Self Assessment - Module 3

- Consider a particle in a one-dimensional harmonic potential with Hamiltonian
$$H = \frac{p^2}{2m} + \frac{1}{2} m\Omega^2 x^2 \tag{1}$$
Compute the linear response of the particle coordinate $\langle x\rangle$ due to an oscillating electric field that couples to the particle via a perturbation term in the Hamiltonian:
$$H_1 = -Ex\cos(\omega t) \tag{2}$$
Express the result in the language of the response function R_{xx} as in Lectures 4 and 5.

- Prove the identity
$$\langle \vec{N}|\vec{S}|\vec{N}\rangle = \frac{\vec{N}}{2} \tag{3}$$
for spin-half coherent states $|\vec{N}\rangle$ as defined in Lecture 7.

- Prove the identity
$$\vec{S} = \frac{3}{2}\int \frac{d\vec{N}}{2\pi} \vec{N}|\vec{N}\rangle\langle\vec{N}| \tag{4}$$
for spin-half coherent states $|\vec{N}\rangle$ as defined in Lecture 7.

- Consider a system of four spin half moments with Hamiltonian
$$H = J_2\vec{S}_2\cdot\vec{S}_3 + J_1\vec{S}_1\cdot\vec{S}_2 + J_3\vec{S}_3\cdot\vec{S}_4 \tag{5}$$
Assume $J_2 \gg J_1$ and $J_2 \gg J_3$, and work in perturbation theory in the ratios J_1/J_2 and J_3/J_2

  a) At zeroth order in perturbation theory, sketch the energy levels of the system, and show that there are 4 degenerate lowest-energy states at zeroth order in perturbation theory

  b) Working to second order in degenerate perturbation theory, compute the effective Hamiltonian whose spectrum correctly reproduces the energies of these 4 lowest states to second order in perturbation theory.