Introduction to Physics of Nanoparticles and Nano structures

Part II: Physics of Nanostructures **Questions on Module 5**

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1. Consider the conduction band of an unbiased GaAs-Al$_x$Ga$_{1-x}$As-GaAs heterostructure, which forms a rectangular barrier of height $V_o = \Delta E_C$ (band offset) and width $w = 2a$, within the effective mass approximation.
(a) Using the transfer matrix method, show that at energy E (measured from E_C) below V_o, i.e., for $E < V_o$ the transmission coefficient is given by

$$T(E) = \frac{\text{transmitted electron flux}}{\text{incident electron flux}},$$

$$= \left[1 + \left(\frac{k^2+\gamma^2}{2k\gamma}\right)^2 \sinh^2(2a\gamma)\right]^{-1},$$

where $k = \sqrt{2m^*E}/\hbar$ and $\gamma = \sqrt{2m^*(V_o - E)}/\hbar$.
(b) Show that for $E > V_o$,

$$T(E) = \left[1 + \left(\frac{k^2-k_1^2}{2kk_1}\right)^2 \sin^2(2ak_1)\right]^{-1},$$

where $k = \sqrt{2m^*E}/\hbar$ and $k_1 = \sqrt{2m^*(E - V_o)}/\hbar$.
(c) Derive the expression for the reflection coefficient defined by

$$R(E) = \frac{\text{reflected electron flux}}{\text{incident electron flux}},$$

and thereby show that $R(E) + T(E) = 1$.

2. Consider now a biased GaAs-Al$_x$Ga$_{1-x}$As-GaAs heterostructure, which forms a barrier of width $w = 2a$, that may be approximated by an asymmetric barrier of height $V_o = \Delta E_C$ (band offset) on the left and height $V_o + V_1$ on the right (as E_C goes down by $-V_1$ on the right).
(a) Using the transfer matrix method, show that the transmission coefficient for transmission from left to right for $E < V_o$ the transmission coefficient is given by

$$T_{(l\to r)}(E) = \frac{4kk_1}{(k+k_1)^2}\left[1 + \frac{(k^2+\gamma^2)(k_2^2+\gamma^2)}{(k^2+k_2^2)\gamma^2}\sinh^2(2a\gamma)\right]^{-1},$$

where $k = \sqrt{2m^*E}/\hbar$ and $\gamma = \sqrt{2m^*(V_o - E)}/\hbar$ and $k_2 = \sqrt{2m^*(E + V_1)}/\hbar$.
(b) Show that the transmission coefficient from right to left is same as that for left to right, i.e.,

$$T_{(l\to r)}(E) = T_{(r\to l)}(E),$$

even though the barrier is asymmetric. (c) Derive the expression for $T(E)$ when $E > V_o$,

3. Consider an unbiased GaAs-Al$_x$Ga$_{1-x}$As-GaAs-Al$_x$Ga$_{1-x}$As-GaAs heterostructure, which forms two rectangular barriers of identical height V_o; the width of the barriers may be different and let they be $2a_L$ and $2a_R$ for the left and right barriers and the separation between the two barriers be b.
   (a) Using the transfer matrix method, show that for identical barriers (i.e., $a_L = a_R = a$) the total transmission coefficient of the double barrier for $E < V_o$ is,

$$T_{total}(E) = \frac{T_1^2}{T_1^2 + 4R_1 \cos^2(kb - \theta)},$$

   where, $k = \sqrt{2m^*E}/\hbar$ and $\gamma = \sqrt{2m^*(V_o - E)}/\hbar$, $\theta = 2ka - \tan^{-1}\left[\frac{k^2-\gamma^2}{2k\gamma}\tanh(2a\gamma)\right]$, T_1 is the transmission of a single barrier, and $R_1 = 1 - T_1$
   (b) Determine the maximum and minimum values of $T_{total}(E)$ and the condition for resonant tunneling states or the quasi bound states.
   (c) Show that around a quasi bound state E_n the total transmission coefficient of the double barrier or the resonant tunnel diode can be approximated as

$$T_{total}(E \sim E_n) \approx \frac{\pi}{2}\Gamma_n \delta(E - E_n),$$

   and in general

$$T_{total}(E) \approx \frac{\pi}{2}\sum \Gamma_n \delta(E - E_n),$$

   where $\Gamma_n = \left[2\hbar^2 E_n T_1^2/(m^* b^2 R_1)\right]^{1/2}$.

4. Consider a biased double barrier. Then using the transfer matrix method for biased barriers, determine the total transmission coefficient of the double barrier.

References:
1. Transport in Nanostructures, David K. Ferry and Stephen M. Goodnick, (Cambridge University Press, Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, Sao Paulo, 1997, paperback edition 1999, reprinted 2001)

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