + 4(d+a)^2}}{4(a+d)(a+d+\delta)} & \text{for } g \ll d \\ \frac{1}{2} + \frac{\sqrt{4(a+d)(a+d+\delta)}}{4(a+d)(a+d+\delta)} & \text{in general} \end{cases} \tag{19.1}
$$

The above expression breaks down when the thickness of the magnetic medium is high. It is clear from the eqn.(19.1) that a small gap length, transition parameter, medium thickness, and magnetic spacing are required to achieve a narrow pulse. An efficient head design to minimize the PW_{50} is given by

$$
g \approx \sqrt{4(a+d)(a+d+\delta)}
$$
 (19.2)

The Fourier transform with respect to \bar{x} of eqn.(18.8), i.e., the voltage pulse of a Karlqvist head reading an arctangent transition, gives

$$
V_x(k) = \frac{2\mu_0 M_r vW}{i} \int_{d+a}^{d+a+\delta} dy FT[H_x(\bar{x}, y)]
$$

$$
V_x(k) = \frac{2\mu_0 M_r vW}{i} \int_{d+a}^{d+a+\delta} dy g H_g \frac{\sin\left(\frac{kg}{2}\right)}{\left(\frac{kg}{2}\right)} e^{-ky}
$$
(19.3)

This gives us

$$
V_x(k) = 2\mu_0 M_r v W \delta \frac{g H_g}{i} \left[\frac{\sin\left(\frac{kg}{2}\right)}{\left(\frac{kg}{2}\right)} \right] \left[\frac{e^{-k(d+a)} - e^{-k(d+a+\delta)}}{(k\delta)} \right]
$$
(19.4)

At low frequencies the voltage is simply due to head differentiation, so it is proportional to velocity *v*, head efficiency, and number of coil turns. At higher frequencies the Fourier component drops due to the Wallace spacing loss and gap nulls.There are three loss terms associated with the above expression,

(1) Spacing loss:

As described in eqn.(19.4), the Fourier transform of the field not only decays with respect to xas exponentially $(e^{k(d+a)})$ as a function of $y=d+a$, but also decays exponentially with the wavevector $(e^{-k\delta})$.

(2) Gap loss:

The term $\sin(kg/2)/(kg/2)$ is originated from the Karlqvist head approximation. The head surface field is assumed to be a single square pulse, so its Fourier transform produces this gap loss term and gap nulls ($kg = 2\pi, 4\pi, ...$). Considering the fact that the Karlqvist approximation is not precise, the first gap null actually occurs at $k g \approx 1.8 \pi$ instead of 2π .

(3) Thickness loss:

The term $(1-e^{-k\delta})/(k\delta)$ is also originated from the Laplace equation. This reveals that the thinner the medium is, the closer the bottom of the medium is from the head, so the less the spacing loss is, i.e., the spacing loss occurs with increasing the medium thickness, as the field from the head may not reach the bottom of the medium.

These loss mechanisms are universal in all the types of magnetic recording. Hence, the necessary considerations have to be taken care when designing and testing a magnetic recording system.

Current Optimization:

In the Williams-Comstock (WC) model discussed in lecture 14, it was assumed that the head field was adjusted such that the maximum in the head field gradient occurred where the head field itself was equal to that coercivity. Now, it would be interesting to deliberate how the output varies as a function of the head field H_g , or, equivalently, the input current.

