MODULE 1

STATISTICAL AND SPECTRAL ANALYSIS OF RANDOM WAVES

1.0 INTRODUCTION

Looking at the realistic sea surface, it is composed of number of waves with different directions, frequencies, phases and amplitudes. For an adequate description of sea surface, understanding of random wave process and analysis are important.

Real field waves

Idealisation into linear superposition of harmonic waves for all practical purposes with discrete in space.

On the assumption of a random wave field follows stationary and ergodic process, the spatial distribution of wave profile is idealized into time series of sufficiently long, however, enough for the analysis.

The stationary property implies the invariant statistical properties if the time window is moving over a random process. And, the probability distribution is same for any sufficiently loner time window over the period of interest.

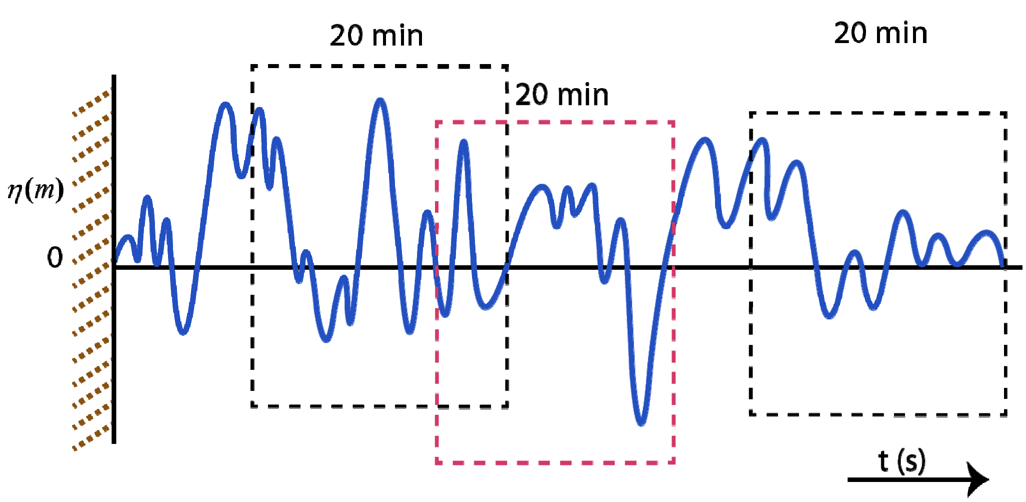
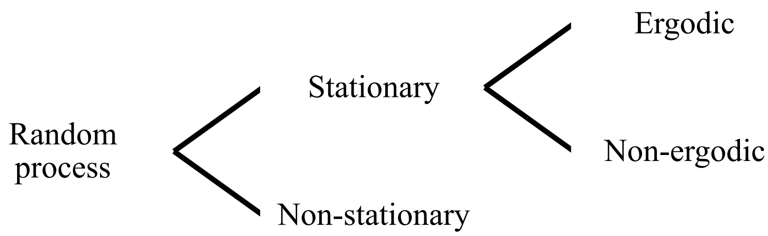


Fig. 1. A typical wave record.

In real ocean wave field, a typical measurement of 20 min. record is sampled to represent a wave field of 3-hr (Fig. 1). The time averaged statistics over 20min time record is equal to the time averaged statistics of every 20min records within the 3-hr time interval. And, in addition, the event averaged statistics (at a particular time frame across all the 20min records) is also equal to the time averaged statistics of one frame following the ergodicity. If the stationary, ergodicity process is not valid, one has to measure for a longer period to take statistical averages to represent a wave climate for 3-hr intervals.



Note some keywords:

Ensemble: Total collection of samples

Sample: Each record

Stationary: All the ensemble average such as mean, standard deviation etc., are independent of absolute time.

Weakly stationary: If only first and second order probability distributions are same

Ergodic: Ensemble average (at any time, t_1) is equal to time average of any record and also time average is same for all records (Fig. 2).

Expected value of x(t) = $\overline{x(t)} = \langle k(t) \rangle$

Average or mean value of a random process:

$$E\left[x(t)\right] = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)dt$$

Mean square value,

$$E\left[x^2(t)\right] = \overline{x^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2 dt$$

Variance σ^2 = mean square value about the mean

$$= \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(x - \overline{x}\right)^2 dt$$

$$\sigma^2 = \overline{x^2} - \left(\overline{x}\right)^2$$

σ – Standard deviation- measure of the spread about the mean.

