

# Principles of Physical Metallurgy

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## Questions and Answers

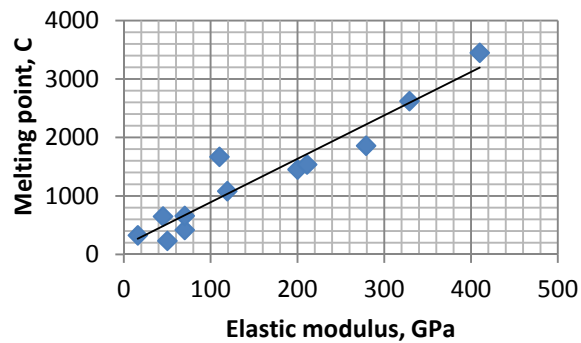
### Exercise:

1. From handbook find out melting points and Young's Modulus of a few common metals (Fe, Al, Cu, Pb, Ni, Zn, Sn, W, Ti, Mg, Cr). Is there a correlation between the two?
2. Find out from handbook atomic weights and density of Au, Ag, Al, Cu, Ni, Pb. Is there a correlation between the two?

### Answer:

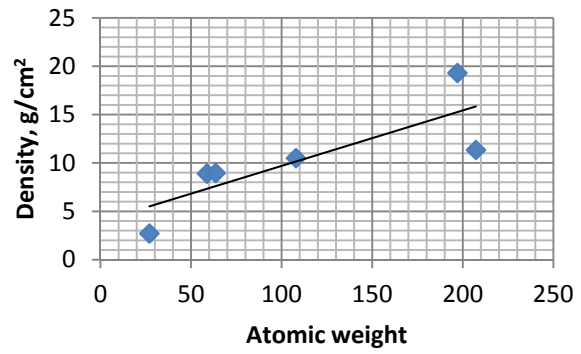
1. The following table gives the melting point and elastic modulus of a few elements. The graph shows that by & large there metals having high modulus has high melting point. Note there is an exception in case of Ti.

	C	Gpa
Sn	232	50
Pb	327	16
Zn	419	70
Mg	650	45
Al	660	70
Cu	1083	119
Ni	1453	200
Fe	1539	211
Ti	1670	110
Cr	1860	279
Mo	2623	329
W	3450	410



2. The following table and the graph display the correlation that higher the atomic number higher is the density (exception Pb. You will know reason in when the concept of lattice defect is introduced)

	At. Wt.	Density
Al	26.98	2.7
Ni	58.69	8.9
Cu	63.55	8.96
Ag	107.87	10.5
Au	196.96	19.3
Pb	207.2	11.34



**Exercise:**

1. What are the three primary bonds in materials? Which is the strongest? Why?
2. What is the electronic configuration of silicon atom? What type of bond do you expect here?
3. Inert gases have completely filled outer cell yet the boiling point of these increases with the atomic number. Explain why it is so.
4. Find out from Handbook the melting points of metals in the 4<sup>th</sup> row of the periodic table (K-Zn). Which has the highest melting point? Explain this in terms of their electronic configurations.
5. Stiffness of C-C bond is around 200N/m. Estimate its elastic modulus assuming the distance between the atoms to be  $2 \times 10^{-10}$ m. Which form of carbon has such a modulus?

**Answer:**

1. Three primary bonds are covalent, ionic and metallic. Boiling or melting points of material are indicators of the strength of the bond. C-C bond (covalent) in diamond is possibly the strongest bond. Its melting point is  $\sim 3700^\circ\text{C}$ . Its atomic diameter is small. It is verified by x-ray diffraction technique.
2. Atomic number of Si is 14. Its electronic configuration is  $1s^2 2s^2 2p^6 3s^2 3p^2$ . It has four atoms in the outer cell like carbon. It also forms covalent bond. Its atomic diameter is larger. The bond is not as strong as C-C bond. Its melting pint is  $1410^\circ\text{C}$ .
3. Inert gases have completely filled outer cell. As you go down the group (from He to Xe) the diameter increases as more orbits are added. As the diameter increases the centre of orbital electrons will no longer coincide with the nucleus. The deviation increases with the increase in diameter. Such atoms will therefore behave like dipoles & promote bonding due to van der Waal force. Boiling point of He ( $Z=2$ ) & Xe ( $Z=54$ ) are  $4.2^\circ\text{K}$  &  $165^\circ\text{K}$  respectively.
4. The fourth row of periodic table has several metals with atomic numbers varying from 19 (K) to 30(Zn). Their melting points are given in the following table. Electronic configuration of Ar (18) is  $1s^2 2s^2 2p^6 3s^2 3p^6$ . K atom has one more electron. Its preferred site is 4s. The next atom Ca has 2 electron in 4s cell. Thereafter electrons occupy 3d cell. Unpaired 3d electrons too participate in bond formation. If the number of such electron increases the cohesive energy and hence melting point increases. (Mn: exception?). In Zn all 3d electrons are paired & do not take part in bonding. In this case number of electrons that take part in bond formation is less. Therefore its melting point is low.

K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn25	Fe26	Co27	Ni28	Cu29	Zn30
64	833	1539	1668	1900	1875	1245	1539	1495	1453	1083	419

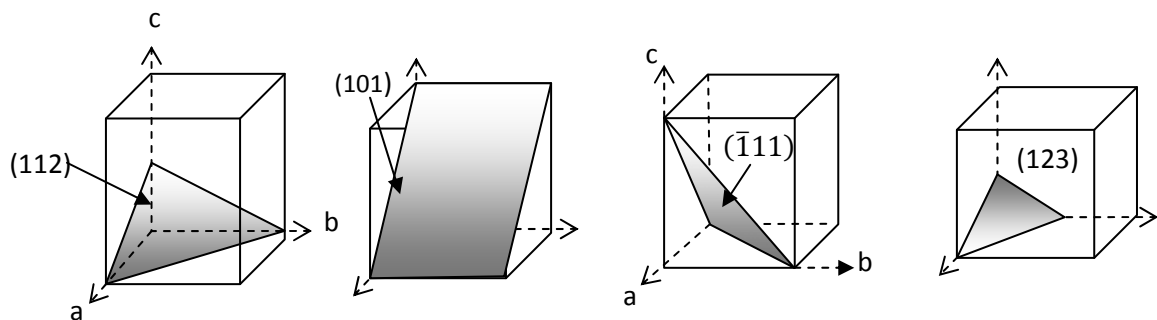
5. The bond is often visualized as a stiff spring which breaks without deformation. The relation between the force (F) needed to snap a bond and the stiffness (strength) of the bond (S) is given by  $F = S\epsilon$  where  $\epsilon$  is the minimum distance of separation at which the bond breaks. Note that the dimension of stiffness is N/m. The cross sectional area is of the order of  $a^2$ . Therefore stress  $\sigma = F/a^2 = (S/a)(\epsilon/a) = E\epsilon$  where E is modulus &  $\epsilon$  is strain. Thus  $E = S/a$ . If  $a$  is of the order of inter atomic distance ( $a \sim 0.2\text{nm}$ ).  $E = 200 / (0.2 \times 10^{-9}) = 10^{12} \text{ N/m}^2 = 1000 \text{ GPa}$ .

**Exercise:**

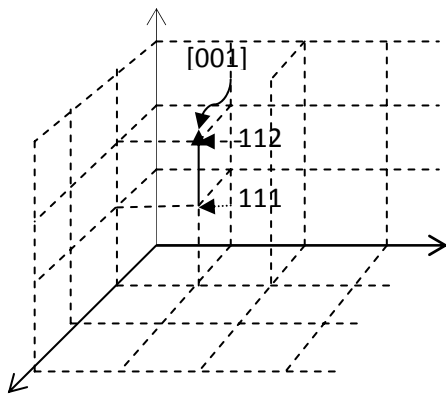
1. Sketch an unit cell and show the following planes (a) (112) (b) (101) (c)  $(\bar{1}11)$  (d) (123)
2. Find out the indices of the direction joining following points in a cubic lattice: (a) 1,1,1 with 1,1,2 (b) -1,1,1 with -3, 2, 1 (c) 1,1,2 with 3,2,-1
3. Show the atomic arrangements in (111) plane of face centre cubic structure and show the following directions  $[\bar{1}10]$ ,  $[\bar{1}01]$ ,  $[01\bar{1}]$ ,  $[\bar{2}11]$ ,  $[1\bar{2}1]$ ,  $[11\bar{2}]$
4. Estimate the density of platinum and lead from their lattice parameters at room temperature. Both are FCC. Compare the theoretical density with experimental values. Which is closer? Why?

**Answer:**

1.

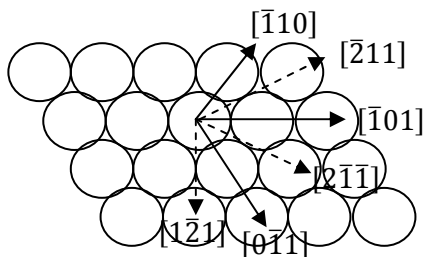


2.



If the coordinate of first point is  $u_1, v_1, w_1$  & the second point is  $u_2, v_2, w_2$  the indices of the line joining the two points can easily be shown with the help of the diagram on the left is  $[u_2-u_1, v_2-v_1, w_2-w_1]$ . Line joining point 111 with 112 is shown try others.

3.



In a cubic crystal a direction  $[uvw]$  lies on a plane  $(hkl)$  then  $hu+kv+lw=0$ . Using this three close packed direction lying on  $(111)$  are  $[-110]$ ,  $[-101]$  &  $\{0-11\}$ . These are shown by firm line. Three  $[112]$  directions are  $[11-2]$ ,  $[-211]$  &  $[1-21]$ . These are shown as dotted lines.

4. The relation between density ( $\rho$ ) & lattice parameter ( $a$ ) is given by  $\rho = \frac{An}{Na^3}$  where  $n$  = number of atom /unit cell,  $A$  = atomic weight &  $N$ =Avogadro number. For platinum  $A=192.09$ ,  $n=4$  fcc,  $N=6.02 \times 10^{23}$  &  $a=3.9239$  Angstrom. On substitution  $\rho = 21.45$  gm/cc. Experimental density of Pt = 21.47. For lead  $A=207.2$ ,  $n=4$  fcc,  $a=4.9502$  Angstrom. On substitution in the expression for density  $\rho = 11.35$  Experimental density = 11.34. The estimation of x-ray density is based on assumptions that all sites are occupied and atoms are hard. If there are vacancies in the lattice real density should be less than x-ray density. If atoms are soft the density should be higher.

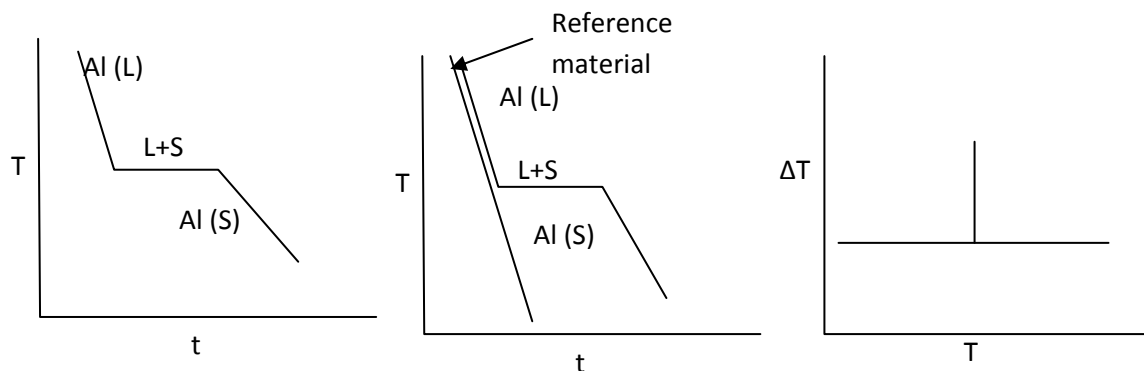


### Exercise:

1. Sketch the cooling curve of pure aluminium as it is cooled from 750° C. Given mp = 660. How would the DTA plot look like?
2. Suggest two simple methods for increasing the resolving power of an optical microscope.
3. Use Bragg law to find out the indices of the first three reflections in a powder diffraction pattern taken from a simple cubic crystal.
4. Use Bragg law to find out the indices of the first reflection in a powder diffraction pattern from fcc structure.
5. Explain why reflection from 100 is absent in diffraction patterns from bcc crystal.
6. Derive a relation between wave length & accelerating voltage in an electron microscope. What is the wave length of electron beam if voltage is 200KV?

### Answer:

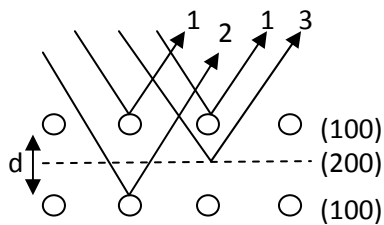
1. In DTA the temperature of reference material keeps going down whereas that of Al at its melting point remains constant until solidification is complete. This is due to latent heat liberation during solidification. Once it is complete the temperature starts dropping. Temperature difference,  $\Delta T$  becomes negligible. Cooling curves are as follows:



2. Resolving power of an optical microscope is given by  $\frac{\lambda}{2 \mu \sin \alpha}$  where  $\lambda$  is wave length,  $\mu$  is the refractive index of the medium between the objective and the sample and  $2\alpha$  is the apex angle of the cone light that gets reflected from a point on the specimen. Clearly the resolving power could be increased by reducing the wave length of the light (use blue) & using oil immersion objective where a drop of oil having high refractive index replaces air between lens & specimen.
3. Powder diffraction pattern are recorded with the help of monochromatic beam of X-Rays where as the samples consists of a large number of tiny crystals of random orientation. The condition of diffraction is given by Bragg's law:  $2 d \sin \theta = \lambda$  where  $2\theta$  is the angle between incident beam and the diffracted beam. The angle ( $\sin \theta$ ) of diffraction is inversely proportional to inter planar spacing (d). The first three reflections therefore must come from the three most widely spaced crystal planes. The d spacing of a simple cubic crystal is given

by  $d = \frac{a}{\sqrt{h^2+k^2+l^2}}$  where (h,k,l) are the Miller indices of the crystal planes. Clearly the lowest sum square value of (hkl) would have the highest d spacing. The first 3 values of  $h^2 + k^2 + l^2$  are 1, 2 & 3 respectively. Therefore the indices of the first 3 reflections are (100), (110) & (111).

4. The most widely spaced plane in fcc crystal is (111). The angle at which the reflection is likely to occur is given by  $\theta = \sin^{-1} \left( \frac{\lambda}{d} \right)$ . Clearly d, the inter-planar spacing for (111) reflection is the lowest. Therefore this is the indices of the first reflecting plane.
5. Diffraction occurs if the path difference between two reflected beams from two parallel planes is equal to the wave length of the beam. In bcc lattice there is a plane having atoms half way between two (100) plane. Therefore this too will reflect the incident beam. The path difference will therefore be  $\lambda/2$ . This would result in destructive interference. This is why there is no reflections from (100) plane in bcc crystal.



Beam 1 represents reflections from the first (100) plane. & beam 2 is the reflection from the next (100) plane satisfying Bragg condition. Let path difference between 1 & 2 =  $\lambda$  If there is a plane halfway between the two having atoms there will be a reflected beam from it. This is shown by path 3. Path diff between 1&3 =  $\lambda/2$ .

6. If V the accelerating voltage energy of electrons = eV = kinetic energy =  $\frac{1}{2}mv^2$  therefore  $v = \sqrt{\frac{2eV}{m}}$ . The particles moving at high speed also have wave nature. The wave length is given by:  $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$ . Where h ( $6.63 \times 10^{-34}$  Js) is Plank's constant, v is the velocity of electron, e ( $1.6 \times 10^{-19}$  Coulomb) is the charge of an electron & m ( $9.11 \times 10^{-31}$  kg) is its mass. If V=100kV,  $\lambda = \frac{1.23}{\sqrt{V}} = 0.0039$ nm. This of course is an approximate relation ignoring relativistic correction for mass of electron moving at high speed.



**Exercise:**

- If tensile stress strain plot beyond elastic limit is given by  $\sigma = k\varepsilon^n$  show that necking (plastic instability) sets in when true strain exceeds n.
- Derive a relation between true strain and engineering strain.
- The size of Brinell indentation taken on a steel specimen was found to be 5mm. Diameter of the ball indenter is 10mm. Estimate its hardness.
- Does necking take place during compressive loading?
- Estimate the size of Vickers indentation on a specimen taken with 10kg load if its hardness is 200VHN. What will be the size of indent if load used were 30kg?
- At what temperature does time dependent deformation become measurable?
- What problem do you anticipate in measuring hardness of lead?
- A specimen having initial length  $l_0$  is deformed under tension in two stages. In stage I it is deformed to a length of  $l_1$  and subsequently it is deformed to a length  $l_2$ . Find out engineering and true strain in each of these stages. Which of these follows additive rule if you have to estimate final strain? Assume that deformation is uniform.

**Answer:**

- Necking sets in during tensile test when the load (P) reaches its peak value. This is given by  $P = \sigma A$  where  $\sigma$  is the stress and A is the cross sectional area. Take log & differentiate to get  $\frac{dP}{P} = \frac{d\sigma}{\sigma} + \frac{dA}{A}$ . This becomes zero when P is maximum.

Since reduction in area ( $-dA/A$ ) is equal to true strain increment ( $d\varepsilon$ ) it gives:

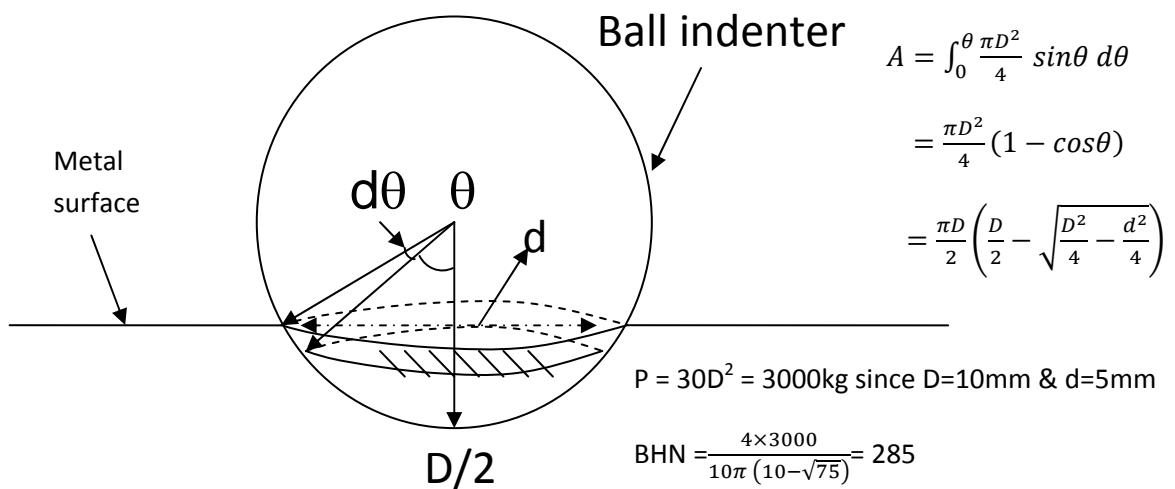
$$\frac{d\sigma}{d\varepsilon} = \sigma. \text{ Since } \sigma = k\varepsilon^n \text{ it can be shown: } \frac{d\sigma}{d\varepsilon} = k n \varepsilon^{n-1} = k \varepsilon^n n / \varepsilon = \sigma \text{ Therefore:}$$

strain at which necking sets in is given by  $\varepsilon = n$ .

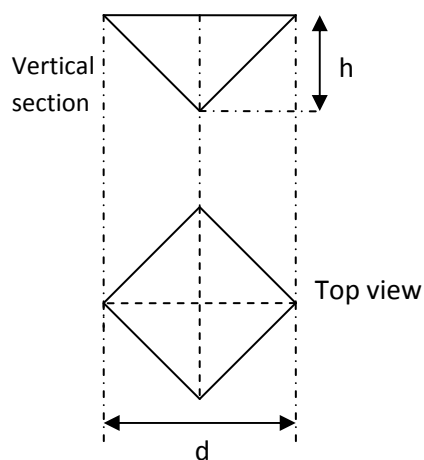
- True strain ( $d\varepsilon$ ) is defined as change in length (dl) over instantaneous length (l). Let initial length be  $l_0$  and final length l so that engineering strain (e) is  $= (l-l_0)/l_0$ . Therefore to obtain true strain one has to integrate the following equation

$$\text{between limits } l_0 \text{ \& } l. \quad \varepsilon = \int_{l_0}^l \frac{dl}{l} = \ln \frac{l}{l_0} = \ln \left( \frac{l+l_0}{l_0} \right) = \ln(1+e)$$

- Relation between indenter & indentation mark is shown in the following figure. Area of the annular strip on the surface of the indenter = dA



4. Necking does not occur under compressive load because stress decreases with strain (note that in this case cross section area increases with deformation). There may be instability of another kind. This is known as buckling. It is determined by length to diameter ratio. Cylindrical test piece with higher height to diameter ratio is prone to such instability.
5. Vickers hardness  $VHN = P/A$  where  $P$  is load and  $A$  is area of indentation mark. The indenter is a square based pyramid with apex angle  $= 136^\circ$ . If the diagonal of the indentation is  $d$  the area of indentation can be obtained as follows:



Length of one side of indentation  $= d/\sqrt{2}$  if the apex angle  $= \theta$  using simple trigonometric relation the area of one of the triangular faces of the indentation can be shown to be =

$$\left( \frac{d}{2\sqrt{2}} \right)^2 \frac{1}{\sin \frac{\theta}{2}}. \text{ There are 4 such faces.}$$

Therefore VHN defined as  $P/A$  is given by:

$$\left( 2 \sin \frac{\theta}{2} \right) \frac{P}{d^2} = \frac{1.854P}{d^2}. \text{ If } P=10\text{kg } d = 0.304\text{mm} \text{ \& if } P=30\text{kg } d=0.527\text{mm}$$

6. Creep is a time dependent deformation. It is a strong function of temperature. It becomes measurable when test temperature is greater than 0.5 times the melting point of the metal in degree Kelvin.
7. Melting point of lead is low ( $\sim 327^\circ\text{C}$ ).  $T_m = 600^\circ\text{K}$ . Room temperature ( $\sim 300^\circ\text{K}$ )  $= 0.5T_m$ . Therefore it would creep and the size of indentation will increase with time. A more precise control of time is required to get reproducible result.
8. Engineering strain in stage I  $= e_1 = \frac{l_1 - l_0}{l_0}$  and that in stage II  $= e_2 = \frac{l_2 - l_1}{l_1}$  whereas total strain  $= e_t = \frac{l_2 - l_0}{l_0}$ . However true strain in stage I  $= \epsilon_1 = \ln \left( \frac{l_1}{l_0} \right)$  and that in stage II  $= \epsilon_2 = \ln \left( \frac{l_2}{l_1} \right)$  and total strain  $= \epsilon_t = \ln \left( \frac{l_2}{l_0} \right)$ . It is easily noted that in case of true strain  $\epsilon_t = \epsilon_1 + \epsilon_2 = \ln \left( \frac{l_1}{l_0} \right) + \ln \left( \frac{l_2}{l_1} \right) = \ln \left( \frac{l_2}{l_0} \right)$ . This is not true for engineering strain.