The longitudinal head field function is

$$
H_x = \frac{H_g}{\pi} \left[\tan^{-1} \left(\frac{x + \frac{g}{2}}{y} \right) - \tan^{-1} \left(\frac{x - \frac{g}{2}}{y} \right) \right]
$$
(19.5)

The centre of the transition occurs when $H_h = -H_I = -H_C$ as discussed in the WC model. For a square loops having $H_{cr} \approx H_C$, the gradient of the head field function is

$$
\frac{dH_x}{dx} = \frac{H_g}{\pi y} \left(\frac{y^2}{\left(x + \frac{g}{2}\right)^2 + y^2} - \frac{y^2}{\left(x - \frac{g}{2}\right)^2 + y^2} \right) \tag{19.6}
$$

If $H_h = -H_c$, then one of the coordinates may be eliminated (consider eliminating *x*), then, eqn.(19.6) turns out to be,

$$
\frac{dH_x}{dx} = \frac{H_g}{\pi y} \sin^2 \left(\frac{\pi H_c}{H_g}\right) \tag{19.7}
$$

The eqn.(19.7) suggests that the H_g must be more than H_c . As H_g/H_c increases beyond 2, the head field gradient goes through a maximum near 2.7. The centre of the transition also moves further from the gap edge as illustrated in Figure 19.1.

Figure 19.1: Longitudinal field intensity at some point above the gap as a function of the deep-gap field.

If we again take an arctangent transition (similar to eqn.(18.5)),

$$
M = \left(\frac{2M_r}{\pi}\right) \tan\left(\frac{x'}{a}\right) \tag{19.8}
$$

then at the centre of the transition, at $x' = 0$ we get

$$
\left. \frac{dM}{dx'} \right|_{x'=0} = \frac{2M_r}{\pi a} \tag{19.9}
$$

Following the demagnetization field (eqn.(12.7)), the gradient of demagnetization field at the top surface of the medium $(y = \delta/2)$ and at the center of the transition is

$$
\left. \frac{dH_d}{dx'} \right|_{\substack{x'=0 \\ y=\delta/2}} = \frac{4M_r \delta}{a(a+\delta)} \tag{19.10}
$$

Using the Williams-Comstock squareness parameter *S** (eqn.(14.3))

$$
\frac{dM}{dH} = \frac{M_r}{H_c (1 - S^*)}
$$
\n(19.11)

The slope criterion is given as

$$
\frac{dM}{dx} = \frac{dM}{dH} \left(\frac{dH_h}{dx} + \frac{dH_d}{dx} \right) \tag{19.12}
$$

Substituting eqns.(19.7), (19.9), (19.10), and (19.11) into eqn.(19.12) results

$$
\frac{2M_r}{\pi a} = \frac{M_r}{H_c(1 - S^*)} \left(\frac{H_g}{\pi y} sin^2\left(\frac{\pi H_c}{H_g}\right) + \frac{4M_r \delta}{a(a + \delta)}\right)
$$
(19.13)

Taking the contribution from the last term in eqn.(19.13) to be small, and assuming $y =$ *d*results,

$$
\frac{2M_r}{\pi a} = \frac{M_r}{H_c(1 - S^*)} \frac{H_g}{\pi d} \sin^2\left(\frac{\pi H_c}{H_g}\right) \tag{19.14}
$$

$$
a = \frac{2(1 - S^*)d}{\left(\frac{H_g}{H_C}\right)\sin^2\left(\frac{\pi H_c}{H_g}\right)}\tag{19.15}
$$

Figure 19.2: Schematic drawing of output pulse amplitude as a function of record current for different record frequencies [1].

The eqn.(19.15) reveals the reciprocal dependence on the head field gradient, i.e., the transition length will have a minimum, when head field gradient is a maximum. A sharper transition means a larger output voltage. Figure 19.2 shows the output maximum as a function of the input current [1].

References:

[1]. B.K. Middleton and P.L. Wisely, IEEE Conf. Proc. 35 (1976) 35.

Module 3: Recording and play back theories Lecture 20: Magnetoresistive readback

In the earlier lectures, we have been discussing the ring and single-pole heads. They all rely on Faraday's law: the output voltage is proportional to the rate of change of magnetic flux linking the head. With increasing the areal densities, the track widths become smaller, and as we move to smaller diameter size disks, these induction signals become weaker. Hence, they are not suitable for reading the signal from high density bits. An alternative and more promising approach for the detection lies in the magnetoresistive (MR) effect [1].

