Department Of Mechanical Engineering

MODULE 9

Radiation Heat Transfer

Thermal energy emitted by matter as a result of vibrational and rotational movements of molecules, atoms and electrons. The energy is transported by electromagnetic waves (or photons). Radiation requires no medium for its propagation, therefore, can take place also in vacuum. All matters emit radiation as long as they have a finite (greater than absolute zero) temperature. The rate at which radiation energy is emitted is usually quantified by the modified Stefan-Bolzmann law:

$$EA = q = dQ/dt = \varepsilon\sigma \cdot T_b^4$$

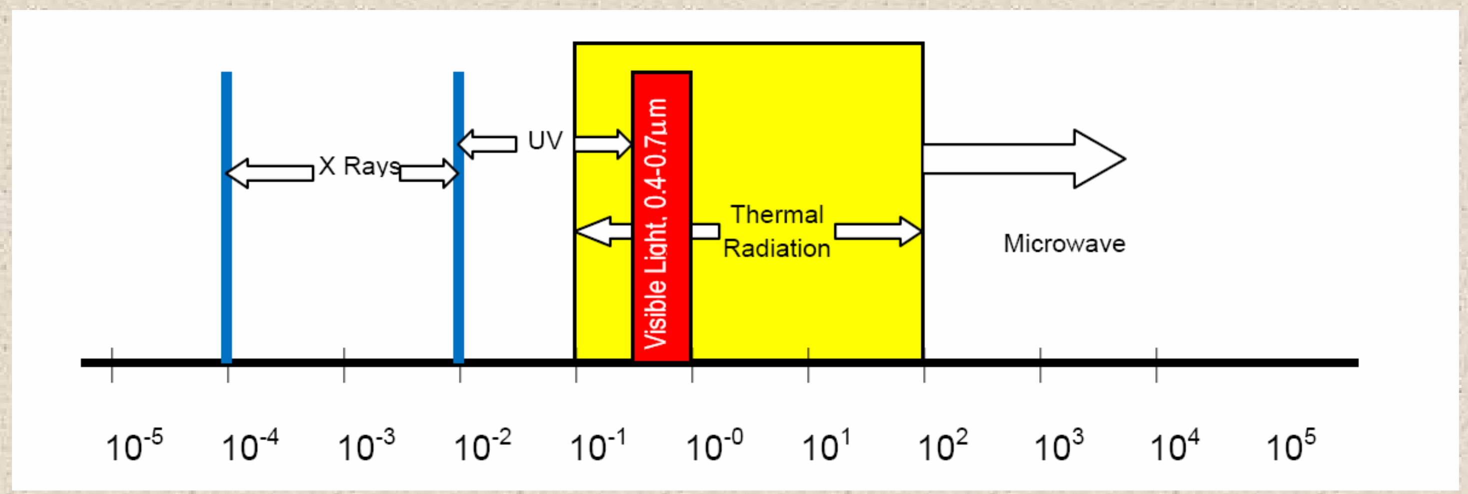
where the emissivity, ε , is a property of the surface characterizing how effectively the surface radiates compared to a "blackbody" ($0<\varepsilon<1$). E=q/A (W/m^2) is the surface emissive power. σ is the Stefan-Boltzmann constant

(s=5.67×10^{-8} $W/(m^2K^4)$). T_b is the absolute surface temp. (in K)

Electromagnetic radiation spectrum

Thermal radiation spectrum range: 0.1 to 100 mm

It includes some ultraviolet (UV) radiation and all visible (0.4-0.76 mm) and infrared radiation (IR).