**Exercise:**

1. Estimate the size of critical nucleus of tin when it is super cooled by 20°C. Assume nucleation to be homogeneous. The enthalpy change for solidification of tin is 0.42 GJ/m<sup>3</sup>. The liquid / solid interfacial energy is 0.055 J/m<sup>2</sup>. The melting point of tin is 232° C.
2. A metal under goes an allotropic transformation at room temperature at high pressure and at lower temperature at atmospheric pressure. Is the volume change associated with this transformation positive or negative?
3. Bismuth has a density of 9.8Mg/m<sup>3</sup> at room temperature. Its coefficient of linear expansion is 14.6x10<sup>-6</sup> /° C. The density of liquid metal at melting point (271°C) is 10.07 Mg/m<sup>3</sup>. Find our dT/dP and estimate its melting point at 100 atmosphere pressure. Latent heat = 10.9 kJ/mole (atomic weight = 209)
4. Derive an expression for critical nucleus size as a function of temperature and show with the help of a schematic graph its variation with temperature. Assuming that a stable nucleus should have at least 100 atoms which correspond to around 1nm radius mark the region of homogeneous nucleation.

**Answer:**

1.  $\Delta f_v = -\Delta H_v \left(1 - \frac{T}{T_m}\right) = -0.42 \times \frac{20}{273+232} = -0.0166 \text{ GJ/m}^3$  and Critical nucleus size =  $r^* = -\frac{2\sigma}{\Delta f_v} = \frac{0.055}{0.0166 \times 10^9} \text{ m} = 3.3 \text{ nm}$

2. The effect of pressure on transformation temperature is given by:  $\frac{dP}{dT} = \frac{\Delta H}{T\Delta V}$ . In this case let the transformation be represented as

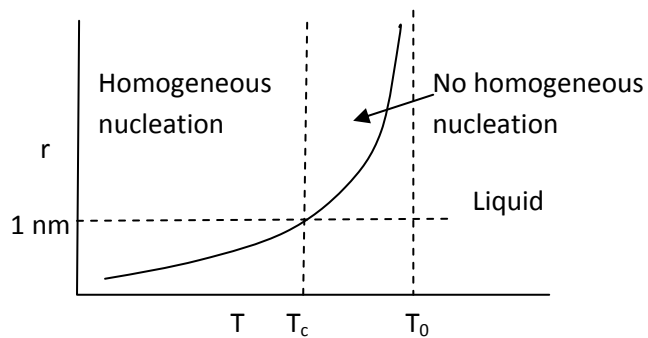
$$\alpha = \beta \text{ at } 300^\circ\text{K \& 10 atmosphere (say) (P1 \& T1)}$$

$$\alpha = \beta \text{ at } 290^\circ\text{K \& 1 atmosphere (say) (P2 \& T2)}$$

$$\Delta H > 0 \text{ reaction is endothermic \& } \frac{dP}{dT} = \frac{P_1 - P_2}{T_1 - T_2} < 0$$

3. Room temperature = 25°C, volume increase due to temperature change =  $3\alpha\Delta T$  where  $\alpha$  is coefficient of linear expansion. Volume of 1gm mass at room temperature =  $1/\rho_0$  & volume of solid Bi at melting point =  $(1/\rho_0) + 3\alpha\Delta T$ . Therefore density of solid Bi at melting point =  $\frac{\rho_0}{1+3\rho_0\alpha\Delta T} = 9.7\text{Mg/m}^3$ . On melting density increases  $\Delta V < 0$ .  $\Delta V = V_L - V_S = \frac{0.209}{1000} \times \left(\frac{9.7-10.07}{9.7 \times 10.07}\right) = -7.9 \times 10^{-7} \text{ m}^3$ .  $\frac{dP}{dT} = -\frac{10.9 \times 1000}{(271+273) \times 7.9 \times 10^{-7}} = -25.31 \frac{\text{MPa}}{\text{K}}$  Note that 1bar = 100kPa and 100 bar = 10MPa. Therefore the change in melting point at 100 bar pressure = 2.53°C.

4. Critical radius =  $r^* = -\frac{2\sigma}{\Delta f_v}$  Free energy change / unit volume for solidification =  $\Delta f_v = \Delta H_v - T\Delta S_v$  where H & S are enthalpy & entropy terms. Suffix v denotes per unit volume. At melting point ( $T_0$ )  $\Delta f_v = 0$ . Thus  $\Delta S_v = \frac{\Delta H_v}{T_0}$  &  $\Delta f_v = \Delta H_v \frac{T_0 - T}{T_0}$  Note that for solidification (it releases heat)  $\Delta H_v$  is negative. Therefore  $r^* = -\frac{2\sigma T_0}{\Delta H_v(T_0 - T)}$  It shows that  $r^*$  approaches infinity as T approaches  $T_0$ . As T approaches zero  $r^*$  becomes exceedingly small since  $|\Delta H_v| \gg \sigma$ . This is schematically shown as follows:



Homogeneous nucleation is possible below a specific temperature  $T_c$

**Exercise:**

1.  $S_{11}$ ,  $S_{44}$  &  $S_{12}$  of tungsten are 0.257, 0.660 & -0.073 (unit:  $10^{-11}$  m<sup>2</sup>/N) respectively. Check is this isotropic?
2.  $C_{11}$ ,  $C_{44}$  &  $C_{12}$  of a cubic crystal with respect to its crystal axes are 267, 82.5 & 161 GPa respectively. Estimate its elastic compliances and Young's modulus along [100]. Will this be the same along [110] or [111]?

**Answer:**

1. If the material is isotropic then  $\frac{2(S_{11}-S_{12})}{S_{44}} = 1$ . On substituting the numerical values it is found that  $\frac{2(0.257+0.073)}{0.66} = 1$  Therefore it is isotropic.

$$2. S_{11} = \frac{(C_{11}+C_{12})}{(C_{11}-C_{12})(C_{11}+2C_{12})} = \frac{(267+161)}{(267-161)(267+322)} = 0.00866 \text{ GPa}^{-1},$$

$$S_{12} = \frac{-C_{12}}{(C_{11}-C_{12})(C_{11}+2C_{12})} = \frac{-161}{(267-161)(267+322)} = -0.00258 \text{ GPa}^{-1} \quad \& \quad S_{44} = \frac{1}{C_{44}} = \frac{1}{82.5} = 0.0117 \text{ GPa}^{-1}$$

Young's modulus =  $1/S_{11} = 1/0.00866 = 146$  GPa. No it represents modulus along cube directions only. Modulus along [110] & [111] will be different.

**Exercise:**

3. In which mode of plastic deformation atomic displacement could be less than inter atomic spacing?
4. Estimate the magnitude of shear strain for (111)  $[\bar{1}\bar{1}\bar{2}]$  twin in fcc lattice.
5. What is the effect of tensile stress on lattice spacing?
6. Show schematic resolved shear stress versus shear strain diagrams of fcc crystal if the tensile axes were (a) [123] (b) [001]
7. What is the difference between simple shear & pure shear? Under which category will you place plastic deformation by slip?
8. What is the effect of plastic deformation on lattice parameter?
9. Draw a standard [001] projection showing all possible slip planes & directions for a bcc crystal. Assume slip can take place only on {110} planes.
10. When does a polycrystalline material have same yield strength along all possible direction?
11. Estimate the ideal cleavage strength and shear strength of pure iron. Given  $E = 211$  GPa and  $G = 83$  GPa.

**Answer:**

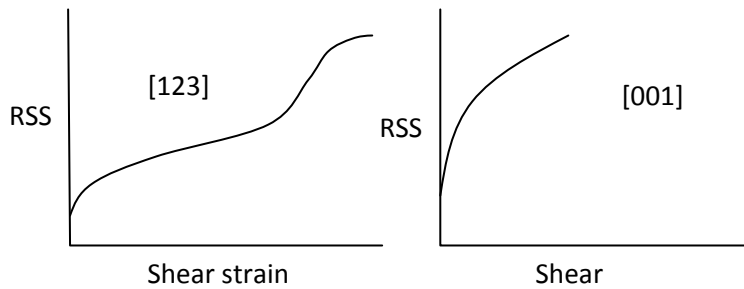
3. Twin.

4. The distance between twin plane =  $\frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a}{\sqrt{3}}$  and the magnitude of slip  $= \frac{a}{6} [11\bar{2}] = \frac{a\sqrt{1^2+1^2+(-2)^2}}{6} = \frac{a}{\sqrt{6}}$  (this represents the distance between two atoms)

along  $[11\bar{2}]$ ). Shear strain is the ratio of magnitude of slip to the distance between the two planes  $= \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} = 0.71$

5. Lattice spacing increases with tensile stress till it reaches its elastic limit. Elastic strain is equal to the ratio of change in lattice spacing along the tensile axis to its original value  $= \left(\frac{\Delta d}{d}\right)$ .

6.



In case of  $[123]$  initially resolved shear stress reaches its critical value for a single slip system. Therefore all 3 stages of deformation are seen. In case of  $[001]$  RSS reaches CRSS simultaneously on several slip system (8 to be precise). Therefore 3 distinct stages are not seen.

7. Simple shear represents displacement or slip on a plane along a specified direction. This is schematically represented as follow:



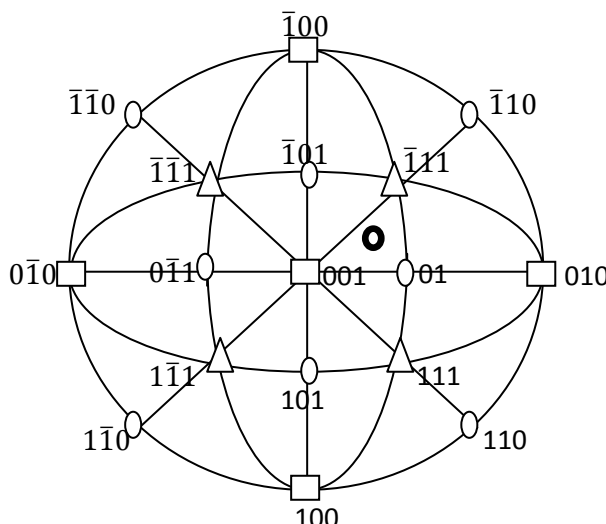
Simple shear:  $x$  is normal to plane on which shear has taken place &  $y$  is displacement. Displacement gradient is:  $e_{xy} = \frac{y}{x} = \gamma$ . Using the notation used it is  $e_{12}$ . All other components are zero. The matrix is not symmetric. Slip is a simple shear

Pure shear: simple shear ( $e_{ij}$ ) is equal to sum of pure shear ( $\epsilon_{ij}$ ) and rotation ( $\omega_{ij}$ ) as shown above. This makes the strain matrix symmetric.  $e_{ij} = \epsilon_{ij} + \omega_{ij}$

$$\begin{bmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \gamma/2 & 0 \\ \gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \gamma/2 & 0 \\ -\gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. Plastic deformation does not alter crystal structure or its dimension. Lattice parameter after deformation is still the same.

9.



Slip systems of bcc crystal are  $\{101\}\langle\bar{1}11\rangle$ . There are 12 such system. These are shown in standard projection. For a pole lying within a stereographic triangle slip plane & direction having maximum resolved shear stress lie in the adjacent triangle. This is shown for one case where the pole is marked as a circle. The slip plane is  $(\bar{1}01)$  and slip direction is  $[111]$ .

10. Strength of a crystal may vary with the direction of loading. Polycrystalline metals may have a large number of grains. Its properties will be a function of the entire group. If these are randomly oriented then one would expect its properties to be isotropic.

11. Ideal shear strength  $\tau_{max} = \frac{G}{2\pi} = \frac{83}{2\pi} = 13.21 \text{ GPa}$ . For estimating cleavage strength please see problem 15 of chapter 1. This gives  $\sigma_{max} = \sqrt{\frac{E\gamma}{a_0}}$  &  $\gamma = \frac{a^2 E}{\pi^2 a_0} \approx \frac{E a_0}{10}$  since  $a$  is nearly equal to  $a_0$ . Thus  $\sigma_{max} = \sqrt{\frac{E\gamma}{a_0}} \approx \frac{E}{\sqrt{10}} = \frac{211}{3.16} = 67 \text{ GPa}$ . These are nearly two orders of magnitude higher than the real strength of iron.

**Exercise:**

1. FCC crystals have more packing density than BCC crystal yet why solubility of carbon in FCC form of iron is higher than in its BCC form?
2. What is the effect of temperature on the concentration of vacancy?
3. If the ratio of iron ions to oxygen ions is 0.994 in FeO, what fraction of Fe sites are filled with Fe<sup>3+</sup> ions? What is the ratio of Fe<sup>3+</sup> to O<sup>2-</sup> ions?
4. What are the major differences between an edge & screw dislocation? Which of these can cross slip?

**Answer:**

1. FCC has the maximum packing density (74%). However the interstitial sites where the carbon atoms are located are larger than those in BCC structure. Packing density in BCC is relatively low (68%). However there is more number of interstitial sites for every Fe atom. The gaps are distributed amongst more number of sites. Therefore these are too small to accommodate carbon atoms. This is why solubility is low.
2. The fraction of vacant lattice sites in a crystal is given by  $\frac{n_v}{n} = \exp\left(-\frac{q_v}{kT}\right)$  where  $k$  is Boltzmann constant,  $q_v$  is the energy needed to create a vacancy and  $T$  is the temperature in degree Absolute. As the  $T$  increases  $\frac{n_v}{n}$  too increases.
3. Ionic crystals must maintain charge neutrality. In this iron oxide fraction of vacant Fe<sup>2+</sup> sites = 1-0.994 = 0.006. The reason that stoichiometry is not maintained indicates that for every O<sup>2-</sup> vacancy there are 2 Fe<sup>3+</sup> ions. Thus the fraction of vacant O<sup>2-</sup> sites = 0.003. Note that to maintain charge neutrality there could have been 0.006 vacant O<sup>2-</sup> sites. In that event stoichiometry would have been FeO.

4.

	Edge	Screw
Burgers vector	Perpendicular to dislocation	Parallel to dislocation
Slip plane	The plane containing both Burgers vector & the dislocation	Any plane containing the dislocation
Cross slip	Not possible	Possible
Climb	Can climb	Cannot climb
Atomic arrangements around dislocation	There is an extra plane of atoms above slip plane	Atoms along the dislocation are arranged in a helix like a screw

**Exercise:**

- There is a dislocation lying along  $[\bar{1}01]$  in fcc crystal. Its Burgers vector is  $\frac{a}{2}[0\bar{1}1]$ . What type of dislocation is it? Determine its slip plane.
- Find out the ratio of elastic stored energy of an edge dislocation to that of a screw. Assume energy of the core to be negligible.
- What is the hydrostatic stress (or strain) field around a screw dislocation?
- What is the hydrostatic stress field of an edge dislocation?

**Answer:**

- Dislocation:  $[t_1 t_2 t_3] = [\bar{1}01]$  & Burgers vector  $[b_1 b_2 b_3] = \frac{a}{2}[0\bar{1}1]$ . Since b does not lie along t it is not a screw dislocation. The angle between the two is given by  

$$\cos\theta = \frac{t_1 b_1 + t_2 b_2 + t_3 b_3}{\sqrt{t_1^2 + t_2^2 + t_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}} = \frac{0+0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$
. Since  $\theta = 60^\circ$  it is a mixed dislocation. It can move only in a plane containing both t & b. If the indices of the plane is (hkl) then:  
 $t_1 h + t_2 k + t_3 l = 0$  &  $b_1 h + b_2 k + b_3 l = 0$ . Therefore  $-h+l = 0$  &  $-k+l = 0$ ; or  $h = k = l$   
 Therefore slip plane is (111)

- Energy of an edge dislocation of unit length:  $E^e = \frac{Gb^2}{4\pi(1-\nu)} \ln\left(\frac{r}{r_0}\right)$  & that of a screw dislocation:  $E^s = \frac{Gb^2}{4\pi} \ln\left(\frac{r}{r_0}\right)$  Therefore  $\frac{E^e}{E^s} = \frac{1}{1-\nu}$  Most metal  $\nu = 1/3$ . Therefore the energy of an edge dislocation is 3/2 time that of a screw dislocation.

- Hydrostatic stress:  $\bar{\sigma} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}$  Each of three terms is zero for a screw dislocation.  $\bar{\sigma} = 0$

- Hydrostatic stress:  $\bar{\sigma} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}$  Since  $\sigma_{33} = \nu(\sigma_{11} + \sigma_{22})$ ,  $\bar{\sigma} = \frac{1+\nu}{3}(\sigma_{11} + \sigma_{22})$   
 for an edge dislocation:  $\sigma_{11} = -\frac{Gb}{2\pi(1-\nu)} \frac{x_2(3x_1^2 + x_2^2)}{(x_1^2 + x_2^2)^2}$  &  $\sigma_{22} = \frac{Gb}{2\pi(1-\nu)} \frac{x_2(x_1^2 - x_2^2)}{(x_1^2 + x_2^2)^2}$  Thus:  

$$\bar{\sigma} = -\frac{Gb(1+\nu)}{3\pi(1-\nu)} \frac{x_2}{(x_1^2 + x_2^2)}$$
 The nature of stress is compressive above the slip plane. This is why the atoms are more closely placed above the slip plane.

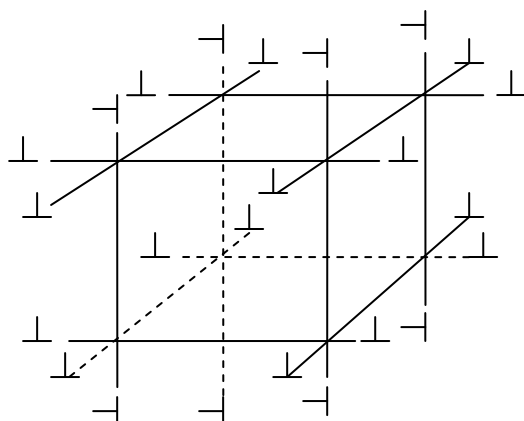


**Exercise:**

9. Assume that dislocations are arranged in an array in three dimensions described as a cubic lattice. If average number of dislocations intersecting a plane is  $\rho$  /unit area show that the average distance between two dislocations is proportional to  $\sqrt{1/\rho}$ .
10. Dislocation density of annealed metal is  $10^{12} \text{ m}^{-2}$ . Find out elastic stored energy per unit volume.
11. Elastic stored energy / unit length of an edge dislocation is given by  $E^e = \frac{Gb^2}{4\pi(1-\nu)} \ln\left(\frac{r}{r_0}\right)$ . Does this mean it can approach infinity as r becomes very large?
12. What is the difference between a kink and a jog? An edge dislocation crosses another dislocation which is perpendicular to the slip plane. Show with neat diagram the effect of such an interaction.
13. A perfect dislocation moving on plane  $(11\bar{1})$  interacts with another moving on  $(1\bar{1}\bar{1})$ . What are the different reactions possible? Which of these are Lomer locks?
14. On what planes can a screw dislocation having Burgers vector  $\frac{a}{2}[111]$  could move in a BCC crystal? What will be the slip plane if it were an edge dislocation?

**Answer:**

9. Let L represents edge of a cube and each of these represent a dislocation line of length L. Repeated array of such a cube would represent a net work of dislocation. This is schematically shown as follows:

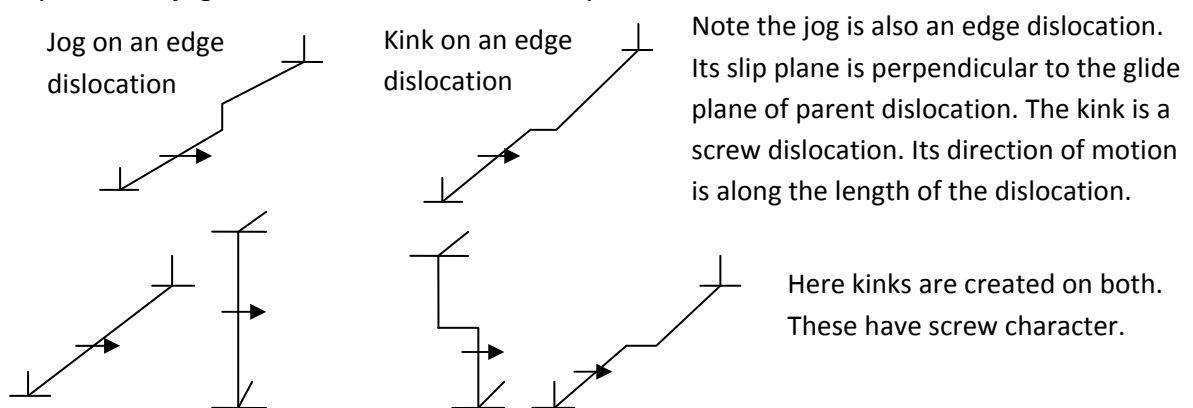


Volume of cube =  $L^3$  Since each edge denotes dislocation of length L the total length of dislocation within the cube =  $12 L / 4$ . This is because each edge belongs to 4 adjacent cubes. Therefore dislocation density  $\rho = \text{total length of dislocation} / \text{volume of cube} = 3L/L^3$ . Or;  $= \sqrt{\frac{3}{\rho}}$ . Therefore the distance between two dislocation is inversely proportional to the square root of dislocation density.

10. Previous problem shows that the average distance between two dislocation = L. The stress field of a dislocation can be assumed to extend over a distance = L/2. Or;  $R = \frac{L}{2} = \sqrt{\frac{3}{4\rho}} = 0.86\rho^{-0.5}$  which is approximately:  $\approx \frac{1}{\sqrt{\rho}}$  Energy of a dislocation consists of two parts.  $U = U_{core} + U_{strain}$  &  $U_{core} = \frac{Gb^2}{10}$  The strain energy =  $\frac{Gb^2(1-\nu \cos^2\alpha)}{4\pi(1-\nu)} \ln \frac{R}{r_0}$  where  $\alpha$  represents angle between dislocation and Burgers vector. For edge dislocation  $\alpha = \alpha/2$  whereas for screw  $\alpha = 0$ . To estimate energy of a dislocation let us assume  $b = 0.25\text{nm}$  and  $r_0 = 5b$  and  $R = \rho^{-0.05} = 10^{-6}$  Therefore  $U = \frac{Gb^2}{10} + \frac{Gb^2}{4\pi} \ln \frac{10^{-6}}{5 \times 0.25 \times 10^{-9}} = Gb^2 \left( \frac{1}{10} + \frac{1}{4\pi} \ln 800 \right) = 0.63Gb^2 \approx 0.5Gb^2$ . Assume  $G = 50\text{GPa}$  Energy / unit length of dislocation =  $0.5 \times 50 \times 10^9 \times 0.0625 \times 10^{-18} \text{ J/m}^2 = 1.56 \times 10^{-9} \text{ J/m}$  Therefore elastic stored energy / unit volume =  $U\rho = 1.56 \times 10^{-9} \times 10^{12} = 1.56 \text{ kJ/m}^3$

11. No. Because dislocations do not occur in isolation. If the average distance between dislocations is L average value of  $r = 0.5L$ . Therefore energy of dislocations is always finite. Approximately this is equal to  $0.5 Gb^2$ .

12. Both kink & jog represent a step on the dislocation. The kink can glide on the slip plane of the parent dislocation. However its direction of motion is different. Glide plane of a jog is different from that of the parent dislocation.