[small 'σ' ⟶ narrower probability curve, p(x)]

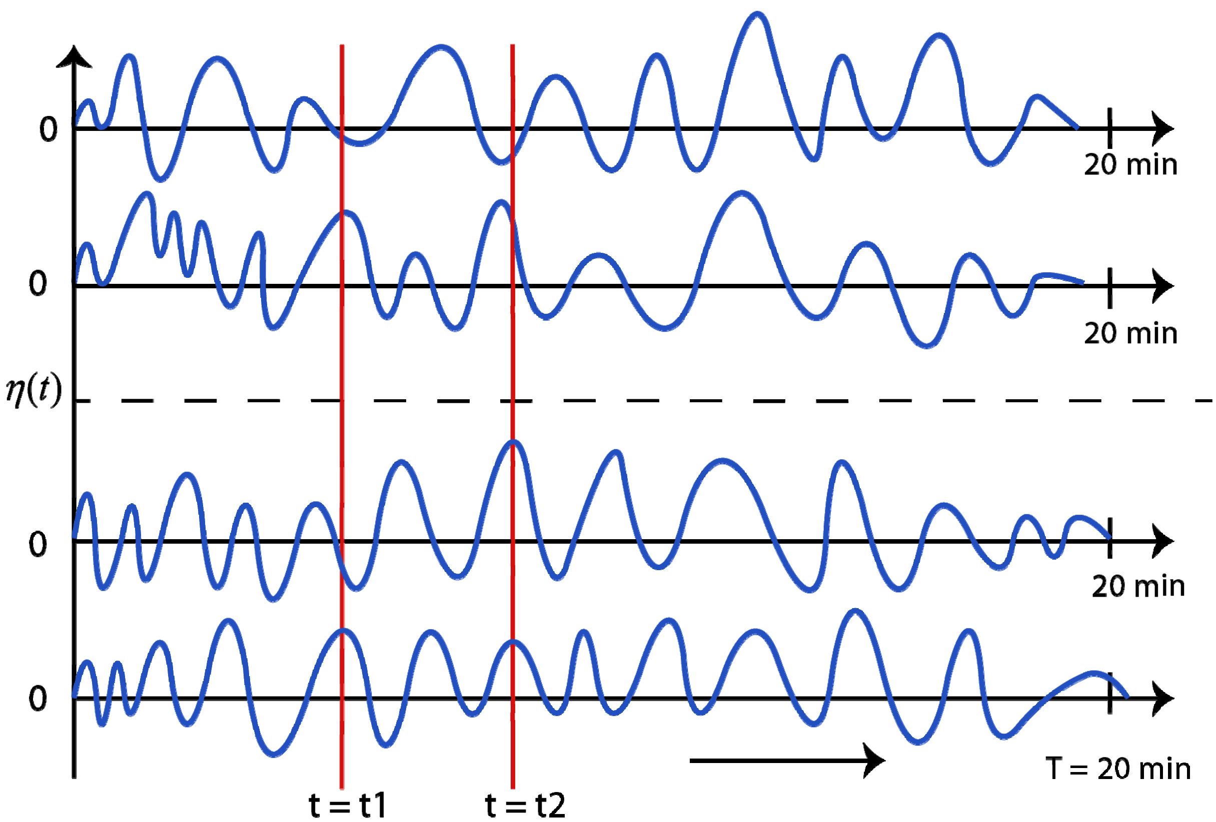


Fig. 2. Discrete 20min wave records from Fig.1.

A simple approach to represent such independent identically distributed random events is the concept of the spectrum of ocean waves. The spectrum gives the distribution of wave energy among different wave frequencies (wave-lengths) on the sea surface. Let us concentrate on statistical and spectral analysis of a random wave process.

Statistical and Spectral Analysis

The random process such as a random wave can be analysed either in the time domain or frequency domain. The assumption of linear superposition (and hence, the process is assumed to be linear) makes a good correlation between two types of analysis such as statistical (time domain) and spectral (frequency domain) analyses.

2.0 STATISTICAL ANALYSIS

It is the direct analysis without subjecting the time series into any conversion process. Hence, it is valuable and can be taken as primary information as far as the physical representation along the time scale and the ordinate scale (it can be wave elevation, pressure, etc.).

See Fig. 3, if we measure the length scale of the time series of consecutive waves, we can have,

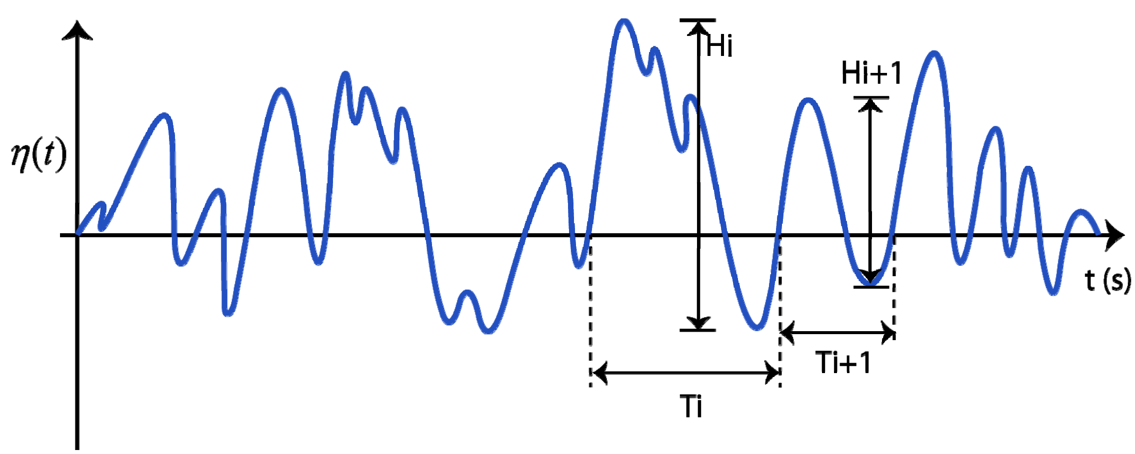


Fig. 3. Representation of individual wave record within the time series.

