Introduction to Physics of Nanoparticles and Nano structures

1. What is the coherent electron tunneling through a generic tunneling barrier. Show that the total coherent tunneling charge current density across a biased tunneling barrier is

$$j_{Total} = -\frac{em_{||}^*}{2\beta\pi^2\hbar^3}\int_0^\infty dE_z T(E_z)\ln\left[\frac{1+e^{\beta(E_{FL}-E_z)}}{1+e^{\beta(E_{FL}-eV-E_z)}}\right],$$

the well known *Tsu-Esaki*-formula and the ln-term is the usually called *Supply Function.*

2. Consider a sample of a small quantum wire structure with only one sub-band (channel) occupied. Ideal (i.e., resistance free) conducting leads (A and B, with electron chemical potentials or quasi Fermi energies μ_A and μ_B respectively) connect the quantum wire sample to the source electrode 1 on the left and drain electrode 2 on the right having quasi Fermi energies μ_1 and μ_2 respectively. Show that the current from left (source) to right (drain) for small bias, is

$$I = \frac{2e}{h}T(\bar{\mu})(\mu_1 - \mu_2),$$

where $\bar{\mu} = (\mu_1 + \mu_2)/2$ and T is the transmission coefficients due to scattering by the quantum wire sample, which is regarded as a barrier.
As a result of transmission and reflection about a barrier, there is a reduction in carrier density on the left of the barrier (i.e., the sample), and a pile up of charges on the right side, when a forward bias is applied. If this charge is approximated by an average density, then the actual voltage drop across the sample is given by $eV = \mu_A - \mu_B < \mu_1 - \mu_2$, so that the difference between $(\mu_A - \mu_B)$ and $(\mu_1 - \mu_2)$ will appear as the potential drop at the contacts. From such a consideration show that in the low temperature limit,

$$\mu_A - \mu_B = [1 - T](\mu_1 - \mu_2),$$

where $T \equiv T(\bar{\mu})$ with $\bar{\mu} = (\mu_1 + \mu_2)/2 \approx (\mu_A + \mu_B)/2$.
Then show that the conductance is given by

$$G_4 = \frac{I}{V} = \frac{2e^2}{h}\frac{T}{1-T}, \qquad \text{(Four − probe formula)},$$

a result relevant for four probe measurement.

3. It may happen that even at low temperature, there are N number of 1d sub-bands (modes) which are populated at the Fermi energy, all of which contribute to the current. Extend the single channel result to the multichannel case with appropriate assumptions (number of channels on left same as that on right) or approximations to show that
(a) when the current and voltage are measured independently as done in a four terminal measurement, the conductance is

$$G_4 = \frac{I_{Total}}{V} = \frac{2e^2}{h}\frac{2\sum_{n=1}^N T_n}{1+\frac{\sum_{n=1}^N [R_n - T_n]v_n^{-1}}{\sum_{n=1}^N v_n^{-1}}},$$

(b) if one performs a two terminal measurement, such that one measures the potential drop across bo the structure and the leads, then the conductance is

$$G_2 = \frac{2e^2}{h} \sum_{n=1}^{N} T_n.$$

References:

1. Transport in Nanostructures, David K. Ferry and Stephen M. Goodnick, (Cambridge University Press, Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, Sao Paulo, 1997, paperback edition 1999, reprinted 2001)

*** END ***