13. The Burgers vectors of dislocations on these planes are given in the following table:

$(11\bar{1})$	$(\bar{1}\bar{1}\bar{1})$
$\frac{a}{2}[101]$	$\frac{a}{2}[101]$
$\frac{a}{2}[\bar{1}10]$	$\frac{a}{2}[110]$
$\frac{a}{2}[011]$	$\frac{a}{2}[01\bar{1}]$

Note: if Burgers vector  $b = \frac{a}{n}[hkl]$  then  $|b|^2 = \frac{a^2}{n^2}(h^2 + k^2 + l^2)$

Lomer lock: The two planes intersect along  $[101]$  Favorable reactions producing edge dislocation are Lomer locks. It is immobile because the plane on which it lies is not a close packed plane on

No.	Reactions	Energy	Remark
1	$\frac{a}{2}[101] + \frac{a}{2}[101] = a[101]$	$\frac{a^2}{2} + \frac{a^2}{2} < 2a^2$	Unfavorable
2	$\frac{a}{2}[101] + \frac{a}{2}[110] = \frac{a}{2}[211]$	$\frac{a^2}{2} + \frac{a^2}{2} < \frac{3}{2}a^2$	Unfavorable
3	$\frac{a}{2}[101] + \frac{a}{2}[\bar{1}\bar{1}0] = \frac{a}{2}[0\bar{1}1]$	$\frac{a^2}{2} + \frac{a^2}{2} > \frac{a^2}{2}$	Favorable
4	$\frac{a}{2}[101] + \frac{a}{2}[01\bar{1}] = \frac{a}{2}[110]$	$\frac{a^2}{2} + \frac{a^2}{2} > \frac{a^2}{2}$	Favorable
5	$\frac{a}{2}[101] + \frac{a}{2}[0\bar{1}1] = \frac{a}{2}[1\bar{1}2]$	$\frac{a^2}{2} + \frac{a^2}{2} < \frac{3}{2}a^2$	Unfavorable
6	$\frac{a}{2}[\bar{1}10] + \frac{a}{2}[101]$ $= \frac{a}{2}[011]$	$\frac{a^2}{2} + \frac{a^2}{2} > \frac{a^2}{2}$	Favorable
7	$\frac{a}{2}[\bar{1}10] + \frac{a}{2}[\bar{1}0\bar{1}]$ $= \frac{a}{2}[\bar{2}1\bar{1}]$	$\frac{a^2}{2} + \frac{a^2}{2} < \frac{3}{2}a^2$	Unfavorable
8	$\frac{a}{2}[\bar{1}10] + \frac{a}{2}[110] = a[010]$	$\frac{a^2}{2} + \frac{a^2}{2} = a^2$	Unfavorable
9	$\frac{a}{2}[\bar{1}10] + \frac{a}{2}[\bar{1}\bar{1}0] = a[\bar{1}00]$	$\frac{a^2}{2} + \frac{a^2}{2} = a^2$	Unfavorable
10	$\frac{a}{2}[\bar{1}10] + \frac{a}{2}[01\bar{1}]$ $= \frac{a}{2}[\bar{1}2\bar{1}]$	$\frac{a^2}{2} + \frac{a^2}{2} < \frac{3}{2}a^2$	Unfavorable
11	$\frac{a}{2}[\bar{1}10] + \frac{a}{2}[0\bar{1}1]$ $= \frac{a}{2}[\bar{1}01]$	$\frac{a^2}{2} + \frac{a^2}{2} > \frac{a^2}{2}$	Favorable; Lomer Lock
12	$\frac{a}{2}[011] + \frac{a}{2}[101]$ $= \frac{a}{2}[112]$	$\frac{a^2}{2} + \frac{a^2}{2} < \frac{3}{2}a^2$	Unfavorable
13	$\frac{a}{2}[011] + \frac{a}{2}[\bar{1}0\bar{1}]$ $= \frac{a}{2}[\bar{1}10]$	$\frac{a^2}{2} + \frac{a^2}{2} > \frac{a^2}{2}$	Favorable: it can glide on (11 $\bar{1}$ )
14	$\frac{a}{2}[011] + \frac{a}{2}[110]$ $= \frac{a}{2}[121]$	$\frac{a^2}{2} + \frac{a^2}{2} < \frac{3}{2}a^2$	Unfavorable
15	$\frac{a}{2}[011] + \frac{a}{2}[\bar{1}\bar{1}0]$ $= \frac{a}{2}[\bar{1}01]$	$\frac{a^2}{2} + \frac{a^2}{2} > \frac{a^2}{2}$	Favorable; Lomer Lock
16	$\frac{a}{2}[011] + \frac{a}{2}[01\bar{1}] = a[010]$	$\frac{a^2}{2} + \frac{a^2}{2} = a^2$	Unfavorable
17	$\frac{a}{2}[011] + \frac{a}{2}[0\bar{1}1] = a[001]$	$\frac{a^2}{2} + \frac{a^2}{2} = a^2$	Unfavorable

14. Let the slip plane be one of the 12 {110} planes. Dislocation & the Burgers vector must lie on slip plane. The Possible glide planes are  $(1\bar{1}0)$ ,  $(10\bar{1})$ ,  $(01\bar{1})$ . The best way to check if dot product of Burgers vector & plane normal is equal to zero ( $b \cdot n = 0$ ). If the slip plane were of type {112} the slip plane for this case would be  $(\bar{2}11)$ ,  $(1\bar{2}1)$  &  $(11\bar{2})$ . Try to find out the possible slip plane of type {123}.

If it were an edge dislocation one must specify its direction. It could glide only if it lies on one of the slip planes. For example an edge dislocation  $\frac{a}{2}[111]$  lying along  $[\bar{2}11]$  could glide on  $(01\bar{1})$ . Find other possibilities.

### Exercise:

15. An fcc crystal is pulled along  $[123]$ . What is the possible combination of glide plane & direction? Estimate the force on the mobile dislocation if applied tensile stress is 100MPa & lattice parameter = 0.36nm.

16. What is the force acting on a dislocation  $\mathbf{b}$   $[010]$  lying along  $[100]$  if the applied stress

is given by  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ? Will this help it glide?

17. An edge dislocation moving on a crystal plane stops at an obstacle. A second dislocation having the same Burgers vector & lying on the same plane approaches the same on application of a shear stress of magnitude 140MPa. Estimate the distance of separation if  $E = 210\text{GPa}$ , Poisson ratio = 0.3 & lattice parameter = 0.362nm. What will be the distance between the two if they were screw dislocation?

18. A dislocation is pinned between two obstacles spaced 1micron apart. What will be the magnitude of stress to bow the dislocation into a semi circle? Hence estimate its yield strength. Given  $G = 100\text{GPa}$ , Burgers vector = 0.25nm

19. Nickel sheet is being rolled at room temperature in a rolling mill (diameter = 50cm, rpm = 200). Initial thickness is 20mm and thickness after rolling is 10mm. Estimate average strain rate, energy that will be stored in material if final dislocation density is  $10^{11}/\text{cm}^2$ , total energy / unit volume spent during rolling (assume flow stress = 300MPa), adiabatic temperature rise if specific heat = 0.49J/g/K

20. Iron ( $a = 0.286\text{nm}$  and  $G = 70\text{GPa}$ ) is deformed to a shear strain of 0.3. What distance a dislocation could move, if dislocation density remains constant at  $10^{14}/\text{m}^2$ ? What will be the average dislocation velocity if strain rate is  $10^{-2}/\text{s}$ ? Estimate its shear strength.

21. Polycrystalline aluminum with average grain size of 10micron is subjected to shear stress of 50MPa. If a dislocation source located at the centre of a grain emits dislocations which pile up at the boundary what is the stress it would experience? ( $G=70\text{GPa}$ ,  $b=0.3\text{nm}$ )

**Answer:**

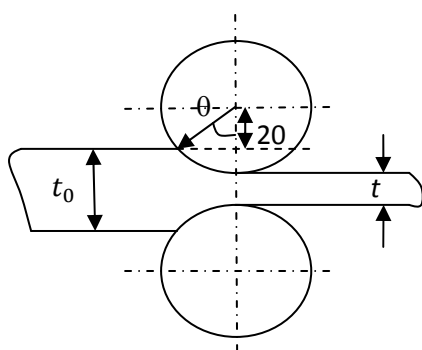
15. Look at the standard project & identify the the slip plane & direction having highest resolved shear stress. In this case it is  $(\bar{1}11)[101]$ . Therefore resolved shear stress:  $\tau = \sigma \cos\phi \cos\lambda$ .  $\cos\phi = \frac{-1 \times 1 + 1 \times 2 + 1 \times 3}{\sqrt{3}\sqrt{14}} = \frac{4}{\sqrt{42}}$  &  $\cos\lambda = \frac{1 \times 1 + 0 \times 2 + 1 \times 3}{\sqrt{2}\sqrt{14}} = \frac{4}{2\sqrt{7}}$   
The force on dislocation which is mobile is given by  $F = \tau b = \sigma b \cos\phi \cos\lambda = 100 \times \frac{0.36}{\sqrt{2}} \times 10^{-9} \times \frac{16}{14\sqrt{6}} = 0.012 \text{ N/m}$

16. The force on a dislocation is given by  $F_i = \epsilon_{ijk} \sigma_{jl} b_l t_k$  The subscripts can have values 1, 2 or 3. Since only non zero components of  $\sigma$ ,  $\mathbf{b}$  &  $\mathbf{t}$  are  $\sigma_{22}$ ,  $b_2$  &  $t_1$ .  $F_i = \epsilon_{i21} \sigma_{22} b_2 t_1$  Therefore subscript i has to be 3 ( $\epsilon_{ijk} = 1$  for  $i \neq j \neq k$  else  $\epsilon_{ijk} = 0$  &  $\epsilon_{321} = -1$ ,  $\epsilon_{123} = 1$ ) Thus  $F_3 = -\sigma_{22} b$  This being an edge dislocation can climb due to a force acting along its Burgers vector. Since the force is against x3 it can help it climb down.

17. Assume that the dislocations lie on (001) plane. Let this be the glide plane. Since  $x_2 = 0$  only non zero force acting on the dislocation is given by  $F_1 = \frac{Gb^2}{2\pi(1-\nu)x_1}$  Since both dislocations have positive b they would repel each other. Here it is balanced by a force acting on the second dislocation which is  $\tau b$ . Therefore the distance between the two is given by  $x_1 = \frac{Gb}{2\pi(1-\nu)\tau} = \frac{210 \times 10^3 \times 0.25 \times 10^{-9}}{2 \times \pi \times (1-0.3) \times 140} = 59.7 \text{ nm}$

18. Shear stress  $\tau$  is given by  $\tau = \frac{Gb}{l} = \frac{100 \times 10^3 \times 0.25}{1 \times 10^3} = 25 \text{ MPa}$  This represents shear strength. YS is usually = 2 x shear strength = 50MPa.

19. Look at the following schematic diagram given below relating different parameters during rolling. Assume that plate width is 1 & initial length is  $l_0$ . Since the volume during rolling does not change it can be shown that strain is given by  $\epsilon = \ln\left(\frac{l}{l_0}\right) = \ln\frac{t_0}{t} = \ln 2$  The time to achieve this strain is estimated as follows:



If R is the roll radius  $\omega$  is the angular velocity, time it takes to give this deformation  $= \theta / \omega$ . Since  $R=25$ ,  $\theta = \cos^{-1}\left(\frac{R - \frac{t_0 - t}{2}}{R}\right) = \cos^{-1}\left(\frac{20}{25}\right) = \cos^{-1}0.8$  &  $\omega = \frac{2\pi \text{ rpm}}{60} = \frac{2\pi \times 200}{60} = \frac{20\pi}{3} \text{ s}^{-1}$  Thus time  $= \frac{3 \cos^{-1}0.8}{20\pi} = 0.031$  & strain rate  $= \dot{\epsilon} = \frac{\ln 2}{0.031} = 22.56 \text{ s}^{-1}$ . Stored energy due to increased dislocation density  $= 0.5 G b^2 \rho = 0.5 \times 200 \times 10^9 \times (0.25)^2 \times 10^{-18} \times 10^{11} \times 10^4 = 6.25 \text{ MJ/m}^3$

Energy spent = flow stress x strain =  $300 \ln(2) = 208 \text{ MJ/m}^3$ . This shows only a very small amount of the total energy is stored within the metal. Bulk of it is dissipated as heat. Assuming the process of rolling is adiabatic temperature increase  $\Delta T = \frac{E}{\rho s} = \frac{(208-6.25) \times 10^6}{8907 \times 0.49 \times 10^3} \approx 46^\circ\text{C}$

20. Total strain ( $\epsilon$ ) is given by  $\epsilon = \rho b \bar{x}$  where  $\rho$  is dislocation density,  $b$  is Burgers vector &  $\bar{x}$  is the average glide distance of dislocation. Burgers vector  $b = a/\sqrt{3} = 0.248 \text{ nm}$ . Therefore  $\bar{x} = \frac{\epsilon}{\rho b} = \frac{0.3}{0.248 \times 10^{-9} \times 10^{14}} = 1.21 \times 10^{-5} \text{ m}$ . This is equal to 1.21 micron which is less than its grain size. The average velocity =  $1.21 \times 10^{-3} \text{ m/s}$ .

21. The force experienced by a dislocation due to another at a distance  $x$  on the slip plane is given by  $F = \frac{Gb^2}{2\pi(1-\nu)x}$ . Since the dislocation source is at the centre of the grain, it is at a distance =  $d/2$  from the dislocation at grain boundary. Therefore  $x = d/2$ . Thus  $F = \frac{Gb^2}{2\pi(1-\nu)\frac{d}{2}}$ . This may be assumed to be the resisting force acting on the source. The net force acting on the source =  $\tau b - F$  which is  $= \tau b - \frac{Gb^2}{\pi(1-\nu)}$   $= 50 \times 10^6 \times 0.3 \times 10^{-9} - \frac{70 \times 10^9 \times 0.09 \times 10^{-18}}{\pi \times 0.7 \times 5 \times 10^{-6}} = 0.015 - 0.00057 \approx 0.014 \text{ N/m}$

### Exercise:

22. Estimate the distance between dislocations in a tilt boundary of aluminium if the misorientation angle is  $5^\circ$ . Given lattice parameter of Al = 0.405 nm. Crystal structure is fcc.

23. A more precise expression for low energy grain boundary is given by  $E = \frac{Gb}{4\pi(1-\nu)} \theta (A - \ln \theta)$  where  $A$  is a constant. This is valid over the range  $0 < \theta < 10^\circ$ . Find a reasonable estimate of  $A$ . Given lattice parameter of Ni (fcc) = 0.35 nm,  $G = 76 \text{ MPa}$  Poisson ratio = 0.3 (Hint: assume dislocation core radius as  $5b$  & the minimum distance between dislocation to be twice this. The dislocation core energy =  $\frac{Gb^2}{10}$ )

24. Estimate the dislocation spacing and energy of a low angle boundary in copper crystal (fcc  $b = 0.25 \text{ nm}$ ) if tilt angle =  $1^\circ$ . Given  $G = 48 \text{ MPa}$  &  $\nu = 0.3$

25. Use the expression given in problem 2 to find out the tilt angle ( $\theta_{max}$ ) at which the energy of a low angle boundary is the maximum. Hence show that  $\frac{E}{E_{max}} = \frac{\theta}{\theta_{max}} \left( 1 - \ln \frac{\theta}{\theta_{max}} \right)$

26. Estimate the energy of the free surface of polycrystalline copper from its heat of sublimation. Does this vary from grain to grain? Given  $L_s = 338 \text{ kJ/mole}$ ;  $a = 0.36 \text{ nm}$

**Answer:**

22. Burgers vector of a dislocation in a tilt boundary  $= \frac{a}{2} [110] = \frac{a}{\sqrt{2}} = \frac{0.405}{\sqrt{2}} = 0.29 \text{ nm}$

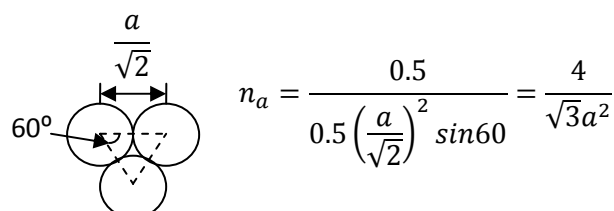
The spacing between the two dislocations is given by  $h = \frac{b}{\theta} = \frac{0.29}{5\pi} \times 180 = 3.32 \text{ nm}$

23. Burgers vector  $= \frac{a}{\sqrt{2}} = \frac{0.35}{\sqrt{2}} = 0.25 \text{ nm}$  When the dislocations are  $10b$  apart energy of the low angle boundary  $= \frac{Gb^2}{10h}$  (since boundary consists of one dislocation of unit length at every distance of  $h$ ).  $h = 2r_0 = 10b$ . Thus  $\frac{Gb^2}{100b} = \frac{Gb}{4\pi(1-\nu)} \theta (A - \ln\theta)$  where  $\theta = b/10b = 0.1 \text{ rad}$  &  $A = \frac{4\pi(1-\nu)}{100 \times \theta} + \ln\theta = -1.42$

24. Since poisson ratio is same as in the previous problem  $E = \frac{Gb}{4\pi(1-\nu)} \theta (-1.42 - \ln\theta) = \frac{48 \times 10^9 \times 0.25 \times 10^{-9}}{4 \times \pi \times (1-0.3)} \frac{\pi}{180} (-1.42 - \ln \frac{\pi}{180}) = 0.13 \text{ J/m}^2$

25. Differentiating the expression for  $E$ :  $\frac{dE}{d\theta} = \frac{Gb}{4\pi(1-\nu)} (A - \theta \frac{1}{\theta} - \ln\theta) = 0$  Thus  $\ln\theta_{max} = A - 1$  On substituting the magnitude of  $A$  from the previous problem  $\theta_{max} = 5.1^\circ$  &  $E_{max} = \frac{Gb}{4\pi(1-\nu)} \theta_{max}$  (note  $A = 1 + \ln \theta_{max}$ )  $\frac{E}{E_{max}} = \frac{\theta}{\theta_{max}} (A - \ln\theta) = \frac{\theta}{\theta_{max}} (1 + \ln\theta_{max} - \ln\theta) = \frac{\theta}{\theta_{max}} (1 - \ln \frac{\theta}{\theta_{max}})$

26. Energy of the free surface depends on the way atoms are arranged. This varies from grain to grain depending on their orientations. If  $Z$  is the coordination number, the number of bonds of type AA in one mole of pure metal  $= \frac{1}{2} ZN_0$  where  $N_0$  is Avogadro number. If  $\epsilon$  is the energy of one bond,  $L_s = \frac{1}{2} ZN_0 \epsilon$  where  $L_s$  is heat of sublimation. The free surface has a set of broken bonds. Energy of a broken bond is approximately  $\epsilon/2$ . The number depends on the indices of the top surface. If it were (111) there will be 3 broken bonds / atom (There are 6 bonds on the plane 3 beneath & 3 above it). Energy of free surface is therefore  $= 3\epsilon/2 \text{ J/atom}$ . If  $n_a$  is number of atom / unit area surcae free energy  $\gamma_s = \frac{3 L_s n_a}{2 ZN_0}$  The arrangements of atom in (111) plane is shown below. On substitution in expression for  $\gamma_s = \frac{3 L_s}{2 ZN_0} \frac{4}{\sqrt{3}a^2} = \frac{2\sqrt{3} L_s}{ZN_0 a^2} = 2.5 \text{ J/m}^2$



**Exercise:**

1. If iron is kept at 1200°K in a carburizing atmosphere for 8hrs to obtain a carbon concentration of 0.75 at a depth of 0.5mm. Find the time it would take to reach same carbon concentration at depth of 7.5mm at 1250°K. (Given  $D_0 = 0.2 \times 10^{-4} \text{ m}^2/\text{s}$  &  $Q = 143 \text{ kJ/mole/}^\circ\text{K}$ )
2. A steel containing 0.2 % carbon was heated to 950°K for 15 hours. Find the depth of layer in which there is no carbide. Assume that steel consists of ferrite and carbide. The solubility of carbon in ferrite at this temperature is 0.015% and % C at the surface is negligible. (Given  $D_0 = 2 \times 10^{-6} \text{ m}^2/\text{s}$  &  $Q = 84.4 \text{ kJ/mole/}^\circ\text{K}$ )
3. The concentration of carbon on the surface of iron is maintained at 1.00% at 1175°K for 2hours. Estimate the depth at which % C would be 0.5%. Use the diffusivity values given in question1. Assume initial carbon content of iron to be negligible.

**Answer:**

1. If iron is kept at high temperature in an environment having high carbon potential it diffuses into iron. The depth of carburization ( $x$ ) is proportional to  $\sqrt{Dt}$ . Therefore  $\frac{x_1}{x_2} = \sqrt{\frac{D_1 t_1}{D_2 t_2}}$  Since  $D = D_0 \exp\left(-\frac{Q}{RT}\right)$   $\left(\frac{x_1}{x_2}\right)^2 = \frac{t_1}{t_2} \exp\left\{\frac{Q}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right\}$  Therefore  $\left(\frac{0.50}{0.75}\right)^2 = \frac{8}{t} \exp\left\{\frac{143000}{8.33}\left(\frac{1}{1250} - \frac{1}{1200}\right)\right\} = 10.14 \text{ hours}$
2. If initial carbon in steel is  $C_i$  & soluble carbon in ferrite is  $C_\alpha$  amount carbon to removed / unit cross section through a distance  $dx = (C_i - C_\alpha)dx = \text{flux of carbon atom}$  in time  $dt = -Jdt = D \frac{dC}{dx} dt$  Assuming carbon concentration at surface as 0  $\frac{dC}{dx} = \frac{C_\alpha - 0}{x}$  Thus  $(C_i - C_\alpha)dx = D \frac{C_\alpha}{x} dt$  or;  $x = \sqrt{\frac{2DtC_\alpha}{(C_i - C_\alpha)}}$   $D = D_0 \exp\left(-\frac{Q}{RT}\right) = 2 \times$



$$10^{-6} \exp\left\{-\frac{84400}{8.31 \times 950}\right\} = 4.55 \times 10^{-11} \text{m}^2/\text{s} \text{ Now } C_i = 0.2 \text{ \& } C_a = 0.015 \text{ Therefore}$$

$$x = \sqrt{\frac{2 \times 4.55 \times 10^{-11} \times 0.015 \times 15 \times 3600}{(0.2 - 0.015)}} = 0.00063 \text{m} = 0.63 \text{mm}$$

3. Carbon content at surface  $C_s = 1.00$ , Initial carbon  $C_0 = 0.0$  & carbon content at a distance  $x$   $C=0.5$

$$\frac{C_s - C}{C_s - C_0} = \text{erf}\left[\frac{x}{2\sqrt{Dt}}\right] \text{ Or, } \frac{1.0 - 0.5}{1.0} = \text{erf}\left[\frac{x}{2\sqrt{Dt}}\right] \text{ Or, } \frac{x}{2\sqrt{Dt}} = 0.477$$

$$\text{Or, } x = 0.477 \times 2 \times \sqrt{0.2 \times 10^{-4} \exp\left(-\frac{143000}{8.31 \times 1275}\right) \times 2 \times 3600} = 0.000425 \text{m}$$

$$= 0.445 \text{mm}$$

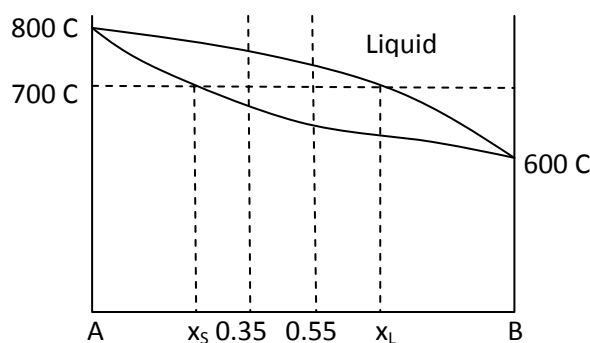
### Exercise:

- Nickel, Aluminium & Copper have face centered cubic structure yet Ni is soluble in copper whereas Al has only a limited solubility. Explain why it is so?
- Two metals A (melting point 800C) and B (melting point 600C) form a binary isomorphous system. An alloy having 35% B has 75% solid and rest liquid whereas an alloy having 55%B has 25% solid at 700C. Estimate the composition of solidus and liquidus at the above temperature.
- Apply phase rule to the two phase field of a binary isomorphous diagram. What conclusion can be drawn?