Along the horizontal time scale, T_1, T_2, ..., T_n for 'n' number of waves.

Along the vertical wave elevation scale, H_1, H_2,...,H_n for 'n' number of waves.

In the above, how *n* is defined? See again in the above figure, the origin of i^{th} wave is defined from the point where the time series is crossing the mean value (here, zero for all the zero mean process) and the slope of time series has positive value at the origin of i^{th} position. This is called up cross analysis.
On the other hand, if our starting point of the time series is such that at the zero crossing, at the mean value, the progressive slope is negative, then it is called down cross analysis.

These are two types of analysis and which one to choose depends on the variable under consideration. For the wave surface elevation, if we observe the time progress of the event, $\eta(t)$, the wave progress from right to left and our time scale is positive from left to right. Hence, in the real field (on our assumption of a conventional wave, crest followed by a trough), the trough accompanying a crest cross the 'time' step forward than the following crest. Hence, downcross analysis is preferred. However, if the convention is different, the up-cross analysis can be adopted.

Now, follow the time series, and pick up the sequences of values, H_i & T_i and we have almost all the information on the time series.

We have a range of values for H & T with two extremes of each.

But the questions in the mind of the engineer or navigator are,
'What is the value to be taken among these?
Shall I take the mean of the above group?', Or,
Shall I pick the maximum value?'
There should not be any ambiguity between two users who want to get the wave field estimate at a same location.

So, it is important to define a unique parameter which is also matching with a visual field observer (good navigator). Always, a visual observer makes a bias in the estimation of the wave height above its mean value? The estimate of the visual observer in general is found to be correlated with the average of the 1/3 rd of the largest wave heights among the group. Hence, for the generality condition, a definition for this value is defined as 'significant wave height' which is defined as the average of the highest one third of the waves.

i.e., Rewrite the group of values, H_i in descending order of n values. Then,

Significant wave height, $H_{1/3} = \frac{\sum_{i=1}^{n/3} H_i}{n/3}$, (1)

where, H_i's are listed in descending order of its magnitude.

It is the most concern for an engineer to find the maximum value and other statistical properties useful for design and operational purposes.

Different statistical properties can be defined as follows from the list of descending order.

Maximum wave height, $H_{max} = H_1$ (2)

Average of highest 0.2% waves, $H_{1/500} = \frac{\sum_{i=1}^{n/500} H_i}{n/500}$ (3)

Average of highest one-hundredth, $H_{1/100} = \frac{\sum_{i=1}^{n/100} H_i}{n/100}$ (4)

Average of highest one-tenth, $H_{1/10} = \frac{\sum_{i=1}^{n/10} H_i}{n/10}$ (5)

Mean wave height, $\overline{H} = H_{av} = \frac{\sum_{i=1}^{n} H_i}{n}$ (6)

The above calculations require a long time series of sufficient records, i.e., to say to calculate $H_{1/500}$ one should have more than 1000 records in the series. What should be the optimum value to find an average? In general, the time series with a record length of 3000 is required.

Now, if you take the typical record of 20min to represent a wave climate, say with an average wave period of 6s in that location, we can have about 200 records in the series. This is not sufficient to adopt in the above form of calculations.

The distribution of wave heights, H_i is found to follow Rayleigh probability distribution which addresses our concern for the estimate. Following Rayleigh distribution, the various statistical parameters can be estimated from the characteristic wave height, i.e., significant wave height, $H_{1/3}$.

Average of highest 0.2% waves, $H_{1/500} = 1.91 H_{1/3}$ (7)

Average of highest one-hundredth, $H_{1/100} = 1.67 H_{1/3}$ (8)

Average of highest one-tenth, $H_{1/10} = 1.27 H_{1/3}$ (9)

Mean wave height, $H_{av} = 0.63 H_{1/3}$ (10)

Since, the Rayleigh distribution has no upper bound, the maximum wave height, H_{max} could not be estimated from the characteristic estimate.
An approximate estimate, however, then can be given as,
Maximum wave height, $H_{max} = 2\ H_{1/3}$. (11)
Another salient parameter, a design engineer looking for is the root mean square wave height (H_{rms}),

$$H_{rms} = \sqrt{\frac{\sum_{i=1}^{n} H_i^2}{n}} \tag{12}$$

Similarly, third and fourth order statistical parameters such as skewness and kurtosis can be evaluated to explore the nonlinearity in the random signal. The analysis of a nonlinear signal will be dealt later.
The above process can be used for any independent variable of interest. However, depending on the distribution of variables, the fitting coefficients in Eqs.(7) to (11) have to be carefully chosen. For eg., the wave surface elevation, η(t) follows Gaussian distribution and extreme values of wave heights for long term statistics follow Weibull or Gumbull distribution.

Exercise: Write a MATLAB code to derive the statistical parameters using Eqs.(1) to (6) & (12) from a given time series. Adopt down-cross analysis. Compare the parameters with the estimate made from Rayleigh distribution (Eqs.(7) to (11)).

