Tutorials

Here, we will consider some problems which can be analysed in terms of the basic elements of quantum mechanics.

8.1 Tutorial 1: Some preliminaries

1. In a photo-dissociation process, an incoming photon of frequency ν is absorbed by a positronium,
and the electron is emitted at an angle θ with the direction of the incoming photon.
What are the possible moments of the electron? Treat the electron and the positron non-relativistically.
2. Obtain the final frequency in Compton scattering by considering the electron non-relativistically.
Carry out an expansion in inverse powers of c to obtain the usual expression for Compton shift.
3. Carry out Planck's analysis for radiation confined to a plane. What is the energy density per unit area,
per unit wavelength? For what values of the wavelength is this a maximum? What is the total energy per unit area?
4. For a particle with charge q, moving in the presence of an external magnetic field, the canonical momentum
is $m\vec{v} - \frac{q}{2}\vec{r} \times \vec{B}$. Use Bohr's quantization condition to obtain the energy levels of a particle with charge q in the presence of $\vec{B}$.

8.2 Tutorial 2: Elements of Quantum mechanics

1. For a hydrogen atom with wave function

$$\psi = A\, r\, sin\theta\, e^{-i\phi} e^{-r/2r_0}$$

normalise the wave function and consider the total current across the plane $x = 0,\ z = (0, \infty)\ y = (0, \infty)$.

Obtain the wave function in the momentum space.
2. For a particle described by a wave function

$$\psi(x) = A\, x\, e^{-|x|/a}, \quad a > 0,$$

in a potential $V(x)$ which vanishes at infinity, obtain the energy and the potential, and calculate the average values of $|x|,\ 1/|x|$. Obtain the wave function in the momentum space and calculate the average values of p and p^2. Relate the average kinetic energy and the potential energy.

3. In the 3-dimensional vector space, write down a complete set of orthonormal basis vectors $a_i,\ i = 1, 2, 3$.

Show that they satisfy the closure property. In this basis, consider an operator A, with $Aa_1 = a_1 + a_2,\ Aa_2 = a_3,\ Aa_3 = 0$.

What are the eigenvalues and eigenvectors of A? Determine the eigenvalues and eigenvectors of $(A - A^{+})/2$.

4. Normalize the 3-dimensional, particle wave function $1/(r^2 + a^2)$. Obtain the average values of $r,\ p^2$.

What is the probability that the particle is found in the region ?

$$r > a$$

5. Which of the operators $\vec{r}$, $\vec{p}$, $\vec{r}\cdot\vec{p}$, $\vec{r}\times\vec{p}$, p^2 are observables? Which of them commute with the Hamiltonian of a particle with charge q, in the presence of a magnetic field in the z direction?

Write down the function which is the eigenfunction of p^2 and $\vec{p}$, and of p^2 and parity operator π.

8.3 Tutorial 3: Problems in 1-D

1. Obtain the wave function which has the minimum value for the product $\sigma_x\sigma_p$.

2. For a particle of mass m in a potential

$$\begin{aligned} V(x) &= -Z\delta(x), \quad Z > 0, \quad x < a, \ a > 0 \\ &= \infty \quad \text{for } x > a, \end{aligned}$$

what is the minimum value of Z for which a bound state exists? For what value of Z is there a bound state with energy $E = -\hbar^2/2ma^2$? For this case, obtain the average value of x.

3. A particle of mass m in a box with potential $V(x)$,

$$\begin{aligned} V(x) &= 0 \quad \text{for } 0 < x < a \\ &= \infty \quad \text{for } x < 0 \text{ or } x > a, \end{aligned}$$

is described by the wave function

$$\psi(x,0) = A sin\frac{(m+n)\pi x}{2a}\, cos\frac{(n-m)\pi x}{2a}$$

where m and n are integers. Obtain the average values of H and x as functions of time.

4. For a particle of mass m described by the potential in

$$H = \frac{1}{2m}p^2 + \frac{1}{2}kx^2 + bx,$$

consider

$$a = (\frac{m\omega}{2\hbar})^{1/2}[x + \frac{i}{m\omega}p + \frac{b}{k}],$$

and express H in terms of a, a^+. Obtain the expressions for $[a, a^+]$, $[a, H]$, and eigenvalues and eigenfunctions of a.

5. Obtain the bound state energies and wave functions for a particle of mass m in a potential

$$V(x) = -\frac{Z}{|x|} + \frac{a}{x^2}, \quad Z > 0, \quad a > 0.$$

Obtain the average value of $1/|x|$ for the ground state.

8.4 Tutorial 4: Problems in 2-D and 3-D

1. For a power-law potential,

$$V(r) = Z\, r^n,$$

obtain the dependence of the energy, its eigenfunctions, and the average values of r, on mass m, Z, n.

2. For a particle of mass μ in a two-dimensional potential

$$V(r) = -\frac{Z}{r} + \frac{c}{r^2},$$

obtain the lowest energy eigenvalue for a given angular momentum quantum number m, and the corresponding normalized wave function. Calculate the average values of $1/r$, $1/r^2$, and use the virial theorem to obtain the average kinetic energy for the state.

3. For a particle of mass m described by the two-dimensional potential

$$V(r) = \frac{1}{2}kr^2[3 + cos(2\phi)] + brcos(\phi),$$

use the creation and annihilation operators to write down the normalized wave function for the two lowest energy eigenstates. Calculate the average value of y for the normalized state

$$\psi = \frac{1}{\sqrt{2}}(|\psi_0> + |\psi_1>)$$

as a function of time.

4. For a particle of mass m in a 3-D potential

$$\begin{aligned} V(r) &= \infty \quad \text{for } r < R, \\ &= -V_0 \quad \text{for } R < r < R + a, \\ &= 0 \quad \text{for } r > R + a, \end{aligned}$$

obtain an implicit expression for the bound state energies for the $l = 0$ states.

5. Obtain the wave functions and bound state energies for a 3-D potential

$$V(r) = \frac{1}{2}kr^2 + a/r^2.$$

For the lowest energy l state, calculate the average values of r^2 and $1/r^2$. Use the virial theorem to obtain the average kinetic energy and verify the result by direct calculation.

6. An electron in a Coulomb potential is described by the wave function

$$\psi(\vec{r}, t) = A[f_0(t)e^{-r/r_0} + f_1(t)\frac{z}{r_0}e^{-r/2r_0}]$$

with $f_0(0) = f_1(0) = 1$. Determine the normalization constant A, and obtain the average values of z, r, L_x and L^2 as functions of time.

7. For an electron in a strong magnetic field in the z direction, evaluate the commutators $[H, \vec{L}]$, $[H, \vec{p}]$.

Which observables are constant in time?