### Answer:

- Both the species must have the same crystal structure, valence & nearly the same atomic diameters. Ni & Cu have nearly the same lattice parameter as well as valence. However lattice parameter of Al is a little longer, it also has a higher valence. Therefore its solubility in Cu is limited.

- A schematic binary phase diagram of this system is shown below:



Apply lever rule to alloys with 35% B & 55%B  $\frac{x_L - 0.35}{x_L - x_s} = 0.75$  and  $\frac{x_L - 0.55}{x_L - x_s} = 0.25$  Solve to get  $x_L = 0.65$  &  $x_s = 0.25$

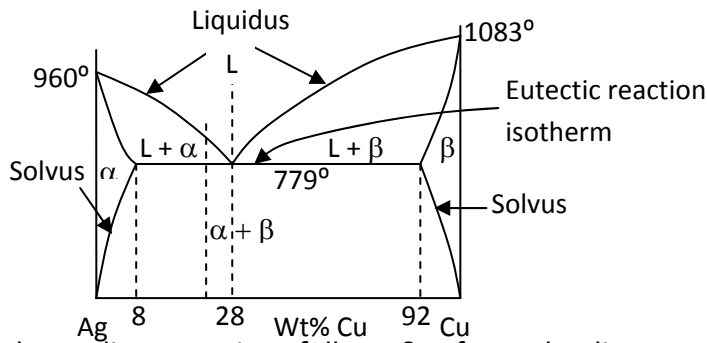
3. Phase rule at constant pressure states  $P+F=C+1$ . In the two phase field  $P=2$ , it is a binary system therefore  $C=2$ . On substitution in the above expression  $F=1$ . It has only one degree of freedom. If temperature is constant compositions of coexisting liquid & solid are fixed. This is given by the intersection of the tie line at the specified temperature. If you specify composition of the solid the composition of coexisting liquid and the temperature are fixed.

**Exercise:**

4. A binary alloy having 28 wt % Cu & balance Ag solidifies at 779°C. The solid consists of two phases  $\alpha$  &  $\beta$ . Phase  $\alpha$  has 9% Cu whereas phase  $\beta$  has 8% Ag at 779°C. At room temperature these are pure Ag & Cu respectively. Sketch the phase diagram. Label all fields & lines. Melting points of Cu & Ag are 1083° & 960°C respectively. Estimate the amount of  $\alpha$  &  $\beta$  in the above alloy at 779°C & at room temperature.
5. A molten Ag-Cu (20%) alloy is allowed to cool slowly till room temperature. Refer to the diagram in problem 1 and plot its cooling curve. Estimate %  $\alpha$  just after it has solidified at 779°C & room temperature. Sketch its microstructure and find % Eutectic .
6. A diffusion couple consisting of Ag & Cu was kept at just below its eutectic temperature. Show with the help of a schematic diagram the concentration profile along its length at different lengths of hold time. Assume that the couple is quenched to suppress any transformation below its eutectic temperature.
7. Two alloys belonging to a binary system have the following microstructures. One having 25% B consists of 50%  $\alpha$  & 50% eutectic and the other having 0.75%B has 50%  $\alpha$  & 50% eutectic. Microstructural examination shows that eutectic is made of 50%  $\alpha$  & 50%  $\beta$ . Estimate the composition of  $\alpha$ ,  $\beta$  & eutectic.
8. Draw the phase diagram for a binary alloy system having following features. Melting point of the two metals (A & B) are widely different. These are partially soluble in each other. There is one three phase reaction isotherm at a temperature higher than the melting point of B but lower than that of A. Write down the equation representing the 3 phase reaction. What is it commonly known as?

**Answer:**

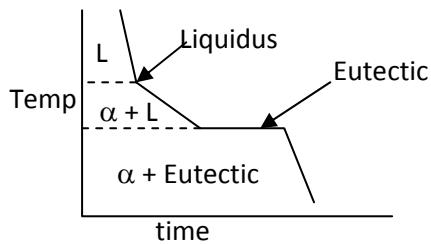
4. The process of solidification at 779°C can be represented as  $L = \alpha + \beta$ . This involves equilibrium between 3 phases. Such a system is known as eutectic. The phase diagram is as follows:



28% Cu is a eutectic alloy. % phases are given by lever rule.  
 $\% \alpha = \frac{92-28}{92-8} \times 100 = 76\%$  at 779°C. At room temperature

$$\% \alpha = \frac{100 - 28}{100 - 0} \times 100 = 72\%$$

5. The cooling curve is as follows & refer to the diagram above.

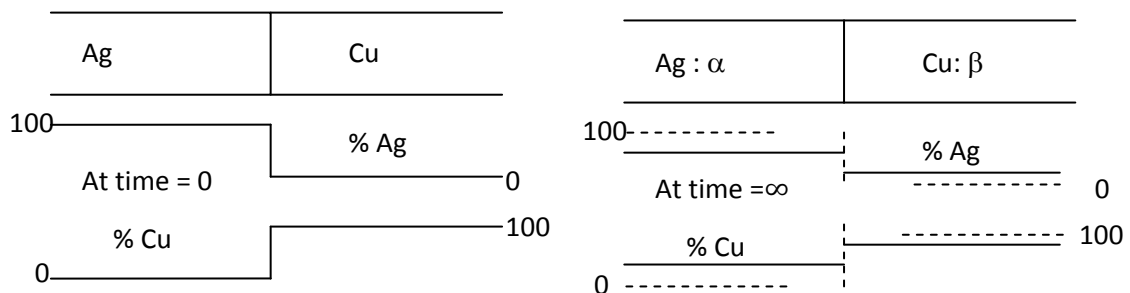


At 779°C:  $\% \alpha = \frac{92-20}{92-8} \times 100 = 86\%$  Note that α is present as primary phase & within eutectic.  
 At room temperature

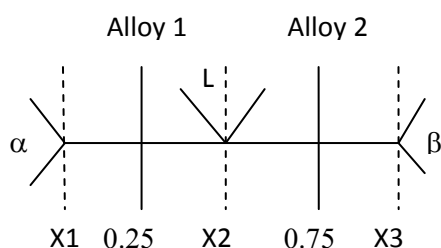
$$\% \text{Eutectic} = \frac{20 - 0}{28 - 0} \times 100 = 71\%$$

The microstructure will consist of proeutectic α & eutectic.

6. Ag will diffuse into Cu & vice versa. The concentration profile will depend on hold time. These are given for different lengths of time assuming that length of both Ag & Cu are infinite.



7. Let the eutectic reaction be represented as  $L (X_1) = \alpha (X_2) + \beta (X_3)$ . Composition of the three phases are given with brackets. The eutectic reaction isotherm is shown in the following diagram. There are 3 unknowns. Therefore we need three equations to find these. These can be formulated as follows.



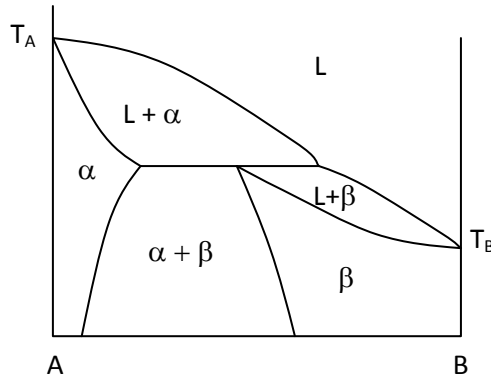
For alloy 1:  $\frac{X_2 - 0.25}{X_2 - X_1} = 0.5$  or;  $2 X_1 + 2 X_2 = 1$

For alloy 2:  $\frac{X_3 - 0.75}{X_3 - X_2} = 0.5$  or;  $2 X_2 + 2 X_3 = 3$

For eutectic:  $\frac{X_3 - X_2}{X_3 - X_1} = 0.5$  or;  $X_1 - 2 X_2 + X_3 = 0$

These equations can be solved by eliminating the unknown variables one after the other. Answer:  $X_1 = 0$ ,  $X_3 = 1$  &  $X_2 = 0.5$

8. The diagram is shown below. The 3 phase reaction in case must be  $L + \alpha = \beta$ . This is called peritectic reaction.

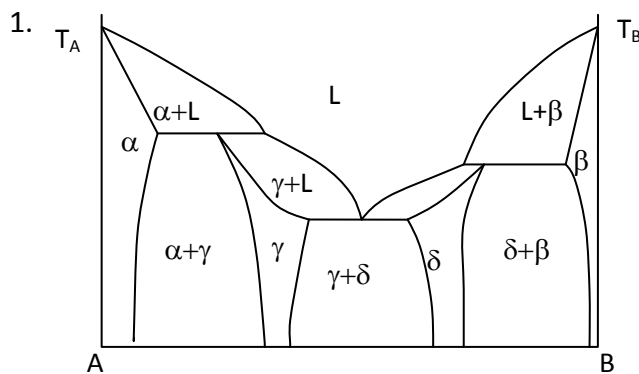


Note the melting points of the two metals are widely different. The peritectic isotherm lies between these two.

**Exercise:**

9. Sketch a binary phase diagram showing 2 peritectic and 1 eutectic reaction isotherm. Assume both metal A & B have nearly same melting point & there are two & terminal two intermediate solid solutions.
10. Draw a binary phase diagram where A undergoes an allotropic transformation (but not B) and there are one eutectic and one eutectoid transformation.
11. Cooling curve of a binary alloy looks exactly similar to that of a pure metal. Is this possible?

**Answer:**

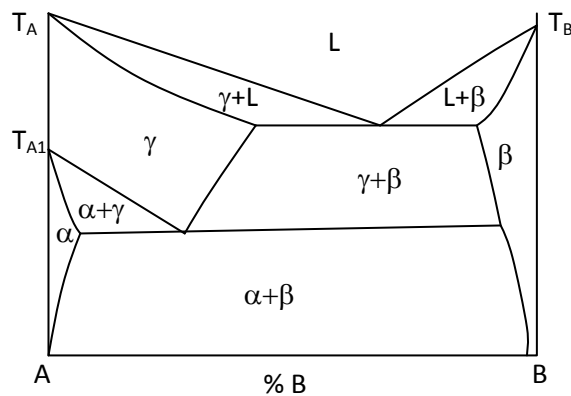


Peritectic 1:  $\alpha + L = \gamma$

Peritectic 2:  $\beta + L = \gamma$

Eutectic :  $L = \gamma + \delta$

2. The diagram is as follows:



Eutectic:  $L = \gamma + \beta$

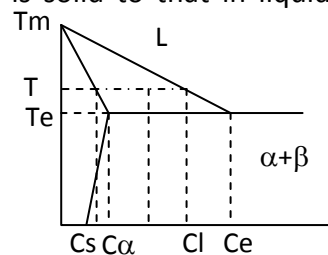
Eutectoid:  $\gamma = \alpha + \beta$

Allotropic transformation at :  $T_{A1}$

3. Yes, if it is an eutectic alloy.

**Exercise:**

12. The left hand side of a binary eutectic phase diagram is given below. The liquidus and solidus can be assumed to be straight lines. Find out expression for segregation coefficient (ratio of solute concentration in solid to that in liquid) in terms of the parameters given in this figure.



13. An alloy having solute concentration C is held at a temperature T within solid – liquid range and the liquid is filtered out. If this process is repeated by heating & filtering out the liquid will it ultimately give pure A? Is this a practical method of purification?

14. Sketch a phase diagram for a binary peritectic system with terminal solid solutions and draw free energy composition diagrams at two temperatures (i) a little above peritectic reaction isotherm (ii) a little below peritectic reaction isotherm.

15. Estimate entropy change during solidification of the following elements and comment on the nature of the interface between solidifying crystal and liquid. The latent heat and melting point are given with brackets. (a) Al [10.67kJ/mole, 660C] (b) Si [46.44kJ/mole, 1414°C]

16. Microstructure of a cast Al-4.5%Cu alloy shows some amount of eutectic when we know for sure that eutectic forms on solidification of 33%Cu alloy giving a having 5.5%Cu and  $Cu_2Al$ . Give reason.

17. A binary alloy has a peritectic reaction isotherm at 500°C given by  $\beta$  (10%A) + L (60%B) =  $\gamma$  (45%B) Estimate % phases present just above & below the peritectic reaction isotherm in an alloy having 50%B under equilibrium cooling. If the alloy were cooled relatively fast what type of structure will you expect on completion of solidification.

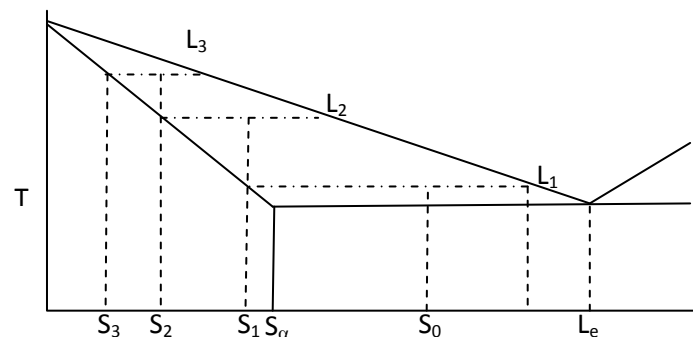
18. Why the eutectic structure does not exhibit coring?

19. When would you expect planar solidification front? (a) When temperature gradient in solid ( $T'_S$ ) and in liquid ( $T'_L$ ) are both positive and  $T'_S > T'_L$  (b) When the liquid is super-cooled ( $T'_L < 0$ ) and the solid is maintained at constant temperature. Assume temperature gradients in both liquid and solid to be constant. Give reasons.

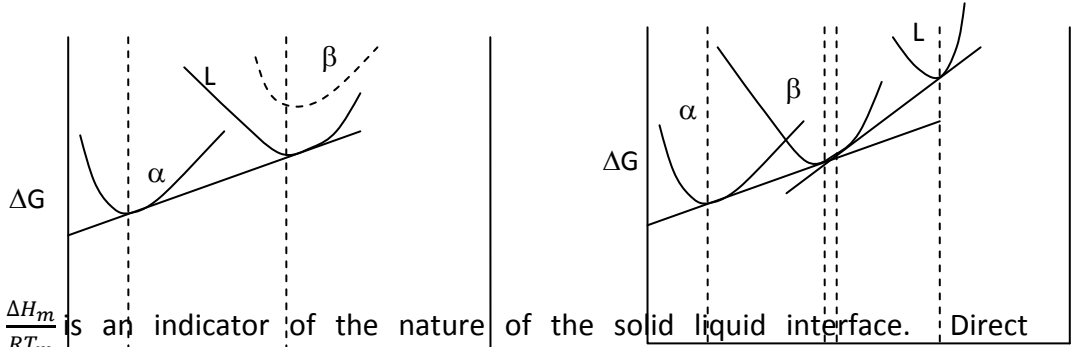
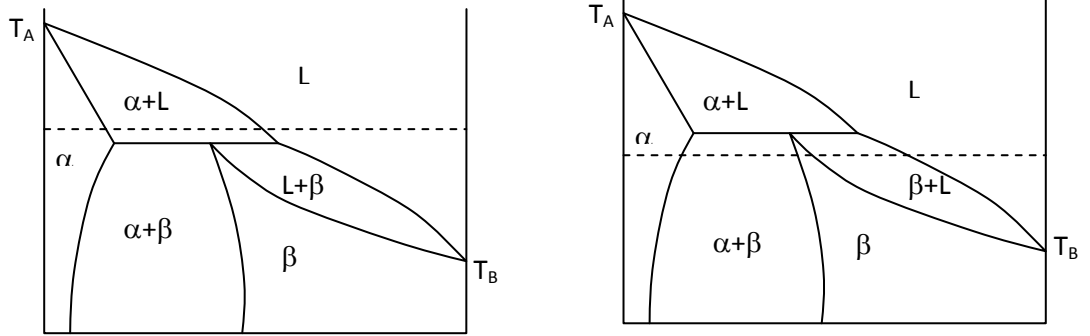
**Answer:**

9. The equation representing the solidus is given by  $\frac{T_m - T}{T_m - T_e} = \frac{0 - C_s}{0 - C_\alpha}$  & the line representing liquidus is given by  $\frac{T_m - T}{T_m - T_e} = \frac{0 - C_l}{0 - C_e}$  Thus segregation coefficient =  $\frac{C_s}{C_l} = \frac{C_\alpha}{C_e} < 1$  Clearly in this case it is a constant independent of temperature.

10. Let us look at an enlarged view of the phase diagram in problem 4. It is seen by holding the alloy at a temperature little above eutectic temperature impurities could be partitioned into the liquid. If this is repeated composition of solid would tend to move towards purer A. In principle purification is possible. However since at every stage significant amount of liquid is rejected. The ultimate yield will be very small. This process of purification is not practical.

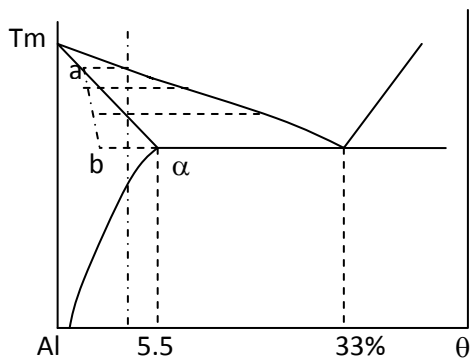


11.



12.  $\frac{\Delta S_m}{R} = \frac{\Delta H_m}{RT_m}$  is an indicator of the nature of the solid liquid interface. Direct substitution of latent heat ( $\Delta H_m$ ) melting point ( $T_m$ ) & universal gas constant ( $R$ ) the above indicator for Al = 1.35 & for Si = 3.26. Therefore S/L interface was planar for Al & faceted for Si.

13. Normal cooling is rather fast. The solid that forms during solidification changes with time. Diffusion in solid being slow the previously formed solid does not get enough time to change its composition. The average solid composition therefore follows a different path as shown in the following diagram. The eutectic reaction is given by L (33%B) =  $\alpha$  (5.5%B) + Cu<sub>2</sub>Al ( $\beta$ ).

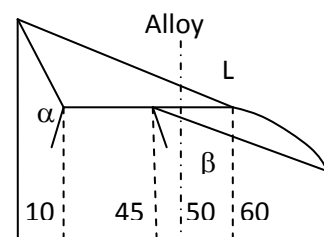


Under equilibrium cooling the alloy on solidification should consist of phase  $\alpha$  only. There should be no eutectic. If cooling rate is fast composition of solid does not follow line  $a\alpha$ . Instead it follows line  $ab$ . This is because fast cooling does not give enough time for the solid to change its composition. Even at eutectic temperature some liquid is left behind. This is why we get some eutectic in the cast structure.

14. The phase diagram is as follows:

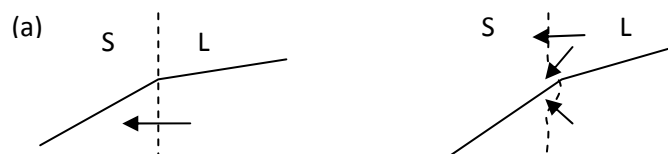
%  $\alpha$  at just above peritectic temp =  $(60-50)/(60-10) = 0.20$   
balance 0.8 is liquid.

Just below peritectic temp: %  $\beta$  =  $(60-50)/(60-45) = 0.66$

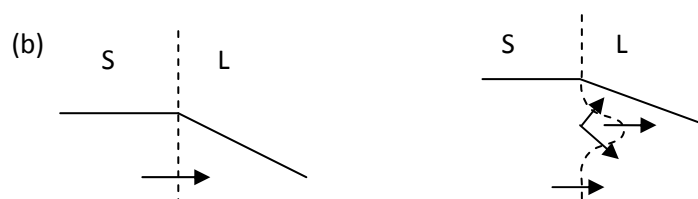


As the temperature drops below peritectic reaction isotherm there is a marked change in % liquid. It decreases from 80% to 33%. Phase  $\beta$  forms as a result of peritectic reaction at liquid /  $\alpha$  interface. In the process  $\beta$  gets surrounded by a layer of  $\alpha$ . B atoms from liquid must diffuse through  $\alpha$  layer to  $\beta$  interface so that peritectic reaction could occur. Therefore due to non equilibrium cooling on solidification the alloy may have some amount of  $\beta$  retained. Final structure may consist of cored  $\alpha/\beta$  dendrites surrounded by cored  $\alpha$ . What is the composition of the last solid? (Answer: pure B)

15. Coring means composition gradient within a phase as a result of relatively fast cooling. Eutectic transformation takes place at a fixed temperature. It is a mixture of two phases of fixed compositions. As per phase rule the degree of freedom is zero. This is why question of change in composition does not arise.
16. The temperature profile across the solid liquid interface in case (a) is given below. The arrow indicates the direction of heat flow. In case any a bulge develops on the solid liquid interface the direction of heat flow is changed. Bulge provides additional area through which heat could flow. Solidification is also accompanied by release of latent heat. Therefore temperature of this zone would rise and the bulge would shrink and disappear.



However in case (b) the situation is opposite. There being no temperature gradient in solid, heat cannot flow through it. It could only flow into super cooled liquid. In case a bulge forms the latent heat released can easily flow into super cooled liquid. Bulge provides additional area through which heat could flow. Unlike the previous situation it would grow. Such instability would exist all along the curved face. This would therefore promote dendritic solidification.



**Exercise:**

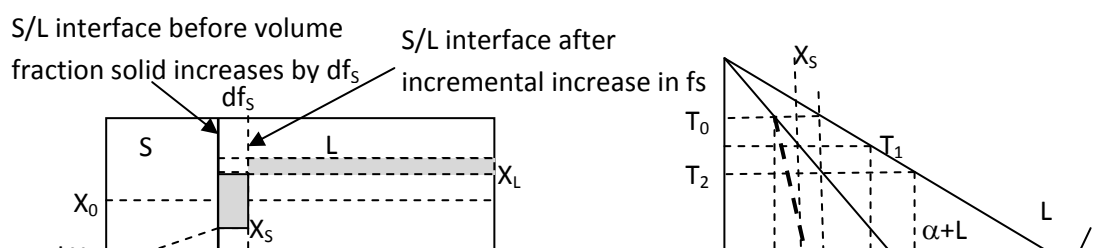
20. Estimate the temperature gradient that is to be maintained within solid aluminum so that the planar solidification front moves into liquid aluminum maintained at its melting point at a velocity of 0.001m/s. Given thermal conductivity of aluminum = 225 W/mK, latent heat of fusion = 398 KJ/kg and density = 2700kg/m<sup>3</sup>.