Given time series: timeser1.dat

Note: In the first step, make the random process 'zero mean'.

Wave period:

Similar to the concept in deriving statistical parameters for the wave height, the characteristic wave period can be derived. Some of the details will be dealt in the next section.

3.0 SPECTRAL ANALYSIS

Even though statistical analysis provides a comprehensive direct data analysis, the information on the distribution of concentration of wave energy at different frequency bands is lacking. It is particularly important, if the offshore system under design has natural frequency of the same order as the wave frequency in which the maximum energy concentrates. This is supplemented by the frequency domain analysis by decomposing the time series into various frequency components using Fourier Transform.

In simple terms, a wave spectrum is the distribution of wave energy as a function of frequency. It describes the total energy transmitted by a wave-field at a given time. Many times a transformation is performed to provide a better or clearer understanding of such a phenomenon. The time representation of a sine wave may be difficult to interpret. By using a Fourier series representation, the original time signal can be easily transformed and much better understood.

Only a brief overview of Fourier Transform to obtain a frequency spectrum has been provided here. The salient aspects that need attention in the analysis, particularly to extract design parameters are dealt here.

Auto correlation function

For a random process, x(t), the auto correlation function is defined as the average value of the product of x(t) and x (t+τ).

$R_x(\tau) = E[x(t).x(t+\tau)] = < x(t)x(t+\tau) >$

If x(t) is stationary, E[x(t)] = E[x(t+τ)] = m

$\sigma_{x(t)} = \sigma_{x(t+\tau)} = \sigma$

Correlation coefficient $\rho = \dfrac{R_x(\tau) - m^2}{\sigma^2}$

3.1. Spectrum

The concept of a spectrum is based on work by Joseph Fourier (1768 – 1830), who showed that almost any function $x(t)$ over the interval ($-T/2 < t < T/2$) can be represented as the sum of an infinite series of sine and cosine functions with harmonic wave frequencies.

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos n\omega' t + b_n \sin n\omega' t\right) \qquad (13a)$$

where,

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos n\omega' t\, dt, \qquad \text{(n=0,1,2,…)} \qquad (13b)$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin n\omega' t dt, \qquad \text{(n=0,1,2,…)} \qquad (13c)$$

$\omega' = 2\pi f' = 2\pi/T$ is the fundamental frequency, and $n f'$ are harmonics of the fundamental frequency. This form of $x(t)$ is called a Fourier series, a_0 is the mean value of $x(t)$ over the interval.

The above equations can be written in complex form,

$\exp(i n\omega' t) = \cos(n\omega' t) + i\sin(n\omega' t)$ and

$$x(t) = \sum_{n=-\infty}^{\infty} Z_n \exp^{in\omega' t} \qquad (14a)$$

where

$$Z_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\exp^{-in\omega' t}\, dt, \qquad \text{(n=0,1,2,...)} \tag{14b}$$

Z_n is called the *Fourier transform* of $x(t)$

The spectrum $S_\eta(f)$ of $x(t)$ is:

$$S_\eta(f) = Z_n Z_n^* \tag{15}$$

where Z^* is the complex conjugate of Z. The computation of ocean wave spectra follows from these forms for the Fourier series and spectra.

Thus in complex form of a random signal, x(t) and its fourier transform can be written as below.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{i\omega t}\, d\omega$$

X(ω)-Fourier transform of x(t)

x(t) – Inverse Fourier transform of X(ω)

Note: The factor $1/2\pi$ can appear in any one of the above equation.

Spectral density

For a stationary process, x(t) goes on forever and the condition,

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty \text{ is not satisfied}$$

so that the theory of Fourier analysis cannot be applied to a sample function. This difficulty can be overcome by analyzing its auto correlation function $R_x(\tau)$; since,

$R_x(\tau \to \infty) = 0$ for non – periodic wave with zero mean process and the condition

$\int_{-\infty}^{\infty} |R(\tau)| dt < \infty$ is satisfied. Given a random process that is stationary and ergodic, with an expected value of zero and autocorrelation $\mathrm{R}(\tau)$, the power spectral density, or spectrum of the random process is defined as the Fourier transform of the autocorrelation.

$$S(\omega)=\int_{-\infty}^{\infty} R(\tau)e^{-i\omega\tau}d\tau \tag{16a}$$

Conversely, the autocorrelation, $R(\tau)$, is the inverse FT of the spectrum

$$R(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}d\omega \tag{16b}$$

Properties of the Spectrum S(ω) of η(x,t):

1. $S(\omega)$ is a real and even function, since $R(\tau)$ is real and even.

2. $$\int_{-\infty}^{\infty} R(\tau)e^{-i\omega t}d\tau = \int_{-\infty}^{\infty} R(\tau)\,(\cos\omega\tau - i\sin\omega\tau)d\tau \tag{17a}$$

   It can be shown that the sine component integrates to zero.