Wavelength, λ, μm

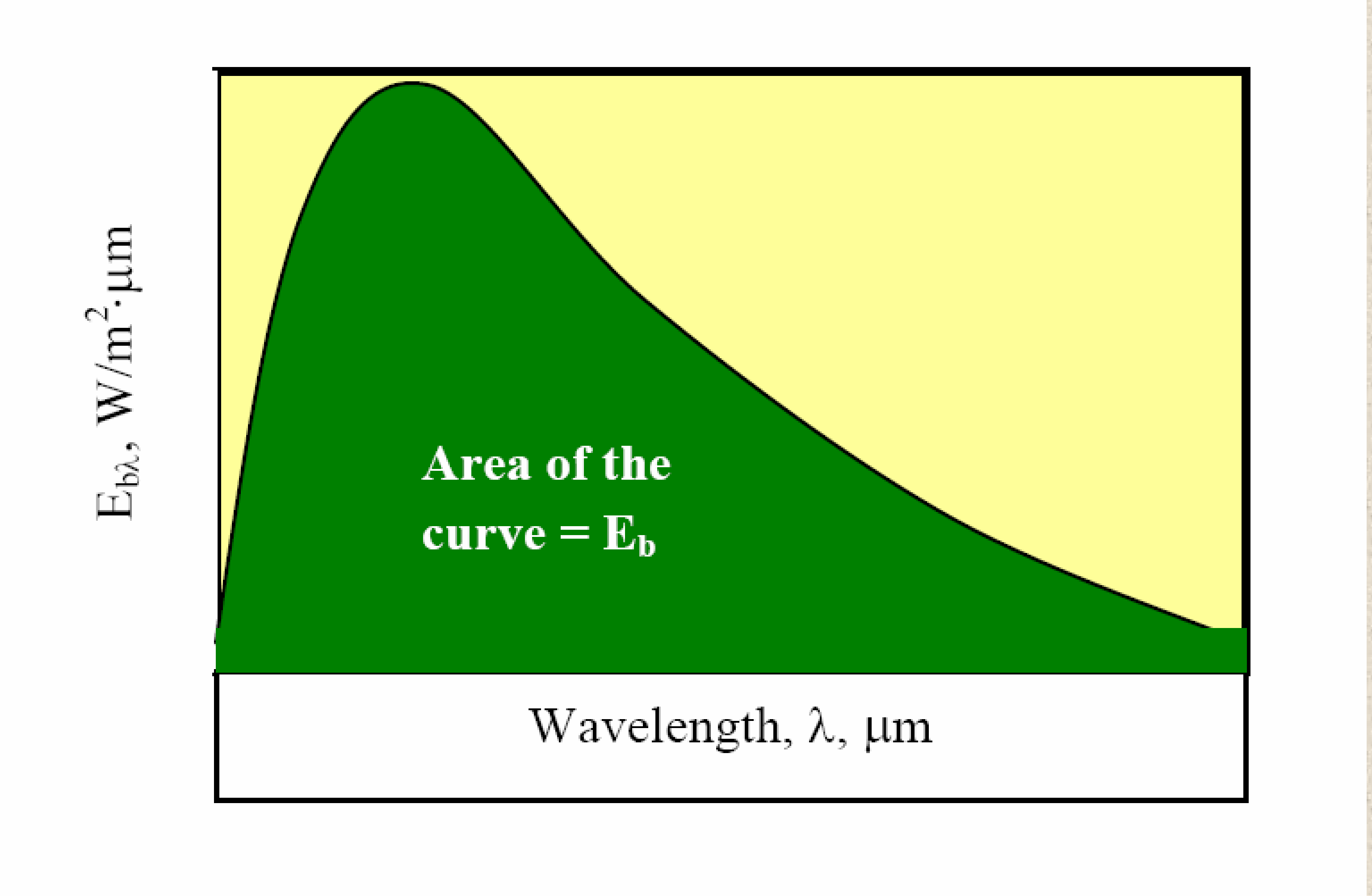
The Planck Distribution

The Planck law describes theoretical spectral distribution for the emissive power of a black body. It can be written as

$$E_{\lambda,b} = \frac{C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}$$

where C_1=3.742x10^8 (W.μm^4/m^2) and C_2=1.439x10^4 (μm.K) are two constants. The Planck distribution is shown in the following figure as a function of wavelength for different body temperatures.

