

Introduction to Physics of Nanoparticles and Nano structures

Part II: Physics of Nanostructures

Questions on Module 3

1. Consider the Drude model for semi-classical electron transport (in 3-dimensional bulk). Derive the expression for mobility μ in terms of the average momentum relaxation time $\langle \tau_m \rangle$. Estimate μ for $m^* \sim m_e/10$ and $\tau_m \sim 10^{-13}$ sec.
2. Consider the semi-classical electron transport due to a small electric field along the z-direction. Using the simple Boltzmann Transport Equation (BTE) with single relaxation time approximation, show that the average momentum relaxation time $\langle \tau_m \rangle$ is determined by

$$\langle \tau_m \rangle = \frac{\beta m^* \int d^3v \tau_m(\vec{v}) v_z^2 f_o(\vec{v})(1 - f_o(\vec{v}))}{\int d^3v f_o(\vec{v})},$$

where $\beta = 1/(k_B T)$, $f_o(\vec{v})$ is the equilibrium electron distribution given by the Femi-Dirac distribution, and the integrations are over the infinite velocity space.

For an isotropic case, where $\tau_m(\vec{v}) = \tau_m(|\vec{v}|)$, simplify the above expression and then assuming $\tau_m(|\vec{v}|) = \tau_o(\beta\epsilon)^r$, show that

$$\langle \tau_m \rangle = \frac{4\tau_o}{3\sqrt{\pi}} (r + 3/2)! \frac{F_{r+1/2}(x_F)}{F_{1/2}(x_F)},$$

where $x_F = \beta\epsilon_F$ and $F_j(x)$ are the so called Fermi integrals defined as

$$F_j(x) = \frac{1}{j!} \int_0^\infty \frac{y^j}{e^{y-x} + 1} dy.$$

Obtain the expressions for $\langle \tau_m \rangle$ in the two opposite limits, $T \rightarrow 0$ and $T \rightarrow \infty$.

3. Using the linearized BTE for a quasi 2-DEG system, and assuming that the scattering rate is elastic, i.e., $\epsilon(\vec{k}') = \epsilon(\vec{k}) = \epsilon$ (say), show that the relaxation time for nth sub-band is given by

$$\frac{1}{\tau_n(\epsilon)} = \sum_m \sum_{\vec{k}'} S_{nm}(\vec{k}, \vec{k}') \left[1 - \frac{\tau_m(\epsilon)}{\tau_n(\epsilon)} \cos \theta_{\vec{k}\vec{k}'} \right],$$

where all quantities have their usual meaning.

Finally assuming $S_{nm} \approx 0$ for $n \neq m$, and $\tau_n = \tau_{no}(\beta\epsilon)^r$ obtain the general expression for $\langle \tau_n \rangle$, and its form in the two extreme temperature limits.

4. Using Fermi's Golden rule for the scattering rate for scattering from charged impurities, show under appropriate assumptions, that for a 2-DEG system $\tau_n(E)$ is proportional to E .
5. Using Fermi's Golden rule for the scattering rate for scattering from roughness at the interface, under appropriate assumptions for a 2-DEG system obtain an expression for $\tau_n(E)$.
6. Using Fermi's Golden rule for the scattering rate for scattering from charged impurities, under appropriate assumptions for a quantum wire system obtain an expression for the scattering rate S_{nn} .

References:

1. Physics of Semiconductor Devices, S.M.Sze and Kwok K. Ng, Third Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2007.
2. Transport in Nanostructures, David K. Ferry and Stephen M. Goodnick, (Cambridge University Press, Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, Sao Paulo, 1997, paperback edition 1999, reprinted 2001).

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