3. The variance of the random process can be found from the spectrum:

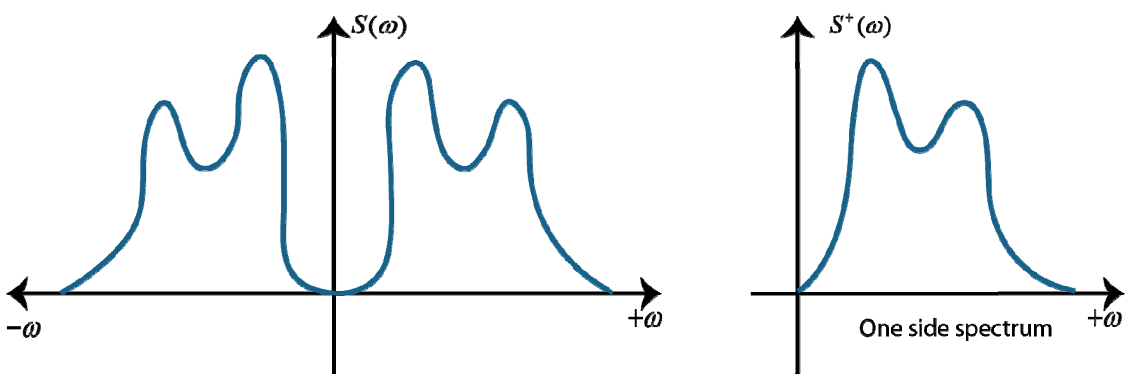
$$\sigma^2=(RMS)^2=R(0)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)d\omega \tag{17b}$$

4. The spectrum is positive always: $S(\omega)\geq 0$

5. With some restriction it can also be established that

$$S(\omega)=\lim_{T\to\infty}\left|\int_{-T}^{T} x(t,x_k)e^{-i\omega t}dt\right| \tag{17c}$$

A spectrum covers the range of frequencies from minus infinity to plus infinity ($-\infty<\omega<+\infty$).



S(ω) $S^+(\omega)$

Fig. 4. Spectral representation

A one-sided spectrum, $S^+(\omega)$, is a representation of the entire spectrum only in the positive frequency domain. It can be obtained by *folding* the energy over $\omega=0$ and introduce the $\frac{1}{2\pi}$ factor we get:

$$S^+(\omega) = \begin{cases} \frac{2}{2\pi} S(\omega) & \text{for } \omega \geq 0 \\ 0 & \text{for } \omega < 0 \end{cases} \tag{18}$$

This representation for the one-sided spectrum comes from the variance, R(0):

$$R(0) = \sigma^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega)\, d\omega = \frac{2}{2\pi} \int_{0}^{\infty} S(\omega)\, d\omega \tag{19}$$

which we can rewrite in terms of the one-sided spectrum

$$\sigma^2 = \int_0^{\infty} S^+(\omega)\, d\omega \tag{20a}$$

where, $S^+(\omega) = \frac{2}{2\pi} S(\omega);\ \text{for } \omega \geq 0$ (20b)

The spectrum provides a distributed amplitude, or "probability density" of amplitudes, indicating the energy of the system. Hereafter, $S(\omega)$ represents $S^+(\omega)$, i.e., '+' sign is conventionally not added.

We can expand the idea of a Fourier series to include series that represent surfaces $\eta(x, y)$ using similar techniques. Thus, any surface can be represented as an infinite series of sine and cosine functions oriented in all possible directions.

Considering the random process, $\eta(x,t)$ follows stationary and ergodic process, it is assumed that the expected value of the random process is zero. However this is not always possible. If the expected value equals some constant a_0, the random process can be adjusted such that the expected value is indeed zero,

$$\eta(x,t) = \eta(t,x) - a_0 \tag{21}$$

Note in our discussion of Fourier series that we assume the coefficients (an, b_n, Z_n) are constant. For times of perhaps an hour, and distances of perhaps tens of kilometers, the waves on the sea surface are sufficiently fixed that the assumption is true. Furthermore, non-linear interactions among waves are very weak. Therefore, we can represent a local sea surface by a linear super-position of real, sine waves having many different frequencies and different phases traveling in many different directions. The spectrum of the wave-height gives the distribution of the variance of sea-surface height at the wave staff as a function of frequency. Because wave energy is proportional to the variance of the spectrum, which is called the *energy spectrum* or the *wave-height spectrum*. Typically three hours of wave staff data are used to compute a spectrum of wave-height.

Window functions

In the above frequency representation of a typical time series, it is assumed that the series is continuous. However, in practice, there is a definite time step between successive data. The time step is small enough such that the event is presented as a smooth functional variation over time. This discrete nature of data forces to adopt discrete Fourier transform and in turn add furious noise in the estimate. In addition, a sudden initiation of the event (represented by the time series at the initial step, say t=0) and also, an abrupt end of the event induce higher frequency noises which is otherwise unwanted change in the energy level. This is avoided by introducing a *windowing function*.

In signal processing, a windowing function is a mathematical function that is zero-valued outside of some chosen compact interval. When another function or a signal (data) is multiplied by a window function, the product is also a window function: all that is left is the part where they overlap.

The following window functions, w(n) are commonly adopted for filtering furious noises.