Spectral blackbody emissive power



Planck Distribution

- At given wavelength, the emissive power increases with increasing temperature

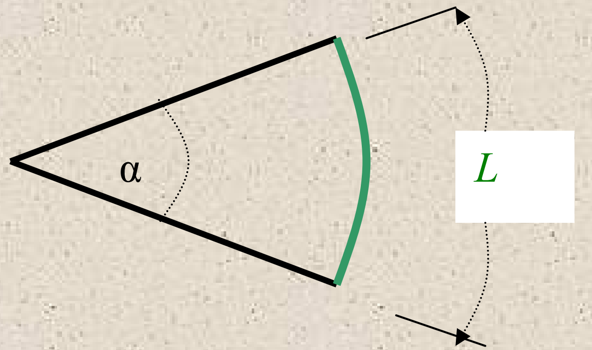
- As the temperature increases,more emissive energy appear at shorter wavelengths

- For low temperature (>800 K), all radiant energy falls in the infrared region and is not visible to the human eyes. That is why only very high temperature objects, such as molten iron, can glow.

- Sun can be approximated as a blackbody at 5800 K

Angles and Arc Length

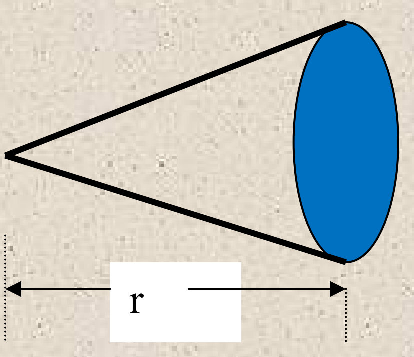
We are well accustomed to thinking of an angle as a two dimensional object. It may be used to find an arc length:



$L = r \cdot \alpha$

Solid Angle

We generalize the idea of an angle and an arc length to three dimensions and define a solid angle, Ω, which like the standard angle has no dimensions. The solid angle, when multiplied by the radius squared will have dimensions of length squared, or area, and will have the magnitude of the encompassed area.

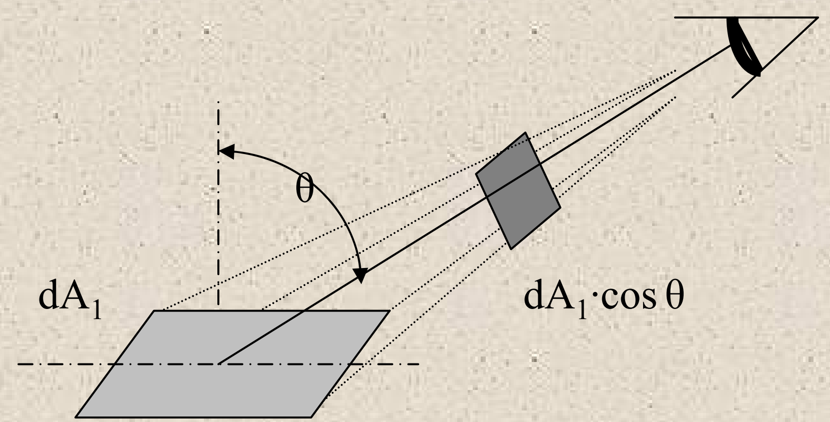


$A = r^2 \cdot d\Omega$

Projected Area

The area, dA_1, as seen from the prospective of a viewer, situated at an angle θ from the normal to the surface, will appear somewhat smaller, as $\cos\theta \cdot dA_1$. This smaller area is termed the projected area.