21. What is partition coefficient? Derive Scheil equation for solidification of binary alloys. State the assumptions made during its derivation. What is the composition of the last solid that forms during solidification of a terminal solid solution of a binary eutectic system?
22. Use the equations derived in the previous question to plot how the compositions at the solid liquid interface keep changing as it moves while an Ag 5% Cu alloy is directionally solidified. Ag-Cu binary phase diagram has a eutectic reaction isotherm at 779°C where an alloy containing 28.1%Cu solidifies as a mixture of  $\beta$  having 8.8%Cu and  $\alpha$  having 92%Cu. The melting point of Ag is 960°C. The solidus and liquidus lines may be assumed to be straight lines. What is the amount of eutectic in the final structure?
23. What is constitutional super-cooling? When does this take place?
24. Derive an expression for solute concentration just ahead of a planar solidification front moving a constant velocity in a binary isomorphous alloy in absence of any convection current in liquid and diffusion in solid.

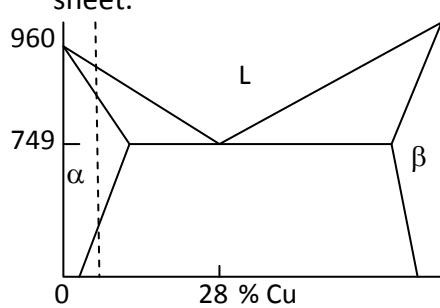
**Answer:**

17. Heat balance per unit area across a planar solidification front is given by  $K_S T'_S = K_L T'_L + L_v V$  where K is thermal conductivity.  $T'$  denotes temperature gradient. Suffix S & L represent solid & liquid respectively.  $L_v$  is latent heat of fusion / unit volume and V is velocity of the planar front. Since the temperature of liquid aluminum is constant  $T'_L = 0$  and thus  $V = \frac{K_S T'_S}{L_v} \therefore T'_S = \frac{398 \times 1000 \times 2700}{225} \times 0.001 = 4776 \text{ K/m}$
18. During solidification of an alloy solute gets partitioned between the solid and liquid. Partition coefficient (k) defines the extent to which it gets segregated between solid and liquid. It is given by  $k = \frac{X_S}{X_L}$  where  $X_S$  &  $X_L$  denote composition of solid and liquid when both coexist. Derivation of Scheil equations are based on the following assumptions: (i) k remains constant (ii) There is no diffusion within solid (iii) there is perfect mixing within liquid (there is no concentration gradient within liquid). Let us apply this to solidification of a terminal solid solution of a binary eutectic system given below. The figure on the left shows the position of solid liquid interface at a temperature  $T_1$  when fraction of solid is  $f_s$ . As the interface moves by an infinitesimal distance corresponding to an increment of volume fraction solid by  $df_s$  an amount of solute given by the shaded area within the liquid moves over to solid. Amount of solute that is added to solid is also shown by a shaded area within the solid.

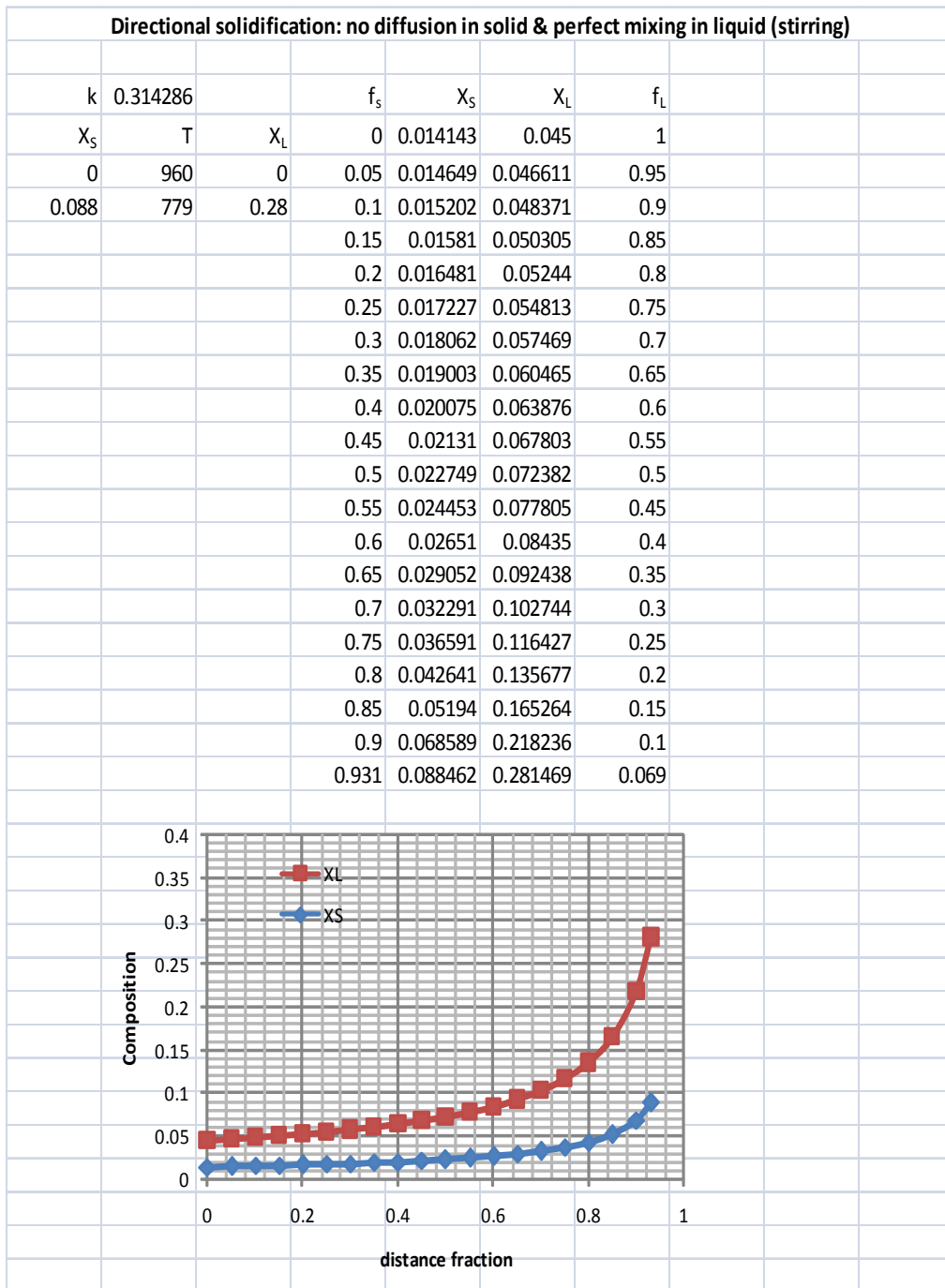


Apply conservation of solute. Equate the filled in area:  $(X_L - X_S)df_S = (1 - f_S)dX_L$   
 Note that  $X_S = kX_L$ . The final expression is obtained by integration  $\int \frac{df_S}{1-f_S} = \int \frac{dX_L}{(1-k)X_L}$   
 $\therefore -(1-k)\ln(1-f_S) = \ln(X_L) + C$ . Substitute initial condition: When  $f_S = 0$ ;  $X_L = X_0$  to evaluate the constant C. This is given by  $C = -\ln(X_0)$ . Thus  $X_L = X_0(1-f_S)^{(1-k)}$  and  $X_S = kX_0(1-f_S)^{(1-k)}$ . These two give the amount of solid and liquid like lever rule in case of equilibrium solidification. These are known as Scheil equation. Note that when  $X_L$  approaches eutectic composition ( $X_E$ ) still there will be some liquid left. This is given by:  $f_L = \left(\frac{X_E}{X_0}\right)^{(1-k)}$ . The last solid that forms has the same composition as that of the eutectic.

19. The phase diagram is shown in the following sketch. The results are given in an Excel sheet.



Note that the entries in first 3 columns give % solute in both solid and liquid at melting point and eutectic temperature. Partition coefficient  $k$  is estimated from these entries. The next 3 columns give fraction solid and solute concentrations in solid and liquid.  $f_L = 1 - f_S$



The first row within columns 4-7 represents the initial condition when the alloy is totally liquid. The magnitude of  $X_L$  in this row is its initial composition.  $X_s$  is estimated by multiplying it with partition coefficient  $k$ . The rest of the cells are evaluated using the expressions derived in the previous problem. Composition of the last liquid to solidify is the eutectic. % eutectic in this case is 6.9%

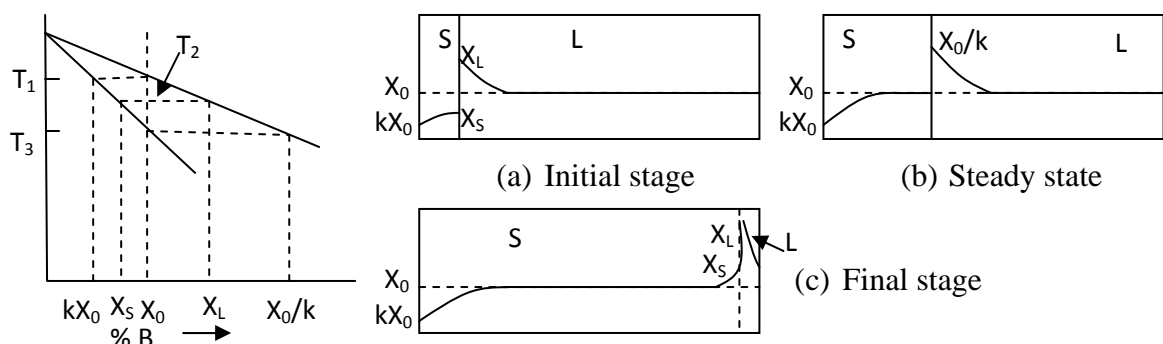
20. During solidification of an alloy solute gets partitioned between liquid and solid. As it cools the composition of liquid and solid keeps changing. While the composition of solid can change only by diffusion that of liquid can change due to both diffusion and convection current (stirring). The latter is much faster. If there is no convection or stirring concentration of solute builds up in the boundary layer. Its melting point

becomes significantly lower than that of the liquid away from the interface. If the temperature gradient beyond the solid liquid interface is less than a critical value constitutional super cooling occurs. This promotes solidification. If the gradient is more than the critical value the solid liquid interface remains planar.

21. The following sketch shows a part of the phase diagram indicating the temperature range over which solidification takes place in an alloy having a given composition ( $X_0$ ). Here it takes place between  $T_1$  and  $T_3$ .  $T_2$  is an intermediate temperature. Composition of solid & liquid at the interface within this range is given by the solidus and the liquidus. If partition coefficient is  $k$  composition of the first solid to form is  $kX_0$ . Subsequent solid would have higher solute content. The composition profile as a function of the distance from the mold face is also shown in the next sketch. As the planar front moves the liquid too at the interface will become richer in solute. Therefore mass transfer within the liquid due to diffusion will keep increasing. This would continue until the temperature reaches  $T_3$ . At this stage the composition of the liquid at the interface would be  $X_0/k$ , whereas that of the solid should be  $X_0$ . Hereafter the composition of the liquid and solid would not change even if the front moves. Velocity ( $v$ ) of the front would be determined by the rate at which solute would diffuse within the liquid. Since the concentration of solute would not change hereafter the velocity of the front should remain constant. This stage, called steady state would continue until the width of the remaining liquid becomes too small to maintain its steady state composition. In such a situation the final transient stage begins. The concentration starts building up. In case the alloy is a terminal solid solution of a eutectic system the last liquid to solidify would be eutectic whereas in case of an isomorphous system it would be pure B. During steady state the growth of solid layer needs to dispose of an additional amount of solute given by  $v(X_L - X_0)$ . This has to diffuse through the stagnant liquid. The rate at which the solute would diffuse is given by  $-D \frac{dX_L}{dx}$  where  $D$  is diffusivity and  $x$  is the distance from the solidification front. The concentration of solute ahead of the interface could therefore be evaluated by equating these two.

$$-D \frac{dX_L}{dx} = v(X_L - X_0) \text{ or; } \int \frac{dX_L}{(X_L - X_0)} = - \int \frac{v}{D} dx + \text{constant}$$

Constant of integration is determined from initial condition that at  $x = 0$ ,  $X_L = X_0/k$ . Thus we get  $X_L = X_0 \left\{ 1 + \frac{1-k}{k} \exp\left(-\frac{v}{D}x\right) \right\}$

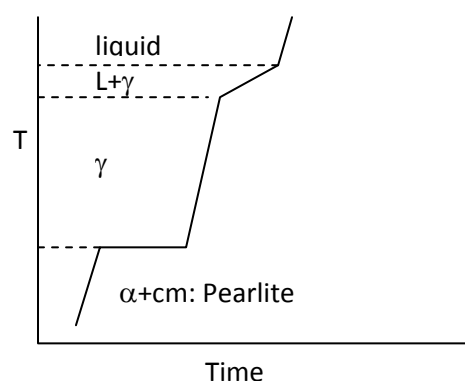


### Exercise:

1. FCC is a more close packed structure yet solubility of carbon in austenite which is FCC is higher than that in ferrite which is BCC. Why it is so?
2. Sketch the microstructure of 0.2% C steel. Calculate %Pearlite % cementite, % proeutectoid ferrite and % total ferrite.
3. Estimate the ratio of the widths of ferrite and cementite plates in lamellar pearlite.
4. Sketch the temperature – time diagram during the heating cycle of a 0.8% C steel. Use standard Fe-Fe<sub>3</sub>C phase diagram.
5. Estimate %C<sub>m</sub> in Ledeburite just below eutectic and just above eutectoid temperatures. What is its structure at room temperature?

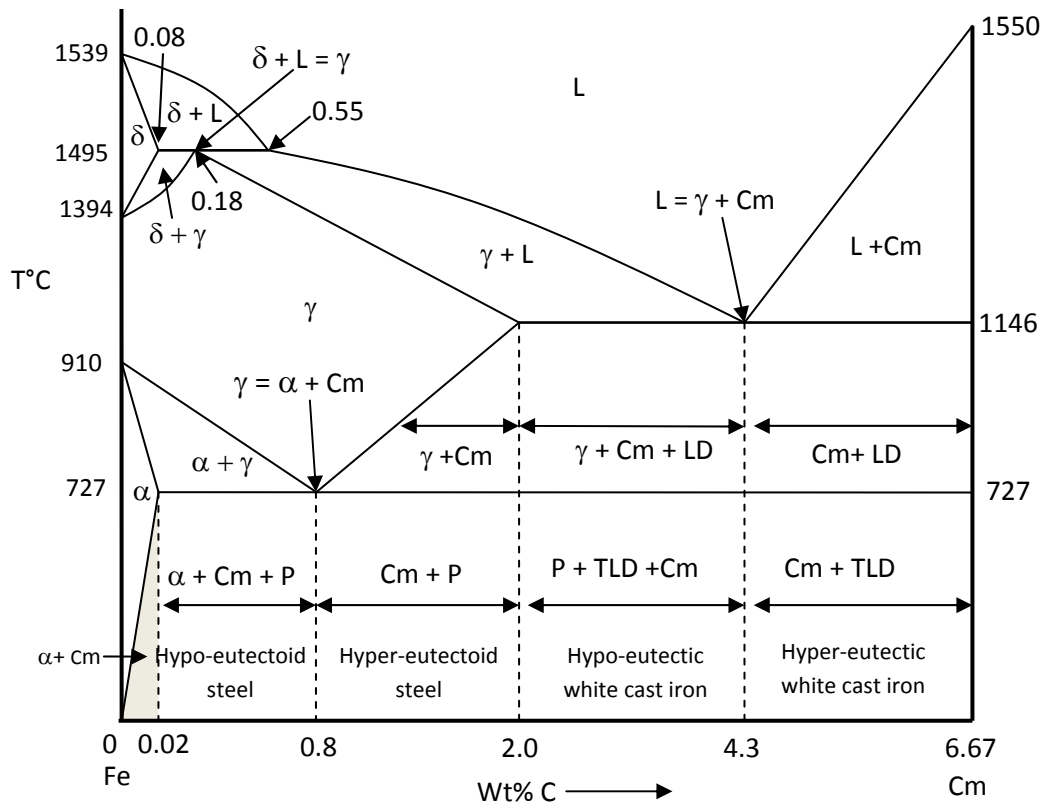
### Answer:

1. There are 2 types of interstitial sites octahedral & tetrahedral. In FCC the former is significantly larger than the latter. Whereas in BCC these are nearly same. The total open space is shared by more number of sites. Therefore interstitial gap in BCC is much smaller than that of FCC. This is why carbon which occupies interstitial site has higher solubility in austenite (FCC).
2. The structure would consist of proeutectoid ferrite and pearlite. Assume solubility of carbon in ferrite is negligible. Refer to phase diagram to get % Pearlite in 0.2% carbon steel =  $\frac{0.2-0}{0.8-0} \times 100 = 25$  Balance 75% is proeutectoid ferrite. % Cementite =  $\frac{0.2-0}{6.67-0} \times 100 = 3\%$  and total ferrite = 97%.
3. Assume density of ferrite and cementite to be same (Note that in reality density of cementite is a little higher than that of ferrite). % ferrite in pearlite =  $\frac{6.67-0.8}{6.67-0} \times 100 = 88$ . Balance 12% is cementite. Therefore the ratio of the widths of the two = 12/88. This is approximately equal to 1:7.
4. 0.8 % carbon corresponds to an eutectoid steel. It would first transform into austenite and then it starts melting when the temperature reaches its solidus temperature. It becomes totally liquid when temperature goes beyond its liquidus. Schematic heating curve is as follows:



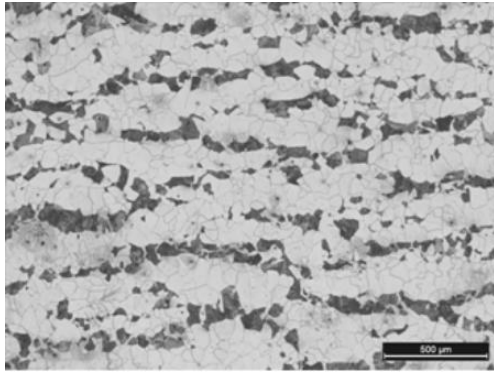
5. Assume % C in eutectic = 4.3. % Cm just below eutectic temperature =  $\frac{4.3-2.0}{6.67-2.0} \times 100 = 49\%$  and just above eutectoid =  $\frac{4.3-0.8}{6.67-0.8} \times 100 = 60\%$  Between eutectic to eutectoid temperatures proeutectoid cementite precipitates from austenite in eutectic. On subsequent cooling the austenite transforms into pearlite. At room temperature what we have is transformed eutectic.

## Iron – Cementite Meta-stable Binary Phase diagram

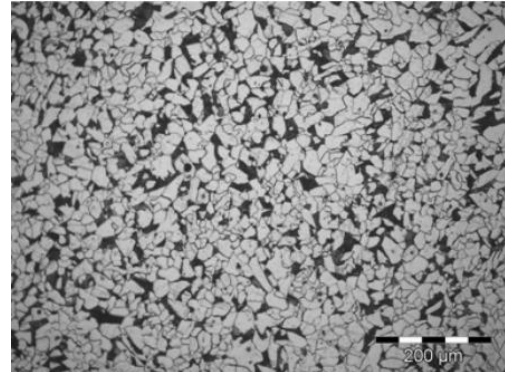


The above sketch is a little enlarged version of the phase diagram. Fe & Fe<sub>3</sub>C (also known as cementite) are the two components. Nomenclature: P denotes Pearlite the product of eutectoid reaction in iron carbon system. It is a mixture of ferrite and cementite. Ferrite =  $\alpha$ ,  $\gamma$  = austenite,  $\delta$  = high temperature ferrite, LD = Ledeburite: the product of the eutectic reaction. Note that austenite can form either by peritectic reaction between  $\delta$  & L and also by direct precipitation from liquid when % C > 0.55. TLD = transformed Ledeburite. Ledeburite is a eutectic mixture of austenite and cementite. At room temperature austenite is not stable. It transforms into Pearlite. TLD therefore consists of tiny nodules of pearlite in cementite matrix. It also describes the main structural features of five types of iron carbon alloys. Low carbon steel consists of ferrite and tertiary cementite. Hypo-eutectoid steel has ferrite, pro-eutectoid cementite and pearlite. Hyper-eutectoid steel has pro-eutectoid cementite (secondary) and pearlite. Hypo-eutectic cast iron has pearlite, transformed ledeburite and pro-eutectoid (secondary) cementite. Hyper-eutectic cast iron has primary cementite and transformed ledeburite. So far evolution of microstructure that takes place during solidification of iron – carbon alloys has been explained with the help of schematic microstructure. The following page gives a few typical microstructures of steel and cast iron. These were prepared by some of my students and colleagues at IIT Kharagpur and CSIR-NML Jamshedpur.

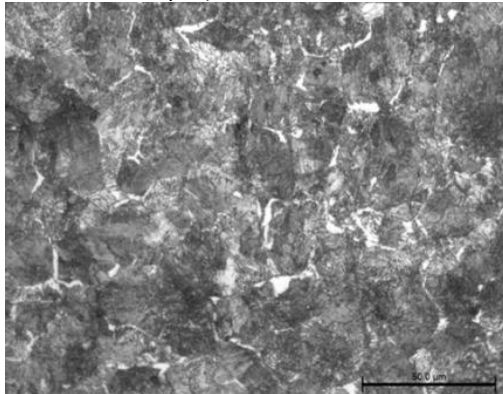
## Typical microstructures of steel and cast iron



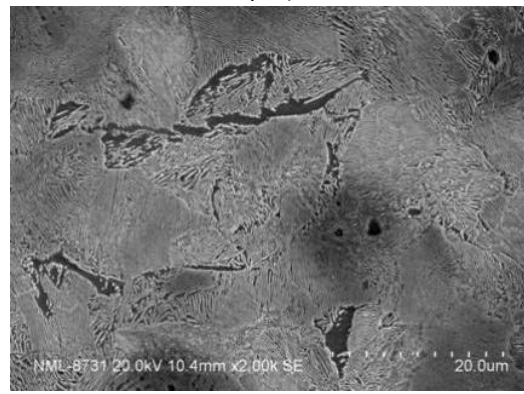
**Fig A1:** Microstructure of 0.15% carbon steel consisting of ferrite (bright) and pearlite (dark). Lamellar feature pearlite is not visible. (Curtsey G Das CSIR-NML Jamshedpur)



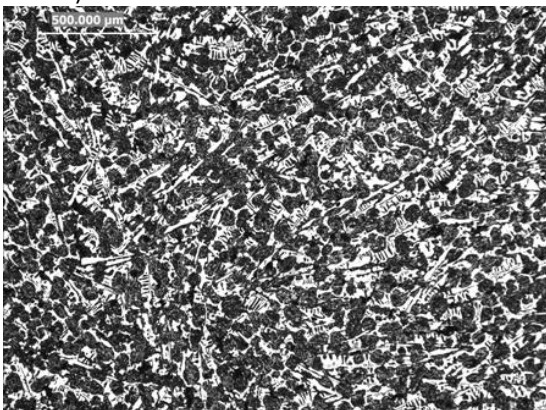
**Fig A2:** Microstructure of medium carbon steel consisting of ferrite (bright) and pearlite (dark). Amount of pearlite increases with % carbon. (Curtsey G Das CSIR-NML Jamshedpur).



