Introduction to Physics of Nanoparticles and Nano structures[1]

Part I: Physics of Nanoparticles[2] **Questions on Module 1**

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1. Starting from the basic laws of electricity magnetism, *i.e.*,

$$\text{Coulomb's law}: \qquad \vec{\nabla}\cdot\vec{D} = \rho_{\text{free}},$$

$$\text{Absence of magnetic monopoles}: \qquad \vec{\nabla}\cdot\vec{B} = 0,$$

$$\text{Ampere's law}: \qquad \vec{\nabla}\times\vec{H} = \vec{j}_{\text{free}},$$

$$\text{Faraday's law}: \qquad \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0,$$

obtain the four Maxwell's equations, after appropriate modifications. Which of the four fields $\vec{D}$, $\vec{E}$, $\vec{B}$ and $\vec{H}$ are external (*i.e.* can be controlled externally)?

2. Assuming the fields to be time harmonic of the form $\vec{F}(\vec{x},t) = \vec{F}(\vec{x},\omega)e^{-i\omega t}$, obtain the Maxwell's equations for the fields $\vec{D}(\vec{x},\omega)$, $\vec{B}(\vec{x},\omega)$, $\vec{E}(\vec{x},\omega)$, and $\vec{H}(\vec{x},\omega)$. What are *Constitutive relations*? Use the constitutive relations to define complex permittivity $\epsilon(\omega)$.

3. For a *uniform isotropic linear and unbounded* medium containing no external charge or current, show that the electric field $\vec{E}(\vec{x},\omega)$, and the magnetic field $\vec{H}(\vec{x},\omega)$ obey the Helmholtz equation (*i.e.*, the wave equation in frequency space) of the form

$$\nabla^2\vec{F}(\vec{x},\omega) + \omega^2\mu\epsilon\vec{F}(\vec{x},\omega) = 0.$$

4. For the previous problem, try the plane wave solutions of the form $\vec{E}(\vec{x},t) = \vec{E}_o e^{i(\vec{q}\cdot\vec{x}-\omega t)}$ and $\vec{H}(\vec{x},t) = \vec{H}_o e^{i(\vec{q}\cdot\vec{x}-\omega t)}$, to show that these are indeed solutions of the wave equations. When are the wave solutions *homogeneous* and when *inhomogeneous*? Show that for homogeneous waves, $\vec{E}_o$, $\vec{H}_o$ and $\vec{q}$ are perpendicular to each other. Relate the *phase velocity* $\vec{v}$ with ω and $\vec{q}$ with the *complex refractive index* $N(\omega)$.

5. In the last problem, writing $N(\omega) = n + ik$, show that for homogeneous waves, the waves are attenuated as

$$I = I_o e^{-\alpha z}, \qquad z = \vec{q}\cdot\vec{x},$$

where I is the *irradiance* (commonly known as intensity) defined as the magnitude of the *time averaged Poynting vector*, *i.e.*, $I = |\langle\vec{S}\rangle| = \frac{1}{2}|Re(\vec{E}\times\vec{H}^*)|$. Determine the *absorption coefficient* α in terms of k.

6. Show that the most general homogeneous plane wave is a linear combination of two *linearly polarized* homogeneous plane waves having the same propagation vector.

7. Prove *Kramer-Kronig's relations*, in general, for the real and imaginary parts of a complex function $\chi(\omega)$. What should be the general properties of $\chi(\omega)$ for such relations to exist? Use them to obtain the Kramer-Kronig's relations between n and k (consult Jackson).

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[2]References : Jackson J D, Classical Electrodynamics, Third Edition, John Wiley & Sons, Inc, 2004.
Bohren C F and Huffman D R, Absorption and Scattering of Light by Small Particles, Wiley Interscience Paperback, 1998.