Introduction to Physics of Nanoparticles and Nano structures

Part I: Physics of Nanoparticles **Questions on Module 7**

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1. Consider a spherical particle of radius R and dielectric constant $\epsilon(\omega)$ in a nonabsorbing medium of dielectric constant ϵ_m (assume $\mu = 1$ for all the media).

   (a) Show that the condition of exciting a well defined dipolar surface mode (to be referred to as Fröhlich mode) of the particle (with very small size parameter) is

   $$\epsilon'(\omega) = -2\epsilon_m \qquad \text{and} \quad \epsilon''(\omega) \approx 0.$$

   (b) Show that at Fröhlich mode frequency ω_F the absorption efficiency Q_{abs} for the particle is

   $$Q_{abs}(\omega_F) = \frac{12x\epsilon_m}{\epsilon''(\omega_F)}.$$

   (c) Show that if the size parameter x is not small, then to order x^2, the condition of exciting a well defined Fröhlich mode of the particle is modified to

   $$\epsilon'(\omega) = -(2 + \frac{12}{5}\, x^2)\, \epsilon_m \qquad \text{and} \quad \epsilon''(\omega) \approx 0.$$

   (d) If the dielectric function of the particle is described by a single oscillator Lorentz model for lattice vibration, find the Fröhlich mode frequency of a small spherical particle.

   (e) Find the Fröhlich mode frequency of a small metallic sphere using the Drude model for its dielectric function with plasma frequency ω_p and damping constant γ_p.

   (f) Find the Fröhlich mode frequency of a small coated sphere (this could be used as a model for C_{60} molecule).

2. Consider an ellipsoidal particle of dielectric constant $\epsilon(\omega)$ in a nonabsorbing medium of dielectric constant ϵ_m (assume $\mu = 1$ for all the media). The effective polarizability of the particle along one of its principal axis is

   $$\alpha = v_p\, \frac{(\epsilon - \epsilon_m)}{\epsilon_m + L(\epsilon - \epsilon_m)},$$

   where v_p is the particle volume and L is the geometric factor.

   (a) If the particle is metallic then, using the Drude model with plasma frequency ω_p and damping constant γ_p, find its Fröhlich mode frequency ω_F.

   (b) Show that for small damping constant ($\gamma_p \ll \omega_p$), the maximum value of $C_{abs}(\omega)$ is

   $$C_{abs}^{max} = \frac{v_p}{c}\,\frac{\omega_p^2}{\gamma_p} f(\epsilon_m, L),$$

   where, c is the speed of light in vacuum and

   $$f(\epsilon_m, L) = \frac{\epsilon_m^{3/2}}{[\epsilon_m + L(1 - \epsilon_m)]^2}.$$

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