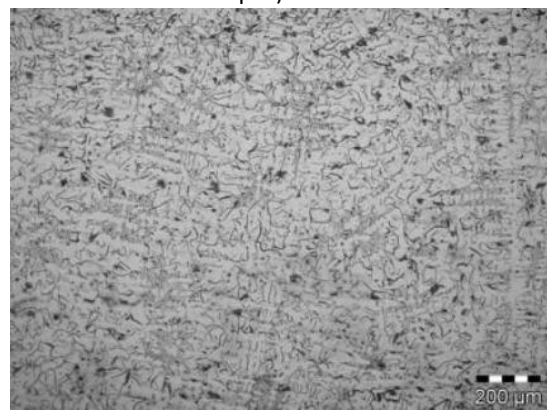
**Fig A3:** Microstructure of a near eutectoid steel having a little pro-eutectoid cementite in a matrix of pearlite. In some of regions finer details of pearlitic (lamellar) structure is visible.



**Fig A4:** SEM image of 0.84% C steel having nearly 100% pearlite. Lamellar feature is clearly visible. The dark region is pro-eutectoid cementite. (Curtsey G Das CSIR-NML Jamshedpur)



**Fig A5:** Microstructure of hyper-eutectic white cast iron showing primary carbides (cm). The rest of the region consists of a mixture of pearlite and cementite or transformed eutectic (ledeburite).



**Fig A6:** Microstructure of grey cast iron having graphite flakes in a matrix consisting of mostly ferrite and some amount of pearlite. (Curtsey G Das CSIR-NML Jamshedpur)

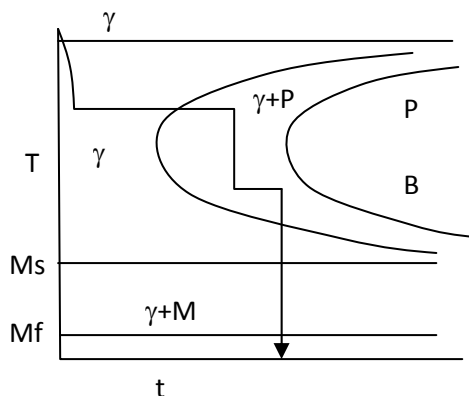


**Exercise:**

1. If an eutectoid steel is kept at 700°C what change do you expect?
2. What is the limitation of phase diagram?
3. If a piece of steel having 0.8 % carbon has martensitic structure can it be converted to fully pearlite structure by holding it at 700° C?
4. Suggest a method of getting a mixture of Pearlite, Bainite & Martensite in an eutectoid steel.
5. Which microstructure in eutectoid steel has maximum hardness? Give reason.

**Answer:**

6. Lamellar structure is unstable as it has large surface area. Initially the cementite plates would break down in to globular structure. Size of the globule will grow with time. Cementite is also metastable on prolonged thermal exposure it breaks down in to ferrite and graphite.
7. Phase diagram does not show the effect of cooling rate. It gives the expected phases in an alloy at a given temperature under equilibrium condition.
8. No. To get pearlitic structure it must be heated back to austenite state then cooled slowly.
9. This is possible through two step isothermal transformation in Pearlitic & Bainitic region followed by queching. The cooling scheme is shown below. Note that no transformation takes place during quenching (fast cooling: the vertical step in cooling curve) if temperature is above Ms.



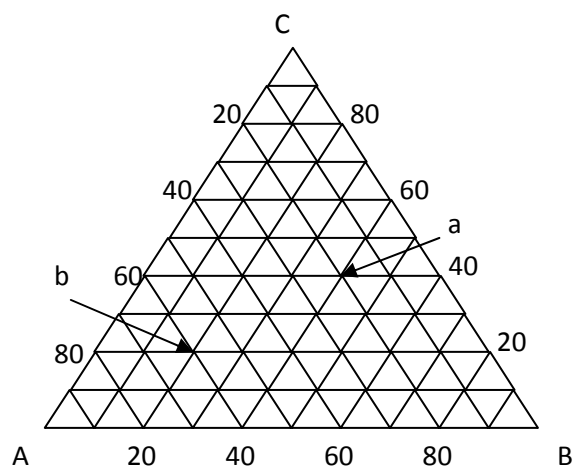
10. Martensitic structure has the maximum hardness. This is because of the presence of carbon atoms in the interstices is far in excess of its normal solubility in ferrite lattice. This results in tetragonal lattice distortion. This makes dislocation movement very difficult. This in conjunction with extremely fine microstructure account for the high hardness of Martensite.

**Exercise:**

1. Sketch a composition triangle to represent composition of a ternary alloy and locate the alloys having following compositions (a) 20A 40B 40C (b) 60A20B20C
2. In a ternary system estimate amount of  $\alpha$  &  $\beta$  in an alloy having 22.5%A, 23.5%B, 50%C at a given temperature T where three phases  $\alpha$ ,  $\beta$  and liquid are in equilibrium. Composition of these three phases are liquid: 15A 15B70C,  $\alpha$ : 60A20B20C,  $\beta$ :20A40B20C
3. Find out the composition of the alloy which has 50%  $\alpha$  & 50%  $\beta$  at temperature T for system described in problem 2.
4. A ternary system consisting of three metals A, B & C has two binary eutectics; one between A & B and the other between B & C and one binary isomorphous system between A & C. The system has two terminal solid solutions ( $\alpha$  and  $\beta$ ). Assume that the temperature of binary eutectic between A & B is higher than that of B & C. (a) Sketch a space model of this system indicating single phase, two phase and three phase regions. (b) Sketch an isothermal section at a temperature where there are single phase, two phase and three phase regions. (c) Sketch a vertical section passing through B and the midpoint between A & C of the composition triangle.

**Answer:**

1.

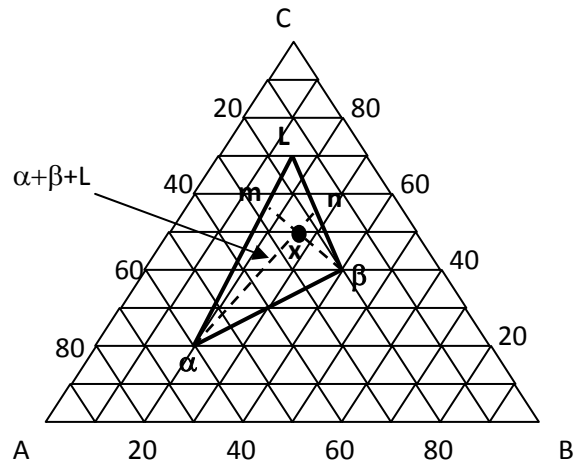


2. Isothermal section at temperature T is given in the following figure. Alloy composition is represented by point x. This lies within a triangle whose vertices

denote the composition of  $\alpha$ ,  $\beta$  and liquid. Join the points  $\alpha$  &  $\beta$  with  $x$  and extend these lines to meet the opposite side at points  $m$  &  $n$ . Measure the lengths  $\alpha n$ ,  $\alpha m$  &  $xm$ .

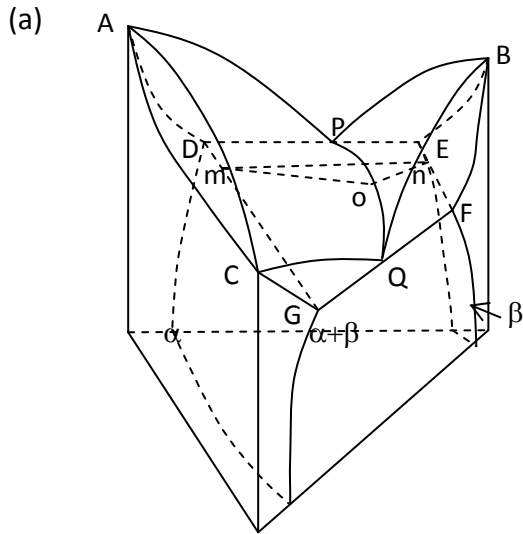
$$\% \alpha = \frac{xn}{\alpha n} \times 100$$

$$\% \beta = \frac{xm}{\alpha m} \times 100$$

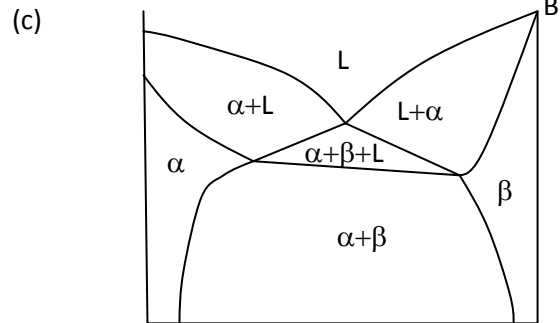
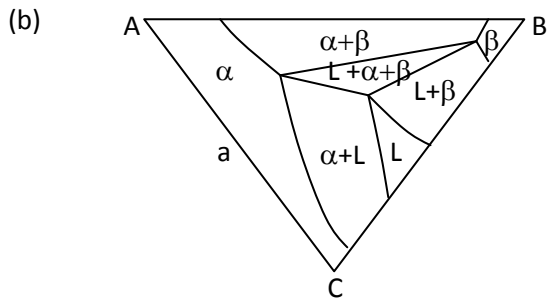


- Alloy having 50%  $\alpha$  is the mid point of tie line  $\alpha\beta$  of the above diagram. In this case it is 40%A 30%B 30%C

4. Sketch of the 3-D model of this system:



Note: line joining eutectic point P with Q is above the surface joining DEFG. There is a surface joining lines DE with FG passing through line PQ. Triangle mno is a section of the region between these two surfaces.



### Exercise:

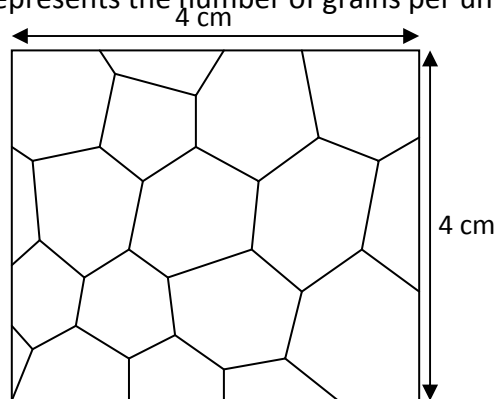
1. List a few advantages & limitation of hot working.
2. List a few advantages and limitations of cold working.
3. Estimate energy required to extrude 250mm diameter billet of aluminium to 25mm diameter rod. Given flow stress of aluminium = 10MPa and therefore find out the amount of force needed. Process efficiency = 70%. Neglect work hardening.
4. If the maximum strain per pass in a wire drawing operation is 0.65 how many drawing passes will be required to draw wire of 0.25mm dia from 5mm wire rod?

### Answers:

1. Advantages: Hotworking is done above recrystallisation temperature where flow stress is very low therefore it is possible to give large deformation with minimum effort (energy required is less). No further heat treatment is required. Product is in annealed / normalised state. Limitation: Metal tends to oxidise. Scaling leads to loss of material. Tool life is shortened because of abrasive scale, high temperature and lack of lubrication. Poor surface finish & difficulty in precise gauge control. It is necessary to pickle hot rolled sheets before cold rolling to remove oxide scale.
2. Advantages: Good surface finish & precise gauge control. Gives higher strength. Limitation: needs intermediate annealing because of work hardening, deformation / pass is limited by flow stress which increases with deformation, more energy intensive, needs more powerful equipment, Makes the material anisotropic.
3. Work done per unit volume =  $\frac{1}{\mu} \int \sigma d\varepsilon = \frac{\sigma}{\mu} \ln\left(\frac{A_0}{A_f}\right) = \frac{\sigma}{\mu} 2\ln\left(\frac{d_0}{d_f}\right) = \frac{10}{0.7} 2\ln\left(\frac{250}{25}\right) \approx 66 \text{ MJ/m}^3$  where  $\mu$  is efficiency. Stress required = 66MPa. Force = stress x area =  $66 \times \frac{\pi}{4} \times 0.25^2 = 3.24 \text{ MN}$
4. Strain in a single pass is  $\varepsilon = \ln\left(\frac{A_0}{A}\right) = 2\ln\left(\frac{d_0}{d}\right)$ . Using this it can be shown that diameter after  $n^{\text{th}}$  pass is given by  $d_n = d_0 \exp\left(-\frac{n\varepsilon}{2}\right)$ . Therefore number of pass =  $\frac{2}{\varepsilon} \ln\left(\frac{d_0}{d_n}\right) = \frac{2}{0.65} \ln\left(\frac{5}{0.25}\right) = 9.3$ . This means successive dies should be so chosen that in 10 passes the wire could be drawn.

**Exercise:**

- While designing rolling process discuss whether large reduction in a single pass is preferable to smaller reductions in successive passes?
- The following figure gives a typical microstructure of a pure metal. Magnification is 100X. Find out its grain size in terms of ASTM number (N) which is given by:  $n = 2^{N-1}$ . Where n represents the number of grains per unit area



- Count the number of grains (F), number of edges (E) and number of points (P) in the microstructure given above. State the type of relationship these must follow.
- Describe how by cold work & annealing the following microstructures can be developed in a single phase alloy (a) fine grain structure (b) coarse grain structure
- Suggest at least two methods to know if a sheet of steel has preferred orientation.

**Answer:**

- During rolling contact length with roll surface is a function of reduction per pass. Let this be  $l$  and plate thickness be  $t$ . The ratio  $l/t$  gives an idea about the deformation zone. If  $l/t \gg 1$  deformation will be inhomogeneous. This would result in residual stresses and may lead to cracking. On the other hand if  $l/t \ll 1$  there will be too much of friction between work piece and the roll. For a given plate thickness the former represents large deformation where as the latter represents small deformation. The optimum rolling schedule is some where in between.
- Note that a large number indicates fine grain structure where as a small number denotes coarse grain structure. The above structure appears to be coarse. Measurement shows following result.

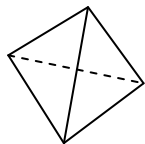
Count the number of grains assuming contribution of corner grain as  $\frac{1}{4}$  & grains at edge as  $\frac{1}{2}$ . Thus  $n = (6 + 3/4 + 9/2)/A = 11.25/A$

$$A = 16/2.54/2.54 = 2.48 \text{ in}^2$$

$$n = 2^{N-1} = 11.25/2.48 = 4.54$$

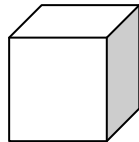
$$N = 3.2$$

7. Number of grains  $F = 18$ ,  $E = 39$  &  $P = 22$ : It follows the relation  $P+F = E+1$ . This is a 2-D form of Euler rule in Topology (There is a striking similarity with phase rule). If you count the number of corners, faces & edges of the following 3D shapes you get the Euler's rule (a) Tetrahedron (b) Cube (c) Octahedron



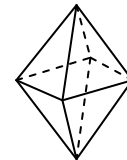
$$P = 4, F = 4, E = 6$$

$$P + F = E + 2$$



$$P = 8, F = 6, E = 12$$

$$P + F = E + 2$$



$$P = 6, F = 8, E = 12$$

$$P + F = E + 2$$

8. Critical nucleus is inversely proportional to amount of cold work. If before annealing if the metal is given small amount of cold work and then annealed at a relatively higher temperature it is likely to have coarse grain structure. On the other hand high amount of cold followed by annealing just above its recrystallisation temperature would give fine grain structure.
9. Preferred orientation would give different properties along different direction. Measurement of elastic modulus along different direction is a common method to determine anisotropy. It can also be established by direct measurement of crystal orientation by X-Ray diffraction technique or SEM using EBSD (Electron Back Scattered Diffraction) .

**Exercise:**

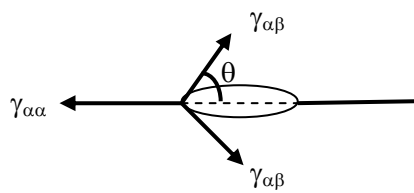
1. What types of alloys would respond to precipitation hardening?
2. Why Aluminium alloy rivets are stored in refrigerator?
3. For a lens shaped nucleus formed at interface estimate the angle  $\theta$  if  $\sigma_{\text{interface}} = 500$  and  $\sigma_{\text{surface}} = 600\text{mJ/m}^2$  and hence find out the shape factor of the precipitate.
4. Critical radii for both homogeneous and heterogeneous nucleation are the same yet the latter is more likely to occur. Give reason.
5. A batch of age hardenable alloy has been overaged by mistake. Is there any way to salvage these?

**Answers:**

1. Alloys which at room temperature has at least two phases and on heating beyond a temperature attains a single phase structure (solid) can respond to precipitation hardening provided it satisfies the following conditions i) on quenching precipitation could be suppressed ii) on heating or aging precipitates that nucleates are coherent with the matrix.
2. Aluminium alloy rivets are first solution treated to get supersaturated solid solution so that they are soft and could be easily deformed during rivetting. If these are not stored at subzero temperature after solution treatment they would age and become strong and relatively brittle. Therefore it would become unusable.



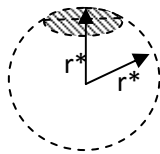
3. It is seen from the following figure



$$\gamma_{\alpha\alpha} = 2\gamma_{\alpha\beta}\cos\theta \therefore \theta = \frac{500}{1200} \text{ or } \theta = 65^\circ$$

$$S(\theta) = (2 + \cos^3\theta - 3\cos\theta)/2 = 0.0105$$

4. Although the critical radii for both homogeneous & heterogeneous nucleii are the same their volumes are significantly different. This is clear from the following sketch. Number of atoms required to form a stable nucleus is much less than that for homogeneous nucleus.



Dotted circle represents homogeneous nucleus. Horizontal dotted line is grain boundary & the hatched region is a lens shaped heterogeneously formed nucleus.

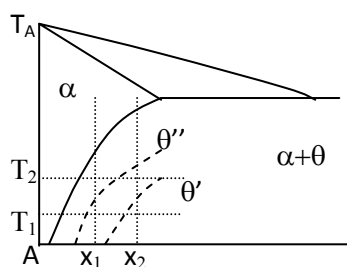
5. Yes. The overaged alloy can be solution treated and quenched before using them as rivets.

**Exercise:**

6. Why non-age hardenable aluminium alloys are chosen for beverage can?
7. When can you get more than one peak in the hardness versus aging time plot of a given alloy at a given temperature?
8. If in an alloy 1nm thick disk of  $\theta''$  has formed, estimate the critical diameter at which its coherency is likely to be completely lost. Given  $\gamma = 10\%$ ,  $\sigma = 500 \text{ mJ/m}^2$  and  $E = 70000\text{MPa}$
9. Ag rich GP zones can form in a dilute Al-Ag alloy. Given that the lattice parameters of Al and Ag are 0.405nm and 4.09nm respectively. What is the likely shape of these zones?
10. Under what heat treatment condition an age harden-able alloy can be machined?
11. Show that the shear stress to move a dislocation in a matrix by cutting dispersed spherical particles is proportional to the cube root of  $f$  where  $f$  represents volume fraction of particles.

**Answer:**

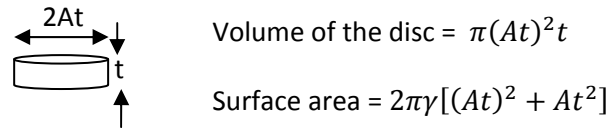
6. Beverage cans are manufactured by cold working. The alloy must have good ductility. Age hardenable alloys have relatively poor ductility. More over these are more expensive.
7. It happens when more than one coherent meta-stable precipitates form during aging. This illustrated in the following sketch where there are two intermediate coherent precipitates  $\theta'$  &  $\theta''$ :



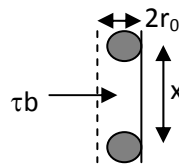
Note if  $x_2$  is aged at  $T_1$  we get one peak for  $\theta'$  another for  $\theta''$  with formation of  $\theta$  which is incoherent hardness starts dropping. Alloy  $x_1$  lies beyond  $\theta'$  solvus here only  $\theta''$  precipitate could form therefore only one peak is expected. Similar situation would arise for a given alloy when aged at different temperatures.

8. Coherent precipitates have disc shape. Its elastic stored energy is given by:  $E_s = \frac{1}{1-\nu} \bar{V} E \delta^2$ . Assuming Poisson ratio  $\nu = 1/3$  and volume of disc shaped precipitate  $\bar{V} = \pi (At)^2 t$  where  $t$  is thickness of disc and  $A$  is its aspect ratio. Therefore radius of disc =  $At$ . On substitution of these in the expression for elastic stored energy:

$E_s = \frac{3}{2} \pi E A^2 t^3 \delta^2$ . Likewise stored surface energy is given by:  $S = 2\pi\gamma[(At)^2 + At^2]$   
 Coherency is lost when  $E_s$  exceeds  $S$ . By equating the two one gets an expressions for critical thickness:  $t_c = \frac{4\gamma}{3E\delta^2} \left(1 + \frac{1}{A}\right)$  or;  $1 \times 10^{-9} = \frac{4 \times 500 \times 10^{-3}}{3 \times 0.001 \times 70 \times 10^9} \left(1 + \frac{1}{A}\right)$  on solving this  $A = 20$ . This means critical diameter =  $2 \times 20 = 40\text{nm}$



9. Al & Ag both have face centered close packed structure. Their atomic radii should be proportional to their lattice parameters. Therefore lattice mismatch =  $100 \times (0.409 - 0.405)/0.405 = 0.99\%$ . If mismatch is less than 5%, the shape is determined by its surface energy. Spherical shapes have less surface energy. If it is greater than 5% it is likely to be disc shaped.
10. It is easy to machine a material alloy when it is soft. Age hardenable material can be easily machined either when it is over aged to a low hardness or under solution treated condition. Some solution treated alloy would age harden during machining in those cases the former option is better.
11. Imagine spherical particles of radius  $r_0$  are arranged in a regular fashion where the inter-particle spacing is  $x$ . The work done to move a dislocation through a distance  $2r_0$  to cut a precipitate is given by  $(\tau b x) 2r_0$ . This is used up in creating new surface having energy =  $\pi r_0^2 \gamma$ .