Magnetoresistivity:

Magnetoresistance (MR) means the increase in the resistance of a metal and/or semiconductor when they are placed in a magnetic field. In the standard geometry, the field is transverse to the current and $\Delta \rho / \rho \sim H^2$. However, in a magnetic material, this MR is anisotropic, depending upon the direction of magnetization alignment with respect to the current direction and the origin of this anisotropy lies in spin-dependent scattering [2]. Here, let us briefly discuss how the resistance depends upon an applied field by considering a film of permalloy which has been deposited in a field so as to give it an easy axis.

The anisotropy energy density may then be written

$$
E_A = K \sin^2 \theta \tag{20.1}
$$

where θ is the angle between the magnetization and the easy axis, as shown in Figure 20.1. If an external field H_0 is applied perpendicular to this axis, then the Zeeman energy is

$$
E_z = -M_s H_0 \sin \theta \tag{20.2}
$$

Figure 20.1: Schematic representation of current, magnetization and easy axis directions in the MR element.

Minimizing the total energy (eqns. $(3.1) - (3.5)$) gives

$$
\sin \theta = \frac{M_s H_0}{2K} \tag{20.3}
$$

Suppose that the current now flows along the easy axis. This may be resolved into components parallel and perpendicular to the magnetization, as shown in Figure 20.1.

Since the resistivities associated with these directions are different, the electric field components are

$$
\mathcal{E}_{\parallel} = \rho_{\parallel} j_{\parallel}
$$
\n
$$
\mathcal{E}_{\perp} = \rho_{\perp} j_{\perp}
$$
\n(20.4)

Therefore, the measured electric field is

$$
\mathcal{E}_z = \mathcal{E}_{\parallel} \cos \theta + \mathcal{E}_{\perp} \sin \theta
$$

$$
\mathcal{E}_z = \rho_{\parallel} j \cos^2 \theta + \rho_{\perp} j \sin^2 \theta
$$
 (20.5)

The resistivity becomes

$$
\rho = \frac{\mathcal{E}_z}{j} = \rho_{\parallel} \cos^2 \theta + \rho_{\perp} (1 - \cos^2 \theta) \implies
$$

$$
\rho = \rho_{\perp} + (\rho_{\parallel} - \rho_{\perp}) \cos^2 \theta \implies
$$

$$
\rho = \rho_{\perp} + (\rho_{\parallel} - \rho_{\perp}) (1 - \sin^2 \theta)
$$

$$
\rho = \rho_{\perp} + (\rho_{\parallel} - \rho_{\perp}) - (\rho_{\parallel} - \rho_{\perp}) \sin^2 \theta
$$

$$
\rho = \rho_0 + \Delta \rho_{max} - \Delta \rho_{max} \left(H_0 \left(\frac{2K}{M_s} \right)^{-1} \right)^2 \tag{20.6}
$$

where, $\rho_0 \equiv \rho_\perp$; and $\Delta \rho_{max} = (\rho_\parallel - \rho_\perp)$

Eqn.(20.6) suggests that the resistivity of the film decreases with increasing the field due to the alignment of the domains in the field directions, resulting less scattering to the electrons travelling through the films.

Analysis of readback signal:

In order to improve the resolution of the MR sensor and to avoid the interference of noise from the external sources, it is usually shielded from the approaching transition as shown in Figure 20.2. The response of MR sensor can be analysed using the reciprocity theorem by calculating the flux in the element. To obtain the field, let us consider a coil wrapped around the MR element. In the absence of the shields, this would correspond to a pole-tip head.In the presence of the shields the magnetic potential along the face of the head has the form (see Figure20.2b), which is the sum of two Karlqvist potentials with opposite signs, one centred at *x=−*(*g+t*)*/2* and another at *x =* (*g+t*)*/2*.Therefore, the field will be,

$$
H_x(x, y) = H_x\left(x + \frac{(g+t)}{2}, y\right) - H_x\left(x - \frac{(g+t)}{2}, y\right)
$$
(20.7)

Figure 20.2: (a) Schematic drawing of MR head with shielding arrangements, (b) magnetic potential along the face of the MR head.

