Prof. MP Gururajan Lec. 46 & 47
Date:- 23/12/2015

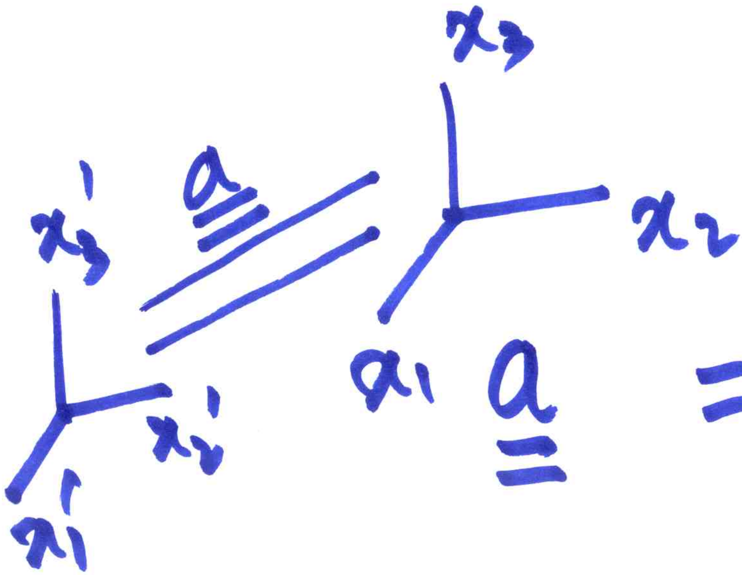
$$
\begin{array}{cc|ccc}
 & & & \text{Old} & \\
 & & x_1 & x_2 & x_3 \\
\hline
 & x_1' & a_{11} & a_{12} & a_{13} \\
\text{New} & x_2' & a_{21} & a_{22} & a_{23} \\
 & x_3' & a_{31} & a_{32} & a_{33}
\end{array}
$$

$a_{11} \Rightarrow \widehat{x_1' \, x_1}$ If $J_i' = a_{ij} J_j$, then

$\overline{J}$ is a vector.

$$D_{ij} = \underline{\underline{D}} \quad \text{Property tensor}$$

$$\text{Property} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}$$



$$\underline{\underline{a}} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$J_i' = a_{ij} J_j \Rightarrow J_i = a_{ji} J_j'$$

$$J_i = -D_{ij} \nabla_j c \qquad \nabla_k c = a_{lk} (\nabla_l c)'$$

$$J_i' = -a_{ij} D_{jk} \nabla_k c$$

$$= -a_{ij} D_{jk} a_{lk} (\nabla_l c)'$$

$$J_i' = -D_{il}' (\nabla_l c)'$$

$$D_{il}' = + a_{ij} D_{jk} a_{kl}$$

$$J'_j = a_{ji} J_i$$

If $D'_{ij} = a_{ik} D_{kl} a_{jl}$, then $\underline{D}$ is a $\underline{\mathrm{II}}$ rank tensor