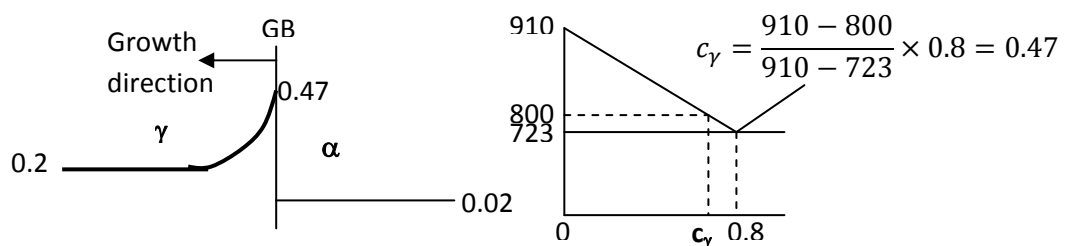
Equating these two you get  $\tau = \pi \frac{\gamma r_0}{2bx}$ . Volume fraction can be expressed in terms of precipitate size and spacing assuming that these are arranged in the form of a cube lattice with spacing  $x$  such that  $f = \frac{4}{3} \pi \left(\frac{r_0}{x}\right)^3$ . On substituting this in the expression for shear stress you get  $\tau = \frac{\pi\gamma}{2b} \left(\frac{3f}{4\pi}\right)^{\frac{1}{3}}$  or  $\tau \propto f^{\frac{1}{3}}$

**Exercise:**

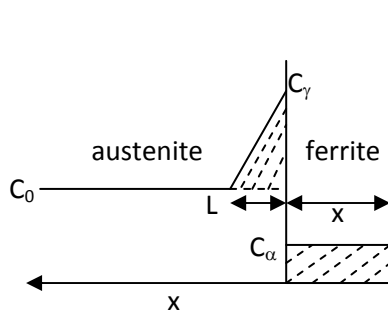
1. If a piece of steel having 0.2% carbon is quenched after soaking at a temperature just above  $A_1$  what type of structure will you get? Estimate approximate amounts of phases present and their compositions.
2. In a hypothetical experiment on steel having 0.2%C, a sample after soaking above  $A_3$  is quenched in a lead bath at 800°C and the structural change is followed with time. Assume that after some time austenite ( $\gamma$ ) boundary is covered by a thin layer of ferrite ( $\alpha$ ) and it continues to grow. This is known to be a diffusion controlled process. (a) Draw carbon profile perpendicular to  $\gamma/\alpha$  interface after some time has elapsed. (b) Derive an approximate expression for the thickness of ferrite as a function of time (c) If diffusivity of carbon in austenite at 800°C is  $3 \times 10^{-12} \text{ m}^2/\text{s}^2$  plot thickness as a function of time.
3. Carbon atoms occupy octahedral interstitial sites in austenite and ferrite. Estimate fraction of these sites that are occupied in these if carbon contents are 0.1 and 0.01wt% respectively.
4. Use Fick's first law to derive an expression for growth rate of pearlite nodule.
5. Show that the inter-lamellar spacing of pearlite is inversely proportional to the degree of under cooling.
6. Microstructure of isothermally transformed pearlite should have identical spacing in all colonies. However often its microstructure often shows that lamellar spacing varies from colony to colony. Why is it so?

**Answer:**

1. A piece of steel when kept at just above  $A_1$  temperature will have ferrite and austenite. % ferrite =  $(0.8-0.2)/0.8 \times 100 = 75\%$ . Assumption: %C in austenite = 0.8% where as that in ferrite is negligible. If quenched 25% austenite present at soaking temperature will convert into Martensite. The structure at room temperature will consist of 75% ferrite and 25% Martensite having 0.8% carbon.
2. (a) Carbon profile on the two sides of  $\gamma/\alpha$  interface is as follows:



(b) To derive a simple relation we assume: densities of austenite and ferrite are the same. Therefore wt % = vol %. Area of interface = 1. Carbon gradient in austenite is constant. This is given by following sketch



Carbon gradient in  $\gamma = \frac{C_\gamma - C_0}{L} \therefore$  flux of carbon crossing the interface in time  $dt$  to move this by  $dx = D \frac{dc}{dx} dt = D \frac{C_\gamma - C_0}{L} dt$  This is equal to flux of carbon rejected by  $\alpha$  as it grows by  $dx = (C_\gamma - C_\alpha) dx$ .  $L$  is obtained by equating the hatched area of the adjoining figure.  $(C_0 - C_\alpha)x = (C_\gamma - C_0)L/2$

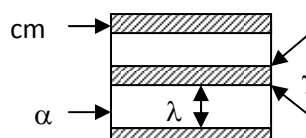
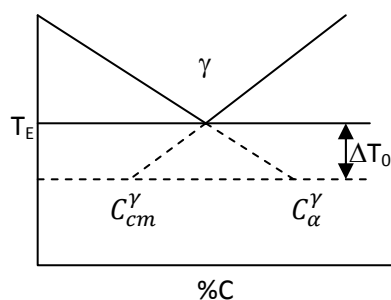
From the above one gets:  $(C_\gamma - C_\alpha)dx = D \frac{C_\gamma - C_0}{L} dt$  On substituting  $L$  :  
 $\frac{dx}{dt} = \frac{D}{2} \frac{C_\gamma - C_0}{C_\gamma - C_\alpha} \frac{C_\gamma - C_0}{C_0 - C_\alpha} \frac{1}{x}$  On integration thickness of ferrite layer at any time  $t = X^2 = D \frac{(C_\gamma - C_0)^2}{(C_\gamma - C_\alpha)(C_0 - C_\alpha)} t$

(c)  $X^2 = 10^{-12} \frac{(0.47-0.2)^2}{(0.47-0)(0.2-0)} t = 2.33 \times 10^{-12} t$  or  $X = 1.53 \times 10^{-6}$  Note that this estimate is valid only for short time.

3. Number of octahedral site in austenite = 1 / atom of Fe. Atomic wt of carbon = 12 and Fe = 56. Atom fraction carbon in austenite =  $\frac{0.1/12}{0.1/12 + 99.9/56} \approx \frac{0.1}{12} \times \frac{56}{100} \approx 0.0047$ . In one unit cell there are 4 Fe atoms and 4 interstitial sites. Fraction of these that are occupied = 0.0047. This mean amongst 100 unit cell the number of carbon atom is approximately 2 (~400x0.0047).

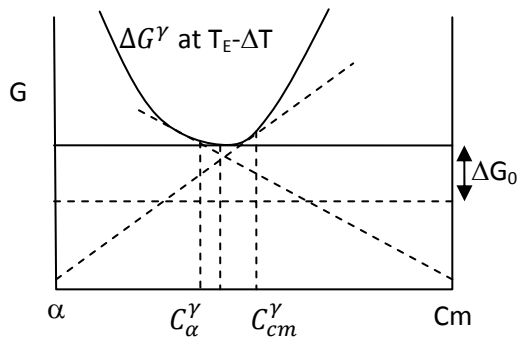
Whereas in ferrite the number of such sites / Fe atom = 3. Atom fraction carbon  $\approx \frac{0.01}{12} \times \frac{56}{100} = 0.00047$ . In one unit cell there are 2Fe atoms. Number of carbon atom = 0.00094. There are 6 sites / unit cell. Fraction of these that are occupied =  $0.00094/12 = 7.8 \times 10^{-5}$ . This means there is approximately one carbon atom in approximately 1000 unit cells.

4. Super cooling is necessary for Pearlite to nucleate:



Pearlite colony grows as carbon diffuses from  $\alpha / \gamma$  interface to  $cm / \gamma$  interface. Flux of  $C$  / unit area =  $G = D_C^gamma \frac{C_\alpha^gamma - C_{cm}^gamma}{\lambda}$

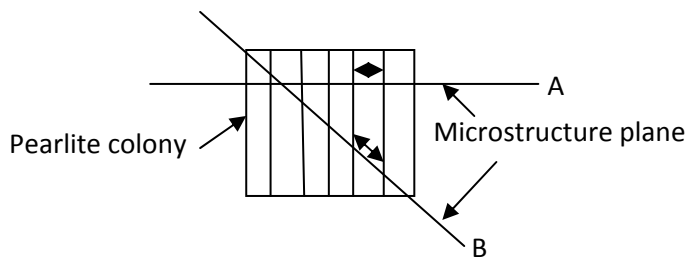
5. Super cooling is necessary to overcome activation hill arising due to creation of new surface.



For inter-lamellar spacing of  $\lambda$  the total interface area =  $2/\lambda \text{ m}^2 / \text{unit volume}$ .  $\Delta G$  for this to form is given by  $\Delta G_\lambda = -\Delta G_0 + \frac{2\sigma V_m}{\lambda}$  where  $V_m$  is molar volume,  $\sigma$  is surface energy and  $\Delta G_0$  is free energy for formation of pearlite having infinite inter lamellar spacing. The transformation will not occur unless  $\Delta G_\lambda < 0$ . Thus:

$$\lambda = \frac{2\sigma V_m}{\Delta G_0} = \frac{2\sigma V_m T_E}{\Delta H \Delta T_0}$$

6. Pearlite is made of alternate layers of ferrite and cementite. Ferrite plates are 7 times wider than those of cementite. Microstructure gives a sectional view. Colonies of Pearlite in a microstructure are randomly oriented. The plane of microstructure may intersect these at different angles. Wherever the plane is perpendicular to ferrite / cementite plates the spacing between two plates will be the minimum. Whereas those intersected at an angle will appear to have larger spacing. This is shown in following sketch:



On plane B spacing would appear significantly larger. If transformation occurs at a fixed temperature the minimum spacing is the correct estimate of lamellar distance.

**Exercise:**

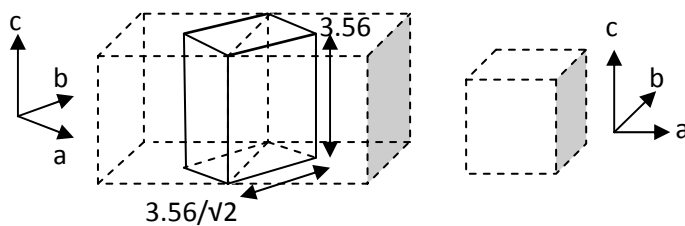
- Use the extended volume concept to show that if diffusion controlled transformation proceeds at constant nucleation rate (N) and constant growth rate (G) to show that fraction transformed (f) in time t is given by  $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$
- The growth of pearlite nodule at a give temperature in eutectoid steel is known to follow the following relationship:  $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$  where f is fraction transformed, t is time, N is nucleation rate and G is growth rate. Use this to explain the characteristic shape of TTT diagram and show that the average nodule size of Pearlite is proportional to:  $\left(\frac{G}{N}\right)^{\frac{1}{4}}$ .
- Use the concept of Bain distortion to estimate maximum displacement experienced by iron atom during martensitic transformation. Lattice parameters of austenite and ferrite are 0.356nm & 0.286nm respectively. Assume c/a ratio of martensite to be 1.15.
- Name the three most important characteristics of martensitic transformation in steel.

**Answer:**

- Assume the shape of nuclei to be spherical. If both N & G are constant at any instant t. Number of new nuclei formed in time dt = Ndt. If G is the average growth rate the average radius of preexisting nuclei = Gt. Therefore increase in extended volume due to transformation:  $df_{ext} = \frac{4\pi}{3}(Gt)^3Ndt$ . Since  $df = (1 - f)df_{ext} \therefore df = (1 - f)\frac{4\pi}{3}NG^3t^3$   
On integration:  $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$ .
- Nucleation rate (N) depends on the degree of super cooling. Lower the transformation temperature higher is the magnitude of N. Whereas G depends on diffusion of carbon atoms in austenite; it becomes slow if temperature is low. Therefore rate of transformation is the higher at intermediate temperatures. It is reflected in the C shape of TTT diagram. To estimate nodule size assume that transformation is nearly complete say 95%. If it happens in time  $t_{0.95}$   $Gt_{0.95}$  is the average nodule radius. Since  $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$  or,  $\left(\frac{\pi}{3}NG^3t^4\right) = \ln\left(\frac{1}{1-f}\right)$

Average diameter of nodule  $D = 2Gt$ . When  $f = 0.95$   $\ln\left(\frac{1}{1-f}\right) \approx 3$  &  $\frac{\pi}{3}NG^3t^4 = \frac{\pi}{48}\left(\frac{N}{G}\right)(2Gt)^4$ . Therefore on equating the two:  $D = \left[\frac{144}{\pi} \frac{G}{N}\right]^{\frac{1}{4}}$ . We get finer nodule if  $G$  is small &  $N$  is large. Finest Pearlite nodule is expected at the knee of the TTT diagram.

3. The following diagram shows how fcc austenite gets transformed into bct martensite. The cubes drawn in dotted line is austenite lattice. The tetragon drawn with firm line



$$\text{Martensite: } c = 1.15 \cdot 2.86 = 3.29 \text{ \AA}$$

$$\Delta c = a_{\gamma} - c_M = 3.56 - 3.29 = 0.27$$

$$\Delta a = \frac{a_{\gamma}}{\sqrt{2}} - a_M = \frac{3.56}{\sqrt{2}} - 2.86 = -0.34$$

is a bct unit cell with higher  $c/a$  ratio. It changes to martensite unit cell by contraction along  $c$  axis and expansion along the other two axes.

$$\text{Maximum displacement} = \sqrt{\Delta c^2 + \Delta a^2} = \sqrt{0.27^2 + 0.34^2} = 0.43 \text{ angstrom}$$

4. Three most important characteristics of martensitic transformation in steel are (a) it is diffusion less there is no change in composition (b) it is athermal: takes place when the temperature goes below a critical temperature and the transformation continues as long as the temperature decreases (c) it results in a product having a very fine structure and high hardness.



**Exercise:**

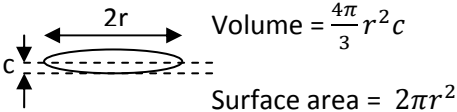
11. Explain why bainite does not form during continuous cooling in plain carbon steel.
12. Why do not you have long carbide plates like Pearlite in Baintite?
13. Use the equation given in the lecture note for total free energy change for nucleation of martensite and find out the expressions for critical nucleus size and activation hill.
14. Use the above expressions to estimate critical nucleus size and activation hill if  $\Delta G_v = -180\text{MJ/m}^3$ ,  $A = 3 \text{ GPa}$ ,  $\sigma = 18 \text{ mJ/m}^2$ .

**Answer:**

1. Bainite is a low temperature eutectoid transformation. For it to form in steel it must be cooled below a critical temperature. This corresponds to the knee of TTT diagram. During continuous cooling once you avoid the knee unless you allow isothermal holding Bainite cannot form as it needs longer hold time since the reaction is very slow.
2. Bainite forms at low temperature where rate of carbon diffusion is very slow although nucleation rate is high because of high degree of super cooling. You have a situation where there are more nuclei and limited (short) diffusion distance over

which movement of carbon atom is possible. This is why you have platelets of cementite in Bainite.

3. Assuming that martensite nucleus has the shape of an oblate spheroid the expression for total free energy change is given by  $\Delta G = \frac{4\pi}{3}r^2c \Delta G_v + 2\pi r^2\phi \gamma + \frac{4\pi}{3}r^2cE_s$  where  $\phi$  is surface energy and  $E_s$  is strain energy / unit volume.  $E_s$  is a function of the shape of nucleus:  $E_s = A \frac{c}{r}$ . Differentiating  $\Delta G$  with respect to  $r$  and  $c$  and equating these to zero one gets the critical size of nucleus.



$$\frac{\partial \Delta G}{\partial r} = \frac{8\pi}{3}rc + 4\pi r \gamma + \frac{4\pi}{3}c^2A = 0$$

$$\frac{\partial \Delta G}{\partial c} = \frac{4\pi}{3}r^2 + \frac{8\pi}{3}c r A = 0$$

- a. Solving the two equations one gets:  $c^* = -\frac{2\gamma}{\Delta G_v}$  and  $r^* = \frac{4\gamma A}{(\Delta G_v)^2}$  On substituting these in the expression for  $\Delta G$  one gets activation hill:  $\Delta G_v^* = \frac{32\pi}{3} \frac{A^2 \gamma^3}{(\Delta G_v)^4}$

4. On substitution of the numerical values:  $c^* = \frac{2 \times 18 \times 10^{-3}}{180 \times 10^6} = 0.2 \text{ nm}$  ,  $r^* = \frac{4 \times 18 \times 10^{-3} \times 3 \times 10^9}{180 \times 180 \times 10^{12}} = 6.6 \text{ nm}$  and  $\Delta G_v^* = \frac{32\pi}{3} \frac{9 \times 10^{18} \times 18 \times 18 \times 18 \times 10^{-9}}{(180 \times 10^6)^4} = 1.7 \times 10^{-18} \text{ J/nucleus} = 17 \text{ eV}$ . Note that such a large amount of energy is difficult to be provided by thermal fluctuation. At  $700^\circ\text{K}$  it amounts to  $kT = 0.06 \text{ eV}$  where  $k$  is Boltzmann constant. This indicates that homogeneous nucleation of martensite is not possible. There is plenty of evidence that it takes place heterogeneously. Dislocations are the preferred sites. It is well known that cold work raises  $M_s$  temperature.

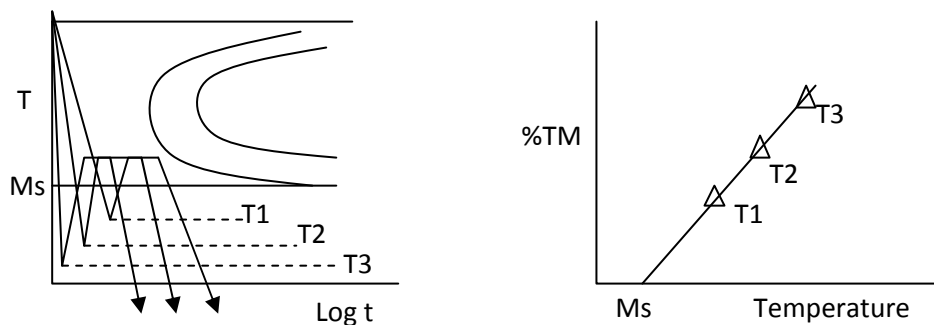
### Exercise:

15. Suggest a simple experimental method for determination of  $M_s$  temperature of steel if you have only facility for heat treatment and metallographic examination.
16. Which of the two would require more severe cooling rate to get a fully hardened structure? (a) 0.8 % carbon steel (b) 1.0 % C steel.
17. A piece of steel which was quenched after prolonged holding at  $700^\circ\text{C}$  was found to have ferrite martensite structure. Explain when would you expect this to happen?
18. List the factors that determine the strength of properly hardened steel.
19. 0.2 % plain carbon steel in annealed condition has 25% coarse pearlite. If it is normalized (heat treatment) what changes would you expect in its microstructure?

20. A thin strip of 1.2% carbon steel is quenched in water from its fully austenitic state. What structure would you expect in this steel?

**Answer:**

1. Austenitize a steel specimen, quench in baths maintained at a temperature close to its expected  $M_s$ , heat it again by placing it in another bath maintained at a little higher temperature, and then quench in water after holding it for some time. Repeat the same steps with different combination of the intermediate quenching temperatures. Polish etch and examine the microstructures under microscope. If martensite forms at the first step cooling it will get tempered when it is put in a higher temperature bath. Tempered martensite etches easily and it appears dark under microscope. Amount of tempered martensite can be estimated. Plot % tempered martensite against intermediate quenching temperature. Extrapolate it to 0% tempered martensite. This gives an estimate of  $M_s$ . The experimental scheme is shown in the following figure for eutectoid steel



2. Austenitization temperatures for both are same for hardening heat treatment. (a) being an eutectoid steel will have homogeneous austenite before quenching. Whereas in (b) there will be cementite globules present which would provide sites for nucleation of pearlite. Therefore the latter would need more severe quenching to get fully hardened structure.
3. Eutectoid temperature of steel is a function of its composition. For plain carbon steel it is around  $720^\circ\text{C}$ . To get martensite the steel should be heated beyond this temperature. However the presence of alloying elements which stabilize austenite (such as Mn, Ni) can bring down eutectoid temperature. In this case steel must be an alloy steel whose eutectoid temperature is less than  $700^\circ\text{C}$ . Therefore it must have been quenched from ferrite – austenite region and the austenite on quenching got transformed into martensite resulting in a ferrite – martensite structure.
4. A steel has maximum strength if has 100% martensite. This is obtained by quenching steel from austenitic state. Strength of martensite depends primarily on its carbon content. However the cooling rate needed to get 100% martensite in plain carbon steel is often difficult to achieve. Presence of additional alloy elements decreases

critical cooling rate. Alloy steels are easy to harden. Therefore strength of hardened steel would depend on its carbon content, section size (thickness) and other alloy elements present & their amounts in the steel.

5. As against furnace cooling in the case of annealing the job is allowed to cool in air while normalizing. This being significantly faster normalized structure would have finer pearlite and a little less amount of ferrite. However the total amount of cementite would still be the same. Only the carbon content and therefore the amount of cementite in pearlite would decrease.
6. On quenching the microstructure of steel will consist of martensite and some retained austenite. Both  $M_s$  &  $M_f$  temperature of steel decreases with % carbon. Steels having beyond 0.7%C has subzero  $M_f$  temperature.

**Exercise:**

21. Cooling rate at the center of a steel rod on quenching in oil from 850°C is given by  $1.4 \times 10^4 \times d^{-1.9}$  °C/s, where d is diameter in mm. Which of the following steel would have 100% martensite at its centre if its diameter is 50mm? Critical cooling rate (CCR °C/sec) of steel to get 100% martensite is a function of composition. Assume that it is given by  $\log(CCR) = 4.3 - 3.3C + (Mn + Ni + Cr + Mo)/1.6$  (a) AISI 4340 steel having 0.4C, 0.7Mn, 1.8Ni, 0.8Cr,0.25Mo (b) AISI 3130: 0.3C,0.7Mn,1.3Ni,0.6Cr
22. Find out the diameters of the above steels (see problem 1) that would have 100% martensite at its center on quenching in the same medium.

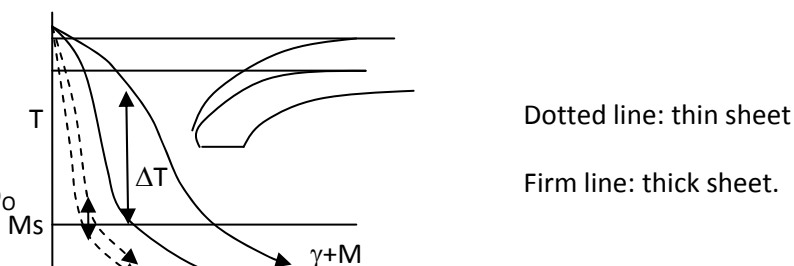
23. Explain why thicker sections are more susceptible to cracking during hardening heat treatment.
24. Critical diameter of a certain grade of steel for oil quench, water quench and ideal quench are  $D_o$ ,  $D_w$  and  $D_i$  respectively. Arrange these in descending order.
25. List the factors that determine hardenability of steel. Which of these are preferred? Give reasons.
26. What is meant by severity of quench? List the factors that determine this. What is its dimension?

**Answer:**

1. Cooling rate at the centre =  $1.4 \times 10^4 \times 50^{-1.9} = 8.28 \text{ }^\circ\text{C/sec}$ . CCR of the two steels are (a)  $5.8 \text{ }^\circ\text{C/s}$  (b)  $48.4 \text{ }^\circ\text{C/s}$ . Cooling rate is higher than CCR of (a) but lower than that of (b). Therefore (a) will have 100% martensite at its center.

2. (a)  $5.8 = 14000 \times d^{-1.9} \therefore d = \left(\frac{14000}{5.8}\right)^{1.9} = 60.5\text{mm}$  (b)  $d = \left(\frac{14000}{48.4}\right)^{1.9} = 20\text{mm}$

3. The difference in cooling rate between the centre and the surface is much more in a thicker section. As soon as the temperature at the surface crosses  $M_s$  temperature, martensite forms accompanied by volume expansion. The core is still soft austenite and can accommodate deformation if required. However later when its temperature goes below  $M_s$  it would expand, when outer core which is already transformed is hard and cannot accommodate deformation. A tensile stress therefore develops at the surface and it becomes prone to cracking. If thickness is less the difference in temperature is not large. Transformation takes place almost simultaneously with little chance of developing high tensile stress at the surface. The following sketch illustrates this.