- Cosine tappered window
- Hanning window
- Welch window

Cosine tapering

$$w(n) = 0.5\left(1 - \cos\left(\frac{\pi n}{M+1}\right)\right) \qquad 0 \le n \le M$$

$$w(n) = 1 \qquad M \le n \le (N - M - 2)$$

$$w(n) = 0.5\left(1 - \cos\frac{(N-n-1)\pi}{M+1}\right) \qquad (N - M - 2) \le n \le N-1$$

where, $M = INT\left[\frac{N-2}{10}\right]$

Hanning

$$w(n) = 0.5\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right) \qquad 0 \le n \le N-1$$

Welch

$$w(n) = \left(\frac{n - 0.5N}{0.5N}\right)^2$$

where, N is the number of time steps in the time series and M is the number of time steps to be windowing.

Spectral smoothening

Similar to the discrete time series, the derived frequency spectrum is also been specified in discrete frequency steps. This resulted in leaking of energy in between discrete steps. This can be rectified by smoothening the spectral curve. 5-point smoothening or higher order smoothening can be carried out to perform this task.

Statistics

Now, the statistics of $\eta(t)$ given by the spectrum $S_\eta(\omega)$ needs to be established. The wave heights, H_i and wave periods of interest T_i are the random variables in this problem. As time statistics are equal to the event statistics almost everywhere, if $\eta(t)$ is a realization of the random process $\eta(t,x)$ then ergodicity says that H_i and T_i will provide the statistics on $\eta(t,x)$ and vice versa.

Before defining the various statistics, let us define the moments of the spectrum as follows:

Zeroth Moment:

$$m_0 = \int_0^\infty S(\omega)d\omega = \sigma^2 = VARIANCE \tag{22a}$$

Second Moment:

$$m_2 = \int_0^\infty S(\omega)\omega^2 d\omega \tag{22b}$$

Fourth Moment:

$$m_4 = \int_0^\infty S(\omega)\omega^4 d\omega \tag{22c}$$

The root mean square wave height, H_{rms} or standard deviation, σ_o is given by,

$$H_{rms} = \sigma_o = \sqrt{m_o}$$

$$H_s = 4\sqrt{m_o}$$

Mean wave height, $\overline{H} = 2.5\sqrt{m_o}$

Average of highest 1/10th waves, $H_{1/10} = 5.09\sqrt{m_o}$

Average of highest 1/100th waves, $H_{1/100} = 6.67\sqrt{m_o}$

The average period, $\overline{T}$ can be found by calculating the centre of the area of the spectrum.

$$\overline{T} = 2\pi \frac{m_o}{m_1}$$

The peak period, T_p is the wave period at which the wave energy is maximum. This can be calculated either by differentiating the spectral function or interpreting the spectral values.

$$\overline{T}_p = 2\pi \sqrt{\frac{m_2}{m_4}}$$

The mean zero crossing period, $\overline{T}_z$ can be estimated as follows.

$$\overline{T}_z = 2\pi \sqrt{\frac{m_o}{m_2}}$$

Hence, $\bar{\eta}$(A), the average frequency of upcrossings past a certain level A (crossing above the threshold elevation A per time) can be estimated as,

$$\bar{\eta}(A) = \frac{1}{T_z} e^{-A^{2/2m_o}} \tag{22d}$$

$$\bar{\eta}(A) = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} e^{-A^{2/2m_o}} \tag{23a}$$

and, $\bar{\eta}$(0), the average frequency of all upcrossings (past a Zero level).

$$\bar{\eta}(0) = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} \tag{23b}$$

Bandwidth of the spectrum

The bandwidth of the spectrum describes how "wide" the spectrum is. For a harmonic signal with single frequency the bandwidth is nearly zero and hence a narrow spectral peak appears. However in a signal that contains multiple frequencies the bandwidth increases. For white noise the bandwidth approaches one.

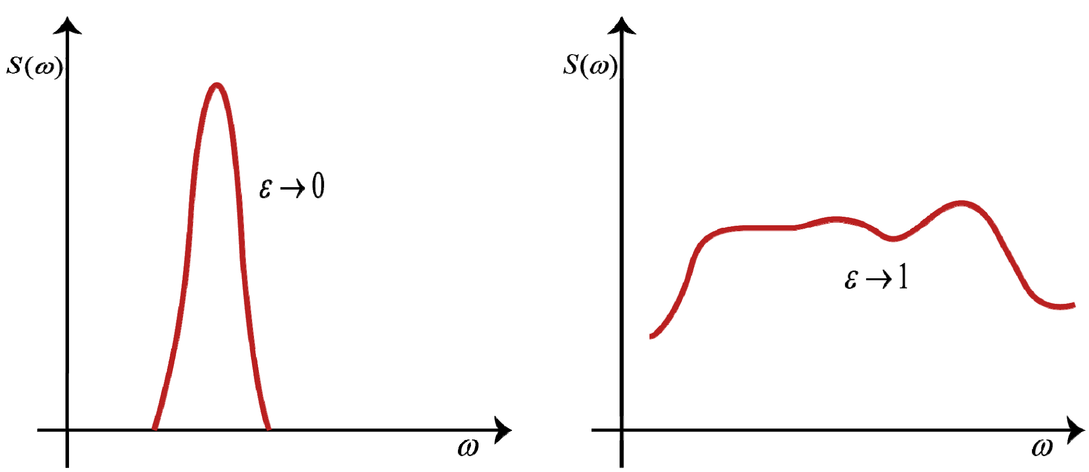


Fig. 5. Narrow and broad band spectrum.