$A_{projected} = \cos\theta \cdot A_{normal}$



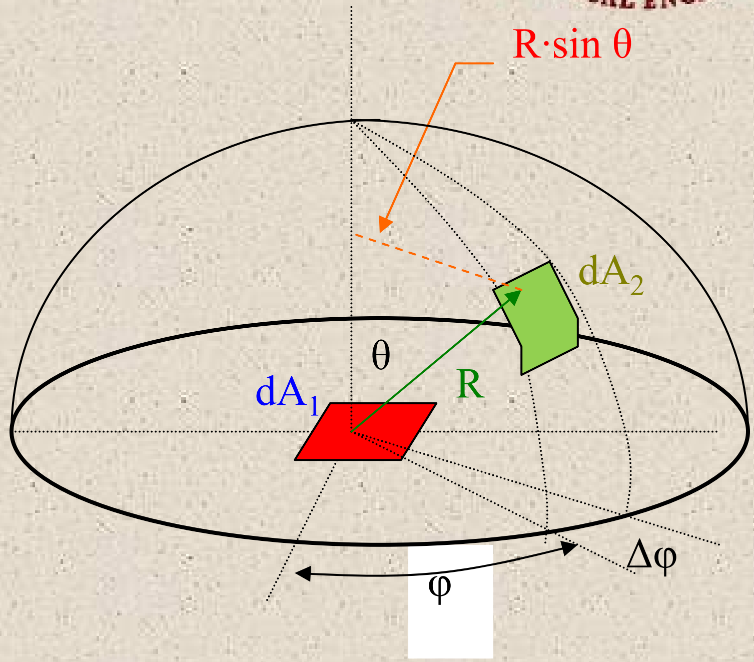
Intensity

The ideal intensity, I_b, may now be defined as the energy emitted from an ideal body, per unit projected area, per unit time, per unit solid angle.

$$I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}$$

Spherical Geometry

Since any surface will emit radiation outward in all directions above the surface, the spherical coordinate system provides a convenient tool for analysis. The three basic coordinates shown are R, φ, and θ, representing the radial, azimuthal and zenith directions.



In general dA_1 will correspond to the emitting surface or the source. The surface dA_2 will correspond to the receiving surface or the target. Note that the area proscribed on the hemisphere, dA_2, may be written as:

$$dA_2 = [(R \cdot \sin\theta) \cdot d\varphi] \cdot [R \cdot d\theta]$$

$$dA_2 = R^2 \cdot \sin\theta \cdot d\varphi \cdot d\theta]$$

Recalling the definition of the solid angle, $dA = R^2 \cdot d\Omega$
we find that: $d\Omega = R^2 \cdot \sin\theta \cdot d\theta \cdot d\varphi$

Real Surfaces

Thus far we have spoken of ideal surfaces, i.e. those that emit energy according to the Stefan-Boltzman law: $E_b = \sigma \cdot T_{abs}^4$

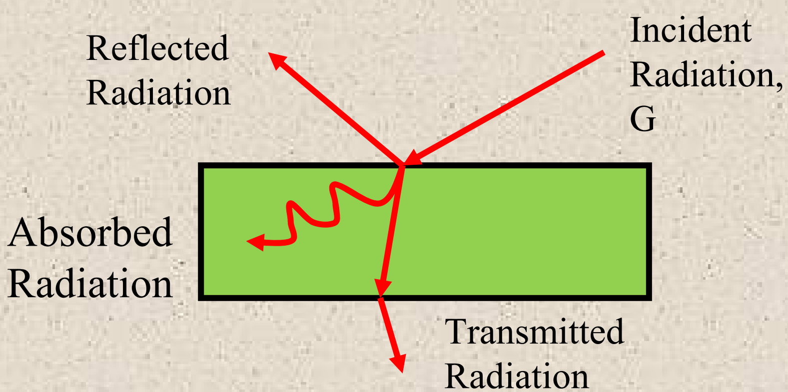
Real surfaces have emissive powers, E, which are somewhat less than that obtained theoretically by Boltzman. To account for this reduction, we introduce the emissivity, ε.

$$\varepsilon \equiv \frac{E}{E_b}$$

Emissive power from any real surface is given by: $E = \varepsilon \cdot \sigma \cdot T_{abs}^4$

Receiving Properties

Targets receive radiation in one of three ways; they absorption, reflection or transmission.