$$\epsilon'_{ijk} = a_{il} a_{jm} a_{kn} \epsilon_{lmn} \quad :$$

$$C_{ijkl} = a_{ip} a_{jq} a_{kr} a_{ls} C_{pqrs}$$

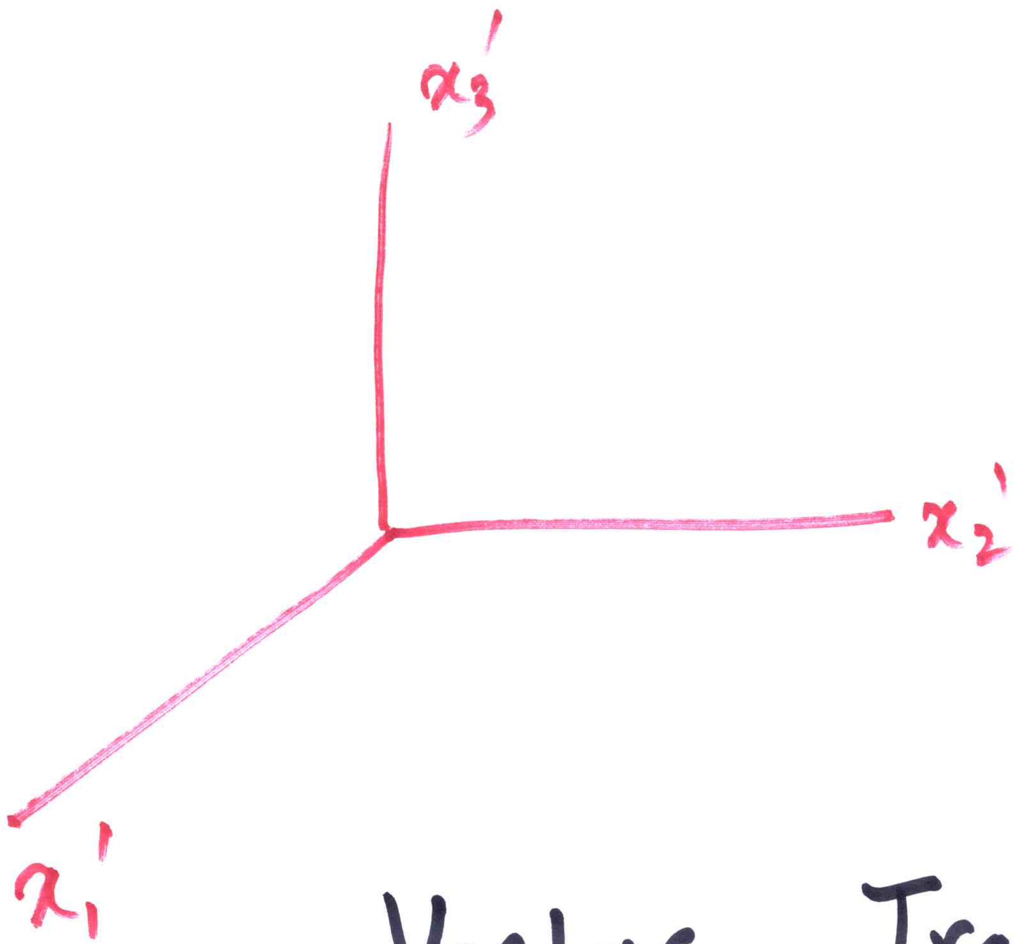
$$\vdots$$

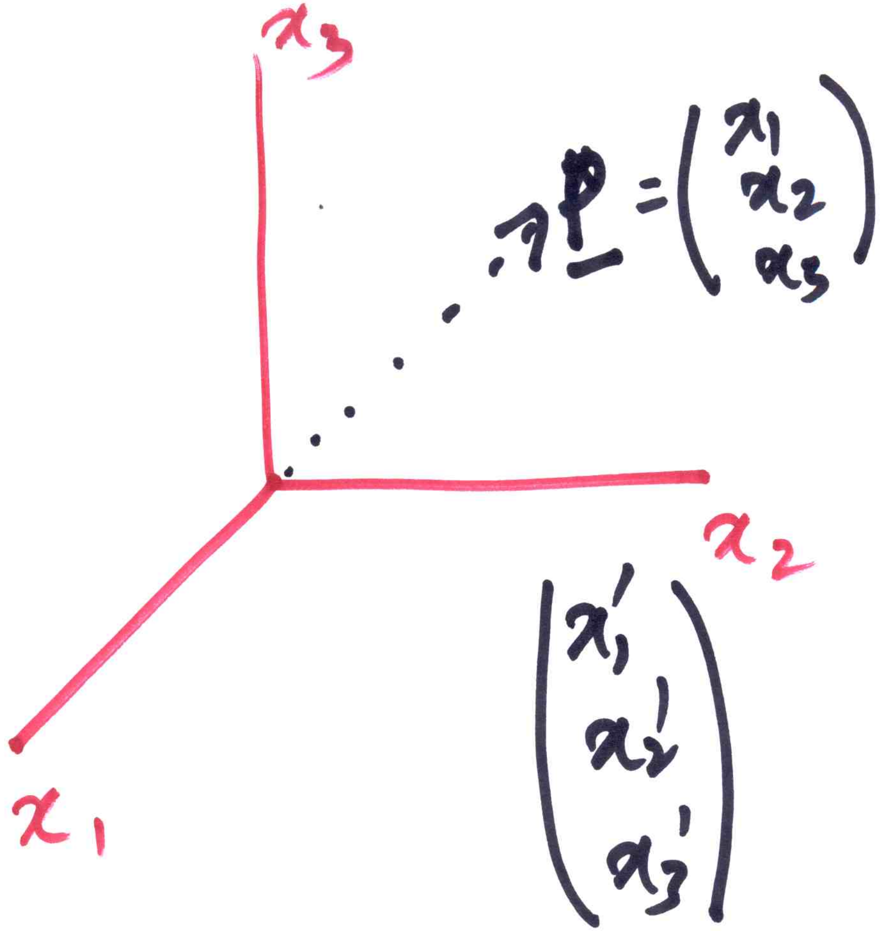
If $D_{ij} = D_{ji}$, then $\underline{\underline{D}}$ is symmetric