The bandwidth parameter, ε, called the spectral width parameter defines the width of the spectrum. It can be estimated from the spread of energy over the frequencies.

$$\varepsilon^2 = 1 - \frac{\overline{T}_p}{\overline{T}_z} = 1 - \frac{m_2^2}{m_0\, m_4} \tag{24}$$

The value of ε is between 0 and 1. In the ocean, a bandwidth between 0.6 and 0.8 is common. In general, $\varepsilon > 0.6$ is called broad band spectrum and $\varepsilon<0.6$ is called narrow band spectrum. In general, it can be said that most sea spectra are relatively narrow banded. It is due to the fact that the very small, high frequency waves (ripples) are of no interest in the prediction of ship response.

It is to be noted that the significant wave height, H_s is dependent on the bandwidth of the spectrum and can be estimated as below.

$$H_s = 4\sqrt{m_o\left(1 - \frac{\varepsilon^2}{2}\right)}$$

On the assumption of narrow bandedness, the significant wave height ($\varepsilon \to 0$) is given by,

$$H_s = 4\sqrt{m_o}$$

If the spectrum is wide band (ε=1),

$$H_s = 2.83\sqrt{m_o}$$

However, the following points have to be noted before continuing to analyse in comparison with statistical analysis that we have seen in the earlier section.

1. Maximum wave period obtained from the above procedure may mislead since it would not correspond to maximum wave energy.

Problem 2:
Given the wave climate, evaluate the frequency spectrum and spectral characteristics.

f (Hz)	a (m)	$S_\eta(f)$ (m^2-s)
0.05	0.0	0.0
0.075	0.0012	3.0E-05
0.100	0.1415189	0.4005518
0.125	0.3610530	2.607185
0.150	0.3915139	3.065662
0.175	0.3352098	2.247313
0.200	0.2684360	1.441158
0.225	0.2122135	0.9006911

0.250	0.1687059	0.5692335
0.275	0.1356967	0.3682719

Ans: $H_s = 2.15m$
$f_p = 0.15\ Hz$

Tutorial 1:
Calculate various statistical averages of wave height ($H_{1/3}$, $H_{1/10}$, $H_{1/100}$, $H_{1/500}$, etc.) and wave period ($T_{1/3}$, $T_{1/10}$, $T_{1/100}$, $T_{1/500}$, etc.) for the given random wave surface elevation time history.

3.0 ALGORITHM

3.1 Statistical analysis

The first step is to make use of zero-mean process for a given time series. For this, the mean of the wave elevations is calculated and then this mean is subtracted from all the individual wave elevation values.

From this zero-mean time-series, locate the time values where the wave elevation becomes zero. Let the elevation be h_1 at time t_1 and h_2 at time t_2. Assume that the time steps are small enough. Then, if the wave elevation reaches a 0 between t_1 and t_2 then h_1 and h_2 must be of opposite sign. We observe that the zero-crossing lie between two time instances t_1 and t_2 such that the product of h_1 and h_2 is negative.

Hence, two points $P_1(t_1, h_1)$ and $P_2(t_2, h_2)$ are obtained between which the curve representing the time-series crosses the x-axis, say at the point P. This point P can be conveniently and accurately obtained by assuming the portion of the curve between P_1 and P_2 to be a straight line provided the time steps are small enough.

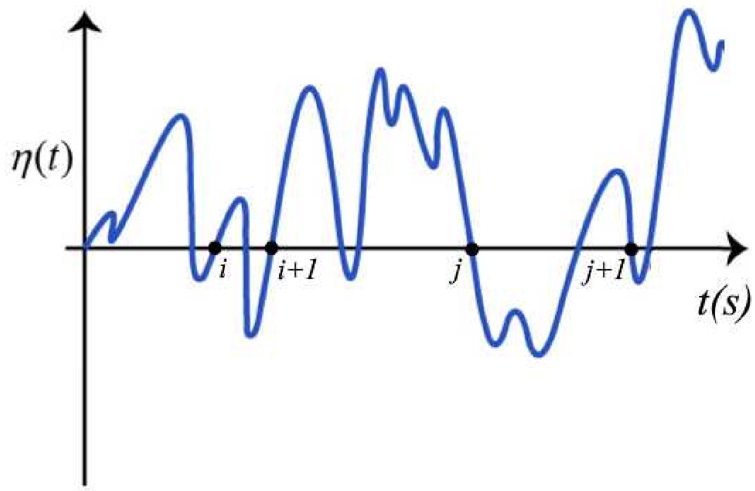


Fig.6. Up crossing and down crossing

Further, if there is a down-crossing between P_1 and P_2, the slope of the line joining P_1 and P_2 will be negative and for an up-crossing the slope will be positive. In the Fig.6, from i to i+1 there is an up-crossing and down-crossing occurs between j and j+1.