- Absorptivity, α, the fraction of incident radiation absorbed.
- Reflectivity, ρ, the fraction of incident radiation reflected.
- Transmissivity, τ, the fraction of incident radiation transmitted.

We see, from Conservation of Energy, that:

$$\alpha + \rho + \tau = 1$$

In this course, we will deal with only opaque surfaces, $\tau = 0$, so that:

$\alpha + \rho = 1$

Relationship Between Absorptivity,α, and Emissivity,ε

Consider two flat, infinite planes, surface A and surface B, both emitting radiation toward one another. Surface B is assumed to be an ideal emitter, i.e. $\varepsilon_B = 1.0$. Surface A will emit radiation according to the Stefan-Boltzman law as:

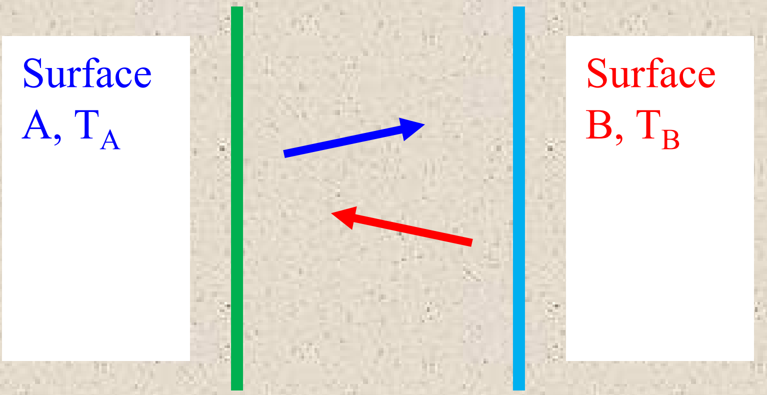
$$E_A = \varepsilon_A \cdot \sigma \cdot T_A^4$$

and will receive radiation as:

$$G_A = \alpha_A \cdot \sigma \cdot T_B^4$$

The net heat flow from surface A will be:

$$q'' = \varepsilon_A \cdot \sigma \cdot T_A^4 - \alpha_A \cdot \sigma \cdot T_B^4$$



Now suppose that the two surfaces are at exactly the same temperature. The heat flow must be zero according to the 2nd law. If follows then that: $\alpha_A = \varepsilon_A$

Thermodynamic properties of the material, α and ε may depend on temperature. In general, this will be the case as radiative properties will depend on wavelength, λ. The wave length of radiation will, in turn, depend on the temperature of the source of radiation.

The emissivity, ε, of surface A will depend on the material of which surface A is composed, i.e. aluminum, brass, steel, etc. and on the temperature of surface A.
The absorptivity, α, of surface A will depend on the material of which surface A is composed, i.e. aluminum, brass, steel, etc. and on the temperature of surface B.

Black Surfaces
Within the visual band of radiation, any material, which absorbs all visible light, appears as black. Extending this concept to the much broader thermal band, we speak of surfaces with α = 1 as also being "black" or "thermally black". It follows that for such a surface, ε = 1 and the surface will behave as an ideal emitter. The terms ideal surface and black surface are used interchangeably.

Diffuse Surface: Refers to directional independence of the intensity associated with emitted,reflected ,or incident radiation.

Grey Surface: A surface for which the spectral absorptivity and the emissivity are independent of wavelength over the spectral regions of surface irradiation and emission.