4.  $D_i, D_w, D_o$
5. Hardenability increases with increasing carbon content, alloy addition (except cobalt), lower inclusion content and coarse austenite grain size. The first three are preferred. Finer grain size gives better crack resistance (toughness). Martensite formed in coarse austenite grain is more prone to cracking.
6. It is a measure of the rate at which heat can be extracted from the quenched steel. It is defined as the ratio of  $h/2k$  where  $k$  is the thermal conductivity of steel and  $h$  is convective heat transfer coefficient between steel and quenching medium.

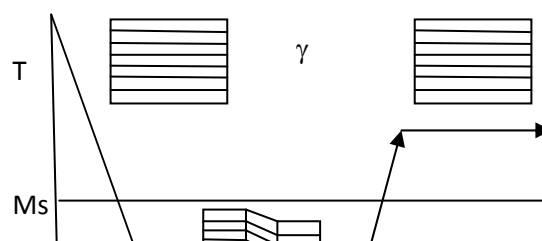
Dimension =  $\frac{\text{Joule}}{\text{m}^2 \text{K s}} \times \frac{\text{m}^2 \text{K}}{\text{Joule} \times \text{m}} = \frac{1}{\text{m}}$ . Therefore note that HD is a dimensionless quantity.

**Exercise:**

27. Give example of a shallow hardening and a deep hardening steel.
28. Martensite in Fe-30% Ni alloy is reversible but in most common grades of steel it is not reversible. Explain why it is so.
29. Hardness of a quenched and tempered steel is reported to be Rc 35. What additional tests will you recommend to know that it has indeed been given this heat treatment?
30. It is apprehended that a hardened and tempered steel has become brittle. Suggest a suitable test to check if it is so.
31. Microstructure of a steel consists of 100% coarse pearlite. Its carbon content is reported to be 0.65%. Is this possible?
32. Diameter of a hardened 1.0% carbon steel rod is found to have increased on tempering. What will you infer from this?
33. A hardened steel has become embrittled on tempering. Can this be de-embrittled?

**Answer:**

1. Most low carbon steels are examples of shallow hardening. Most alloy steels for tools and dies are examples of deep hardening steel. Many of these exhibit air hardening characteristics.
2. Martensitic transformation takes place through shear when temperature goes below  $M_s$ . There is no diffusion or change in composition. Therefore if the temperature is again raised beyond  $M_s$  it should go back to austenitic state. Most commercial steel has carbon. Martensite in these steel is susceptible to tempering if temperature is raised. Carbon precipitates as carbide. As a result composition changes. Therefore question of reverse martensitic transformation does not arise. However Fe-30Ni has little carbon. Here there is no precipitation. This is why it exhibits reverse martensitic transformation where it regains its original shape. Such alloys are known as shape memory alloys. This is shown schematically in the following schetch.



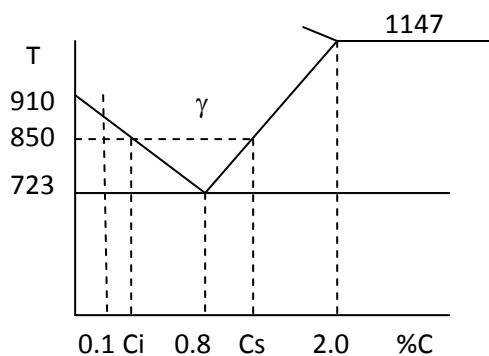
3. Examine its microstructure to see if it has tempered martensitic structure. Fine pearlite can also give similar hardness.
4. Best test to know if it has been embrittled to find out is the Charpy V notch impact test. If it has become brittle its CVN value will be low and fractured face will be predominantly cleavage with little notch root contraction.
5. Only eutectoid steel can have 100% pearlitic structure. Carbon content of eutectoid steel is a function of other alloying elements present in steel. It must be an alloy steel. Relatively fast cooling or isothermal transformation could give 100% pearlite. This possibility is ruled out since the pearlite is coarse.
6. 1% carbon steel is austenitized at 760°C and subsequently quenched during hardening. Since it has high carbon content it is likely that its  $M_f$  temperature is lower than room temperature. Therefore it would have some amount of retained austenite having a much more close packed structure than martensite. On tempering retained austenite would decompose to martensite. This transformation is accompanied by volume expansion. This is the most likely reason for increase in diameter.
7. Certain alloy steel on tempering at 350°-550°C may become embrittled. This is known as temper embrittlement. To de-embrittle the steel it must again be hardened and tempered at 600°C followed by quenching. Fast cooling will help avoid exposure to the temperature range susceptible to such embrittlement.

### Exercise:

1. Induction heating followed by quenching is a common method of surface hardening of steel. Can it be applied to an alloy steel having 18Cr8Ni0.15C?
2. Can steel having 0.1% carbon be case carburized at 850°C?
3. Cite three main reasons for surface hardening of steel.
4. Explain why core refining heat treatment may not be required for case carburized aluminium killed steel.
5. What is the white layer on steel that forms during nitriding?

### Answers:

1. No. 18Cr8Ni0.15C is austenitic steel. It cannot be hardened by heating & quenching.
2. It would carburize but the process would be too slow. At 850°C it will have ferrite austenite structure. Solubility of carbon in ferrite is very small. Only the austenitic region will pick up carbon. The concentration gradient for carbon to diffuse into austenite is also less. Since both temperature & concentration gradients are low rate of carbon pick up will be extremely slow. Therefore carburization at 850C is not recommended. The following figure illustrates how %C at the interfaces can be estimated.



$$C_i = \frac{910 - 850}{910 - 723} \times 0.8 = 0.26$$

$$\frac{2 - C_s}{2 - 0.8} = \frac{1147 - 850}{1147 - 723} \text{ or; } C_s = 1.16$$

$$\Delta C = 1.16 - 0.26 = 0.9$$

3. Three most important reasons for surface hardening are: (i) to have hard surface but soft (tough) core in components like gears, shafts etc. (Hardening is often accompanied by loss of toughness.) (ii) to overcome section size effect which makes it difficult to get the required surface hardness in large sections by quenching and (iii) to get a favorable residual stress on the surface which would inhibit crack initiation.
4. Purpose of core refining treatment is to get fine austenite grain in case carburized steel. Aluminum killed steel are resistant to austenitic grain growth. Aluminum



reacts with dissolved oxygen to form oxide particles during solidification. These are located along austenite grain boundaries and restrict their movements. In such steel grain growth during carburization heat treatment may not be significant. This is why core refining treatment may not be necessary. Steels having micro alloying elements like Nb, V, and Ti too are resistant to grain growth. These too do not need core refining treatment.

5. Nitriding is done on steel after it has been hardened and tempered. Sample is heated to around 500°C which is lower than the eutectoid transformation temperature in Fe-N phase diagram. The eutectoid consists of ferrite and Fe<sub>4</sub>N. While some N would diffuse through ferrite to form fine carbo-nitrides some may form a white nitride layer made of Fe<sub>4</sub>N at the surface. This is hard and brittle. It is harmful and it must be removed by lapping.

#### Exercise:

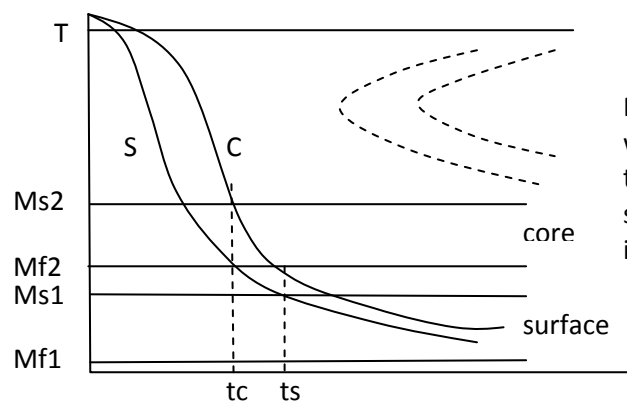
1. Microstructure of a cold rolled annealed mild steel sheet shows elongated pearlitic region in equiaxed ferritic matrix. Comment on the processing route that will give this type of microstructure.
2. Name three common grades of steel produced by ingot route. Which of these has higher yield?
3. Why does compressive stress develop at the surface in a case carburized and subsequently case hardened steel?
4. Why low carbon content is preferred in most structural application? Is high strength not a major criterion?
5. What is the composition of the steel designated as 25Mn1S14? What is the role of sulfur in this steel?

#### Answer:

1. Elongated pearlite is a sign of cold work whereas equiaxed ferrite means annealed structure. It looks that the steel was cold worked when both pearlite and ferrite got elongated. Later on annealing just below eutectoid temperature ferrite has recrystallized but pearlite remains untransformed. To modify pearlite shape it must be heated above the eutectoid temperature.
2. Steel produced by ingot route is classified as killed, semi-killed and rimming steel. This is based on the way the dissolved oxygen in molten steel is removed. Rimming steel has the highest yield. The oxygen gets removed by the reaction between dissolved carbon and oxygen with evolution of CO bubbles as the steel cools in mold. This gives the rimming action. Formation of solid crust at the top of the ingot does not allow the gas to go out. It remains entrapped as porosity and balance the shrinkage associated with solidification. There is no shrinkage pipe. Therefore the

entire ingot without cropping can go for hot rolling. The pores get welded during hot working.

- This is illustrated with the help of following diagram. Here we have high carbon steel with lower  $M_s$  &  $M_f$  temperature at the surface whereas core is a low carbon steel with higher transformation temperatures ( $M_s$  &  $M_f$ ). Therefore transformation occurs first in core with expected expansion when surface is still soft austenite and therefore it can allow the core to expand (due to transformation) by plastic deformation. When martensite starts forming at the surface accompanied by volume expansion the core which is now strong will inhibit this. This is why there is a residual compressive stress at the surface.



Note that transformation would start at  $t_c$  within core although it is at a higher temperature whereas when transformation starts at surface at  $t_s$  transformation at core is complete.

- Often structural steel is supposed to have additional properties like weldability & formability apart from high strength. Low carbon content gives better formability & weldability. Loss in strength due to lower carbon is made up by grain refinement through controlled thermo-mechanical processing with addition of micro-alloy elements like Nb, V, & Ti. They prevent grain growth during hot rolling.
- This is a free cutting steel having 0.25% carbon, 1% Mn and 0.14% S. In presence of Mn, S is present as MnS inclusion. This improves machinability. It is mostly used for making nuts & bolts. If Mn is not present then S forms a low melting eutectic between Fe & FeS. This segregates along austenite grain boundary leading to hot shortness. This is undesirable. This is why in most steel S is less than 0.05%.

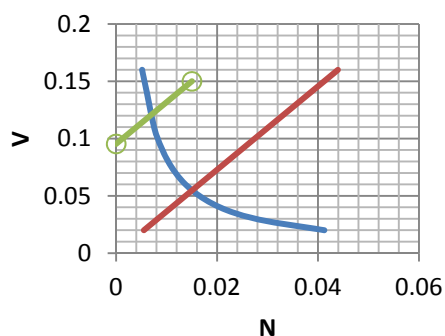
#### Exercise:

- Why is it possible to get much finer grain structure in steel than in aluminum?
- Estimate the amount of martensite that can be obtained in 0.2% carbon steel by quenching from inter-critical temperature regime (between  $A_1$  &  $A_3$ ) as a function of temperature. Which of these is likely to give maximum strength?

8. The concept of solubility product is used to estimate the amount of MX precipitate in micro-alloyed steel (Fe-M-X) in terms of its composition in wt%. If solubility product of VN is given by  $\log k_{VN} = -\frac{8330}{T} + 3.46$  estimate the amount of V present as VN precipitates in alloy having 0.15 wt% V and 0.015wt% N at 1273°K.
9. How is ausforming different from austempering? Is this a hot working process?
10. What is the effect of micro-alloy addition on rolling load?

**Answer:**

6. Steel unlike aluminum offers additional opportunity for grain refinement because of austenite to ferrite transformation. In addition to grain refinement during solidification there is further refinement because of nucleation & growth of new ferrite grains at prior austenite boundaries.
7. Apply lever rule to estimate %  $\alpha$  as a function of inter-critical temperature. Amount of  $\alpha$  will be minimum near  $A_1$  and maximum (100%) at  $A_3$ . On quenching this gets converted to martensite. Hardness of martensite depends on carbon content. Therefore Martensite formed on quenching from  $A_1$  will have maximum hardness where as that formed on quenching from  $A_3$  will have minimum hardness. Hardness of ferrite remains constant. Apply rule of mixture to get the hardness of ferrite martensite aggregate. Assume hardness of martensite change linearly from 200VHN at 0% C to 950VHN at 0.8% C. Assumption: there is no retained austenite in final structure.
8. Since  $\log k_s = -\frac{8330}{1273} + 3.46$  or  $k_s = 0.000825$  Use this to generate solubility plot at 1273K. This is shown by line k in the graph. Atomic weights of V & N are 51 & 14 respectively. Plot line VN passing through origin with slope = 51/14. This represents amount of V needed to form VN (Stoichiometry).



Mark the composition of the steel by a point (0.15V, 0.015N). Draw a line parallel to VN through this point. The point of intersection with k represents amount of V that is present in solution. Thus soluble V = 0.12wt% Balance 0.03 is present as VN.

9. In austempering the steel is austenitized and then quenched in a bath kept at a little above  $M_s$  temperature for sufficiently long time to get 100% Bainitic structure. Whereas in ausforming the steel is taken to austenitic state followed by quenching in

a bath kept at a temperature a little above  $M_d$  and subjected to plastic deformation before quenching to get martensitic structure.  $M_d$  temperature corresponds to a temperature above which martensite does not form even if austenite is being deformed. Since plastic deformation during ausforming takes place below re-crystallization temperature of austenite it is a cold working process.

10. In the case of Nb containing steel grain boundaries get pinned by nano sized NbCN precipitates formed soon after rolling at high temperature. Finish rolling is done below its re-crystallization temperature. This gives rise to elongated austenitic grains which ultimately transform to ferrite pearlite structure. This process would need higher rolling load. Whereas steels having V as micro alloy addition precipitation occurs during finish rolling stage to inhibit grain growth. Therefore rolling load here is low.

#### Exercise:

1. Compare the crystal structures of two alloy steels having nominal carbon content (a) 18%Cr (b) 18%Cr8%Ni. Can these be hardened by conventional quenching from a high temperature?
2. High speed cutting tools are made of 18W4Cr1V0.6C and it is used in hardened and tempered condition. Unlike normal hardened and tempered plain carbon tool steel it does not lose its cutting ability even when it is red hot. Explain why it is so.
3. How is the mechanism of strengthening in maraging steel different from that in conventional ultra high strength steel?
4. Explain why commercial steels / alloys having lower stacking fault energy exhibit higher creep resistance.
5. Does all steel suitable for ausforming exhibit transformation induced plasticity?

#### Answers:

1. (a) BCC: ferrite (b) FCC: austenite. 18%Cr steel cannot be transformed to austenite which is the main criterion for conventional hardening process by heating & quenching. This is because amount of ferrite stabilizer present is beyond  $\epsilon$  loop. Therefore it cannot be hardened by this method. 18Cr8Ni on the other hand is austenitic at room temperature. Steels that could be hardened by conventional process develop martensitic structure on quenching to room temperature. Therefore this too cannot be hardened as above. However through cold work and subzero quenching some martensite may form in 18Cr8Ni steel.
2. High speed tool steel retains its hardness even when it is red hot because it has significant amount of strong carbide former which precipitates during the 4<sup>th</sup> stage

of tempering (gives secondary hardening) before these are put to use. The precipitate is coherent with the matrix. This gives it excellent micro-structural stability to resist particle coarsening and retain its hardness.

3. Martensite in normal steel is hard primarily due to carbon. Presence of excess carbon is responsible for tetragonal distortion. This is why martensite in normal steel has BCT structure. Maraging steel has very little carbon. Therefore martensite that forms here is BCC. In virtual absence of carbon it is soft. It can be cold worked to give desired shape and increase its strength. On aging at around 500°C inter-metallic compounds precipitate. This results in further increase in strength. Therefore apart from solid solution strengthening unlike conventional steel martensite in maraging steel derives its strength from cold working and precipitation hardening.
4. All creep resisting alloys have precipitates which obstruct movement of dislocation that is responsible for deformation. Usually precipitates are strong. Stress required to shear these is high. Therefore dislocations held up at obstacles can move either by climb (restricted to edge dislocation) or cross slip (restricted to screw dislocation). There are also restrictions imposed by crystal structure on slip planes and burgers vectors of mobile dislocations. If stacking fault energy is high the partials join to form a perfect dislocation that could either cross slip or climb depending on its character. If SFE is low the distance between the two partial is large. It would require higher stress to join these. Therefore lower SFE gives higher creep resistance.
5. Not all. Only steels whose  $M_s$  temperature is little lower than room temperature and  $M_d$  temperature is higher than room temperature would respond to transformation induced plasticity if it is heavily deformed above  $M_d$  temperature. Such steels are in metastable state during service. If a crack develops in the steel martensite would form at crack tip. Such a transformation is accompanied by volume expansion resulting in local deformation. The region surrounding the crack tip would resist such deformation. Therefore more work has to be done so that the crack would grow. This gives the steel high yield strength and toughness. Such steels are known as TRIP steel.

#### **Exercise:**

1. Why there is excessive grain growth in the heat affected zone of a welded structure even if it is made of micro alloyed steel?
2. Most structural steels have some amount Mn primarily to take care of the hot shortness problem because of residual sulfur in steel. Some amount of Si is also present. It is added to remove dissolved oxygen in steel. ASTM A516 Grade 60 is a common grade of steel used for the construction of welded vessels. It has 0.021C,

0.66Mn, 0.45Si, S <0.035, P < 0.035. Examine if it is suitable for construction of welded pressure vessel.

3. What are the precautions that need to be taken to weld creep resistant steel tubes having 0.15C, 0.5Mn, 0.54Si, 2.25Cr, 1.0Mo (this corresponds to ASTM T 22 grade steel)?
4. There is no chance of forming martensite in heat affected zone while welding austenitic steel. Should therefore all grades of austenitic steels be easily weldable? Give reasons for your answer.
5. Derive an expression for the time it takes to reach maximum temperature at a distance  $r$  from the heat source during arc welding of a thick plate. Estimate maximum temperature at a point 5mm away from the heat source during welding of carbon steel. Material and process parameters are as follows: thermal conductivity = 50W/m/K, density = 7850kg/m<sup>3</sup>, specific heat = 0.45kJ/kgK, initial temperature of the plate = 300K, weld current = 150 A, voltage = 20volts, welding speed = 2.5mm/s, heat lost = 0.4.
6. Use the data given in problem 5 to estimate the distance at which temperature would reach 1500°K (the melting point of the steel).

#### Answers:

1. Micro-alloy elements are present in steel as carbo-nitrides. These are effective in pinning grain boundaries during normal thermo-mechanical processing. However during welding temperature near fusion zone goes beyond their solvus temperature. Absence of precipitates to pin austenite boundaries in this region leads to excessive grain growth.
2. Medium carbon steels are more prone to heat affect zone related problem. Although % C is 0.21 its carbon equivalent =  $0.21 + 0.66/6 = 0.33$ . This too is less than 0.4% C. Therefore it can be welded without any special precaution. There is little chance of formation of hard and brittle martensite from where crack could initiate. Experience shows that failures often do not take place in the main welds of a structure rather it initiates from points where a small weld bead may have been placed to attach ancillary equipments temporarily during fabrication.
3. Carbon equivalent for this steel =  $0.15 + 0.54/6 + (2.25+1.0)/5 = 0.89$ . In this case martensite is likely to form unless special precautions are taken to preheat the tubes before welding. This would help reduce cooling rate encountered in the HAZ during welding. In many cases post weld heat treatment is also required. Note that most steel specifications allow a range of composition. To decide whether any special precaution is necessary carbon equivalent should be estimated using the maximum permissible alloy content.

4. Totally different kinds of problem are faced during welding of austenitic steel. One is related to solidification shrinkage which is much more than in ferritic steel. This is overcome by suitable control of the composition of weld consumables so that on solidification it has some amount of delta ferrite. Ferrite meters, a non destructive testing tool to detect if there is some delta ferrite. The second problem is associated with the precipitation of  $Cr_{23}C_6$  at grain boundaries in the heat affected zone. The carbides have very high amount of chromium. Whenever such precipitates are formed chromium content of the surrounding matrix drops below the minimum amount needed to form protective oxide layer. These regions become prone to corrosive attack. Often it leads to inter granular failure. This phenomenon is known as sensitization. Stainless characteristics of sensitized steel can be restored by giving post weld annealing by heating to  $950^{\circ}C$  followed by rapid cooling to suppress carbide precipitation. Alternatively select special low carbon austenitic steels or those having additional alloy elements like Nb. This is a strong carbide forming element. It does not allow any carbon to be present in the matrix to form  $Cr_{23}C_6$ .
5. Temperature at distance  $r$  from a moving point heat source is given by the following expression:  $T = T_0 + \frac{q}{2\pi\lambda vt} \exp\left(-\frac{r^2}{\alpha t}\right)$  where  $T$  is temperature,  $t$  is time,  $\alpha$  is thermal diffusivity,  $v$  is welding speed,  $\lambda$  is thermal conductivity,  $T_0$  is initial temperature of the plate,  $q = \eta iV$  where  $i$  = current,  $V$  = voltage,  $\eta$  = efficiency,  $\rho = \frac{m}{V}$  where  $m$  = density and  $c$  = specific heat. To find out time at which  $T$  reaches maximum at a given point  $r$  equate:  $\frac{dT}{dt} = 0 = \frac{d\ln(T-T_0)}{dt}$ . Since  $\ln(T - T_0) = \ln\left(\frac{q}{2\pi\lambda v}\right) - \ln(t) - \frac{r^2}{\alpha t}$  or  $\frac{1}{(T-T_0)} \frac{dT}{dt} = \left(\frac{r^2}{\alpha t^2} - \frac{1}{t}\right) = 0 \therefore t_{max} = \frac{r^2}{\alpha}$  On substituting this in the expression for  $T$ :  $T_{max} = T_0 + \frac{q}{2\pi\lambda v t_{max}}$  On substituting numerical values one directly gets both  $t_{max}$  and  $T_{max}$ .  $t_{max} = \frac{r^2}{\lambda \rho c} = \frac{(0.005)^2}{50} \times 7850 \times 450 = 1.77 \text{ second}$  and  $T_{max} = T_0 + \frac{q}{2\pi\lambda v t_{max}} = 300 + \frac{0.6 \times 150 \times 20}{2\pi \times 50 \times 0.0025 \times 1.8 \times 2.718} = 777^{\circ}K$ .
6. The expression for  $T_{max}$  can be rewritten as  $T_{max} = T_0 + \frac{\epsilon iV \alpha}{2\pi\lambda v r^2}$  or  $r^2 = \frac{\epsilon iV \alpha}{2\pi\lambda v (T_{max} - T_0)} = \frac{(0.6 \times 150 \times 20 \times 1.42 \times 10^{-5})}{(2\pi \times 50 \times 2.718 \times 0.0025 \times 1200)} = 0.00316m$