When this result is used in the reciprocity relation (as in eqn. (18.3)), the voltage will be the difference of two Wallace-like expressions. Since the voltage is the time rate of change of the flux, the flux can be obtained by integrating the voltage,

$$
\Phi(\bar{x}) = \frac{1}{V} \int_{-\infty}^{\bar{x}} \left(V\left(x' + \frac{(g+t)}{2}\right) - V\left(x' - \frac{(g+t)}{2}\right) dx' \right) \tag{20.8}
$$

where $x = vt$. In practice, the MR sensor is biased with a transverse field H_0 such that $\theta =$ $\pi/4$ where the response is linear. Thus, the last term in eqn.(20.7) turns out to be,

$$
\left(H_0 \left(\frac{2K}{M_s}\right)^{-1}\right)^2 \to \left(H_0 \left(\frac{2K}{M_s}\right)^{-1}\right)^2 + \left(2H_0 \Delta H \left(\frac{2K}{M_s}\right)^{-2}\right) \tag{20.9}
$$

Since $\theta = \pi/4$, the term, $sin^2 \theta = \left(H_0 \left(\frac{2K}{M}\right)\right)$ $\left(\frac{2K}{M_S}\right)^{-1}\bigg)$ 2 $=\frac{1}{2}$ $\frac{1}{2}$. Hence, the eqn.(20.9) turns out to be,

$$
\rightarrow \frac{1}{2} + \frac{2\Delta H}{H_0} \left(\frac{1}{2}\right) = \frac{1}{2} + \frac{\Delta H}{H_0}
$$
 (20.10)

Also, H_0 induces the transverse magnetization, $M_v = M_s / \sqrt{2}$, and since $M_v >> H_0$,

$$
\frac{\Delta H}{H_0} = \frac{\sqrt{2}\Phi(\bar{x})}{4\pi M_s \text{tw}}\tag{20.11}
$$

Substituting the eqns. (20.10) and (20.11) in eqn. (20.7) results us

$$
\rho = \rho_0 + \Delta \rho_{max} - \Delta \rho_{max} \left(\frac{1}{2} + \frac{\sqrt{2} \Phi(\bar{x})}{4 \pi M_s t w} \right)
$$

$$
\rho = \rho_0 + \frac{1}{2} \Delta \rho_{max} - \sqrt{2} \Delta \rho_{max} \left(\frac{\Phi(\bar{x})}{4 \pi M_s t w} \right)
$$
(20.12)

Here, $\Phi(\bar{x})$ is the flux entering the sensor and ρ is the resistivity along this face. As one moves up the sensor element the flux leaks out of the element into the shields. If the height of the element is of the order of this characteristic decay length then the average flux is $\Phi/2$. The voltage across the element is then

$$
V = -\frac{j}{\sqrt{2}} \left(\frac{\Delta \rho_{max} \ \Phi(\bar{x})}{4\pi M_s t} \right) \tag{20.13}
$$

The peak to peak amplitude of the permalloy based MR head is of the order of 90 μ V/ μ m track width. This is about ten times that of an inductive head and is the reason that such sensors are being important and developed for recording high density bits in magnetic recording.

References:

[1]. R.P. Hunt, IEEE Trans. Magn. 7 (1971) 150.

[2]. T.R. McGuire, R.I. Potter, IEEE Trans. Magn. 11 (1975) 1051.

Quiz:

(1) Explain the mechanism to get the readback voltage from the written information?

(2) What is pulse width? What are the parameters required to achieve a narrow pulse width?

(3) Describe the different types of losses controlling the readback voltage?

(4) How does the readback voltage in the magnetoresistive based head enhance as compared to inductive head?