Relationship Between Emissive Power and Intensity

By definition of the two terms, emissive power for an ideal surface, E_b, and intensity for an ideal surface, I_b.

$$E_b = \int_{hemisphere} I_b \cdot \cos\theta \cdot d\Omega$$

Replacing the solid angle by its equivalent in spherical angles:

$$E_b = \int_0^{2\cdot\pi} \int_0^{\pi/2} I_b \cdot \cos\theta \cdot \sin\theta \cdot d\theta \cdot d\varphi$$

Integrate once, holding I_b constant:

$$E_b = 2\cdot\pi\cdot I_b \cdot \int_0^{\pi/2} \cos\theta \cdot \sin\theta \cdot d\theta$$

Integrate a second time. (Note that the derivative of sin θ is cos θ·dθ.)

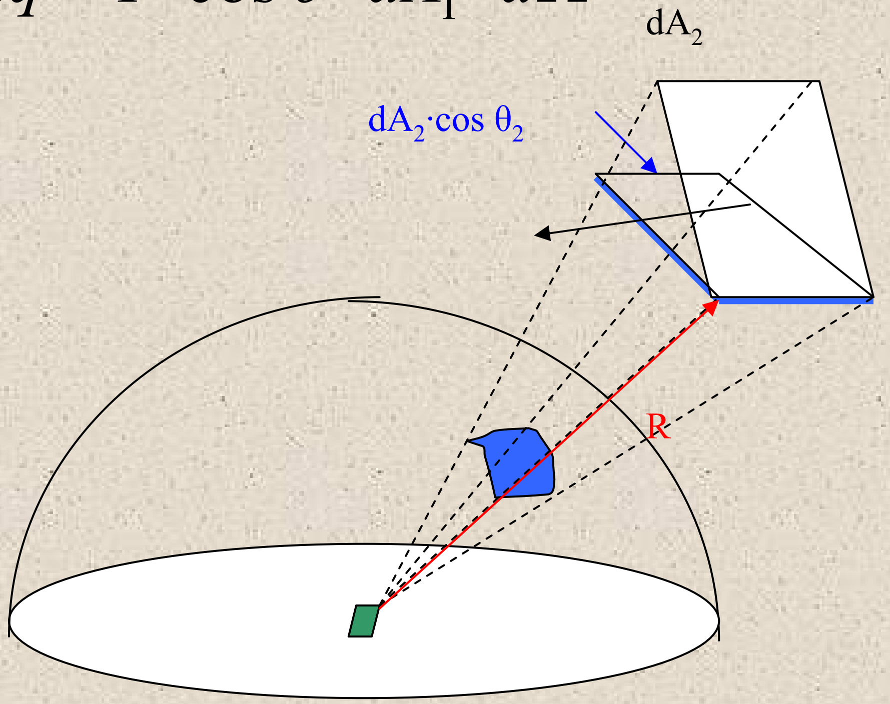
$$E_b = 2\cdot\pi\cdot I_b \cdot \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2} = \pi\cdot I_b$$

$$\boxed{E_b = \pi \cdot I_b}$$

$$I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}$$

$$dq = I \cdot \cos\theta \cdot dA_1 \cdot d\Omega$$

Next we will project the receiving surface onto the hemisphere surrounding the source. First find the projected area of surface dA_2, $dA_2 \cdot \cos\theta_2$. (θ_2 is the angle between the normal to surface 2 and the position vector, R.) Then find the solid angle, Ω, which encompasses this area.



To obtain the entire heat transferred from a finite area, dA_1, to a finite area, dA_2, we integrate over both surfaces:

$$q_{1\to 2} = \int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}$$

Total energy emitted from surface 1: $q_{emitted} = E_1 \cdot A_1 = \pi \cdot I_1 \cdot A_1$