The following chunk of MATLAB code reads the time series from start to end (N = total number of points in the time-series), finds all the zero-crossings and then stores the zero down-crossings in a variable 'downcross'.

```
while(next <= N)
  if(height(current)*height(next) < 0)
     p1 = [time(current) height(current)];
     p2 = [time(next) height(next)];
     slope = (p2(2) - p1(2))/(p2(1) - p1(1));
     if(slope < 0)
        time_value = p1(1) + p1(2)*(p1(1) - p2(1))/(p2(2) - p1(2));
        downcross(index,1) = time_value;
        points(index,1) = current;
        index = index + 1;
     end
  end
  current = current + 1;
  next = current + 1;
```

```
end
------------------------------------------------------------------------
```

The time period is the difference between any two consecutive zero down-crossings. A number of time periods $T_1, T_2, T_3, ..., T_{n-1}$ etc. are obtained from n zero down-crossings. Also, the maximum and minimum heights between any two down-crossings are noted and then the wave-height for that time-range is $H_{\max} - H_{\min}$. A total of $n-1$ wave-heights are obtained corresponding to $n-1$ time periods. The MATLAB code that calculates this is shown below:

```
-----------------------------------------
for i = 1:(length(downcross)-1)
   T_value(i,1) = downcross(i+1,1) - downcross(i,1);
end

for i = 1:(length(points)-1)
   point1 = points(i);
   point2 = points(i+1);
   temp = height(point1:point2);
   h_min = min(temp);
   h_max = max(temp);
   H_value(i,1) = h_max - h_min;
end
-----------------------------------------------------
```

Simple calculations from standard formulae follow after arranging both, the wave heights and the periods, in descending order. The results are noted in the table below. All the values have been rounded to one decimal place.

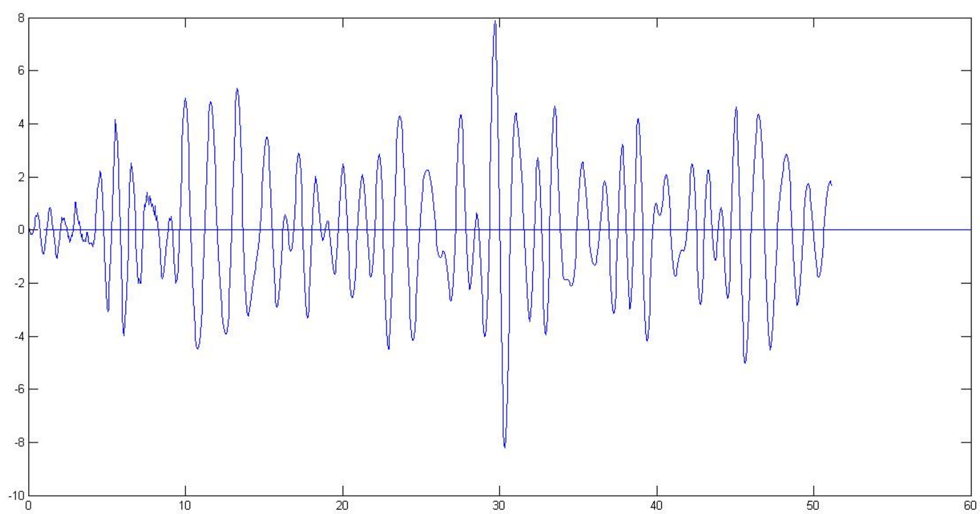


Fig. T1. Random wave surface elevation time history.

$T_{1/3}$	1.7s	$H_{1/3}$	8.7m

$T_{1/10}$	1.9s	$H_{1/10}$	10.8m
$T_{1/100}$	2.8s	$H_{1/100}$	14.5m
$T_{1/500}$	3.2s	$H_{1/500}$	16.6m
T_{mean}	1.1s	H_{mean}	5.5m

Spectral Analysis:

1. Read the given data using 'LOAD' command.
2. The given time series has first column time (t) and second column, the wave elevation (η).
3. Transform it to zero mean process.
4. Apply windowing function
5. Take 'Fourier transform' of η by using *fft* command.
6. Evaluate the spectrum using,

ffty = fft(η)
f=(0:N/2)/(Δt.N)
sf=2*dt*abs(ffty).^2/(N*Δt*0.875);

7. Smoothing using 5 point smoothening.
8. Find zero, second and fourth order moments of spectrum.
9. Evaluate H_{mean} , H_{rms} $H_{1/3}$, $H_{1/10}$ using the moments
10. Find out spectral width parameter

Result
Mean_WAVE_Height = 5.4684 m
WAve_Height_rms = 6.1726 m
H_one_third = 9 m
H_one_tenth = 11 m
Mean time period = 1.2463 s
Time rms = 1.2998 s
Spectral width parameter- 0.78
It is a broad band spectrum

References

Time series analysis
http://userwww.sfsu.edu/~efc/classes/biol710/timeseries/TimeSeriesAnalysis.html

Random waves
http://folk.ntnu.no/oivarn/hercules_ntnu/LWTcourse/partB/2randomwaves/random.htm