$$D_{12} = D_{21} \; ; \; D_{13} = D_{31} \; ; \; D_{23} = D_{32}$$

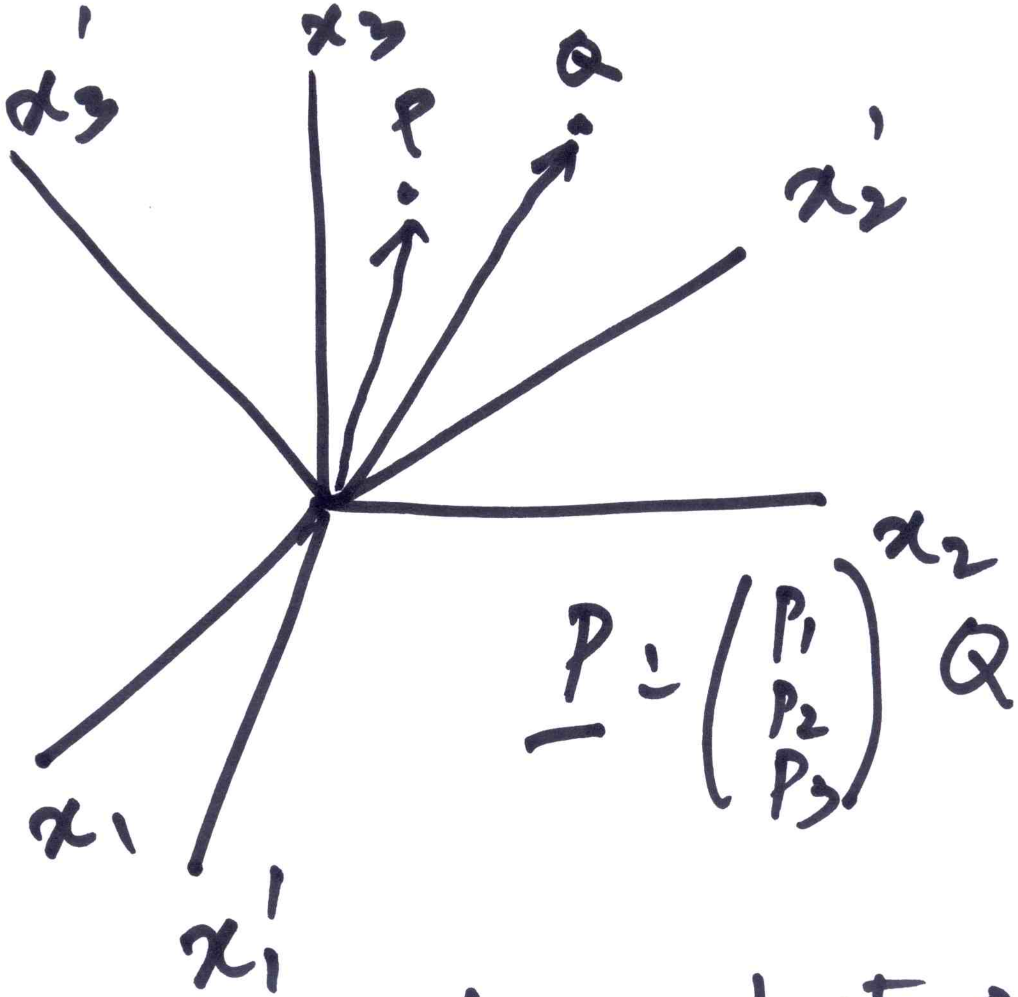
$$\begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix}$$

If $D_{ij} = -D_{ji}$, then $\underline{\underline{D}}$ is anti- Symmetric
skew-





Vectors – Transformation of the coordinates of a point



$\underline{P} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad Q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$

II rank tensor – Transformation of products of coordinates

Neumann's principle.

The symmetry elements of any physical property of a crystal must include the symmetry elements of the point group of the crystal.

Symmetric tensor $\Rightarrow D_{ij} = D_{ji}$