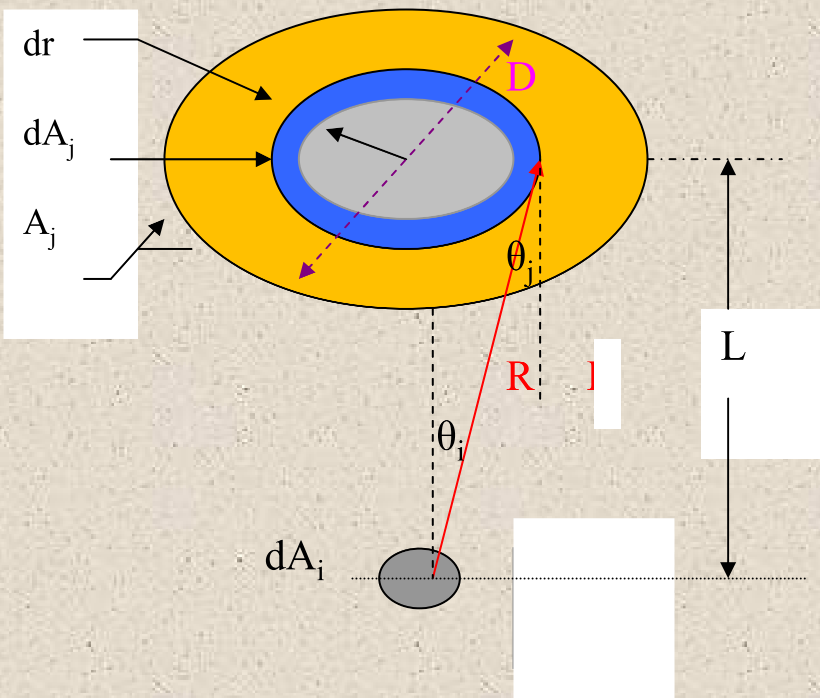
View Factors-Integral Method

Define the view factor, $F_{1\text{-}2}$, as the fraction of energy emitted from surface 1, which directly strikes surface 2.

$$F_{1\to2} = \frac{q_{1\to2}}{q_{emitted}} = \frac{\int_{A_2}\int_{A_1} \frac{I\cdot\cos\theta_1\cdot dA_1\cdot\cos\theta_2 dA_2}{R^2}}{\pi\cdot I\cdot A_1}$$

$$F_{1\to2} = \frac{1}{A_1}\cdot\int_{A_2}\int_{A_1}\frac{\cos\theta_1\cdot\cos\theta_2\cdot dA_1\cdot dA_2}{\pi\cdot R^2}$$

Example Consider a diffuse circular disk of diameter D and area A_j and a plane diffuse surface of area $A_i \ll A_j$. The surfaces are parallel, and A_i is located at a distance L from the center of A_j. Obtain an expression for the view factor F_{ij}.



$$F_{1\to 2} = \frac{1}{A_1} \cdot \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2}$$

Since dA_1 is a differential area

$$F_{1\to 2} = \int_{A_2} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_2}{\pi \cdot R^2}$$

$$F_{1\to 2} = \int_{A_2} \frac{\left(L/R\right)^2 \cdot 2\pi \cdot r \cdot dr}{\pi \cdot R^2}$$

$$F_{1\to 2} = \int_{A_2} \frac{L^2 \cdot 2 \cdot r \cdot dr}{R^4}$$

Let $\rho^2 \equiv L^2 + r^2 = R^2$. Then $2\cdot\rho\cdot d\rho = 2\cdot r\cdot dr$.

$$F_{1\to 2} = \int_{A_2} \frac{L^2 \cdot 2 \cdot \rho \cdot d\rho}{\rho^4}$$

$$F_{1\to 2} = -2 \cdot L^2 \cdot \left.\frac{\rho^{-2}}{2}\right|_{A_2} = -L^2 \cdot \left[\frac{1}{L^2 + \rho^2}\right]_0^{D/2}$$

$$F_{1\to 2} = -L^2 \cdot \left[\frac{4}{4 \cdot L^2 + D^2} - \frac{1}{L^2}\right]_0^{D/2} = \frac{D^2}{4 \cdot L^2 + D^2}$$

Enclosures

In order that we might apply conservation of energy to the radiation process, we must account for all energy leaving a surface. We imagine that the surrounding surfaces act as an enclosure about the heat source which receive all emitted energy. For an N surfaced enclosure, we can then see that:

$$\sum_{j=1}^{N} F_{i,j} = 1$$

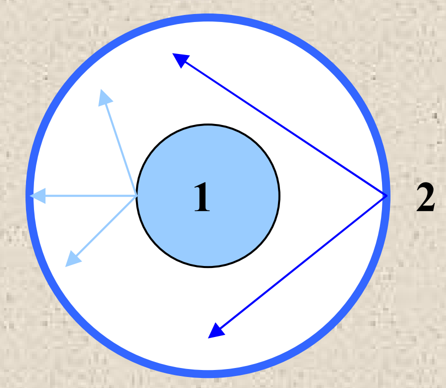
This relationship is known as the Conservation Rule".

Reciprocity

$$A_i \cdot F_{i \to j} = A_j \cdot F_{j \to i}$$

This relationship is known as "Reciprocity".

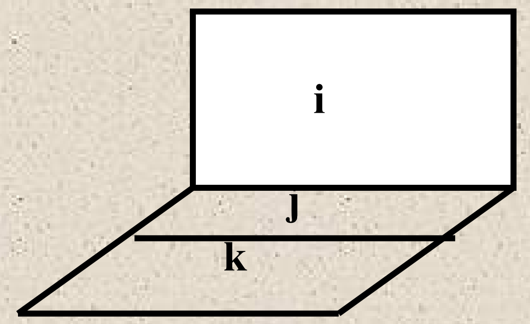
Example: Consider two concentric spheres shown to the right. All radiation leaving the outside of surface 1 will strike surface 2. Part of the radiant energy leaving the inside surface of object 2 will strike surface 1, part will return to surface 2. Find $F_{2,1}$. Apply reciprocity.



$$A_2 \cdot F_{2,1} = A_1 \cdot F_{1,2} \Rightarrow F_{2,1} = \frac{A_1}{A_2} \cdot F_{1,2} = \frac{A_1}{A_2} = \left[\frac{D_1}{D_2}\right]^2$$

Associative Rule

Consider the set of surfaces shown to the right: Clearly, from conservation of energy, the fraction of energy leaving surface i and striking the combined surface j+k will equal the fraction of energy emitted from i and striking j plus the fraction leaving surface i and striking k.



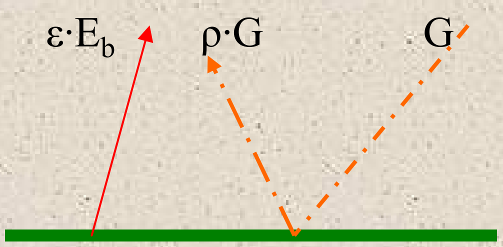
$$F_{i \Rightarrow (j+k)} = F_{i \Rightarrow j} + F_{i \Rightarrow k}$$

This relationship is known as the "Associative Rule".

Radiosity

Radiosity, J, is defined as the total energy leaving a surface per unit area and per unit time.

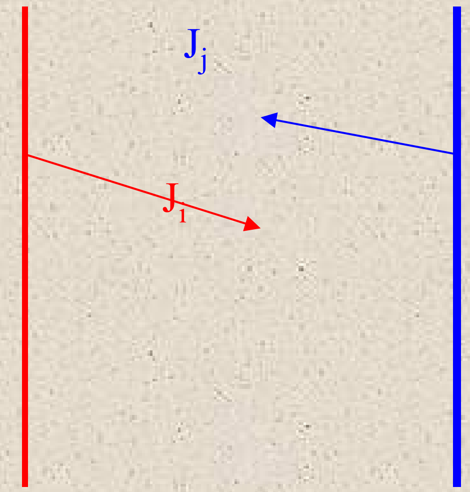
$$J \equiv \varepsilon \cdot E_b + \rho \cdot G$$



Net Exchange Between Surfaces

Consider the two surfaces shown. Radiation will travel from surface i to surface j and will also travel from j to i.

$$q_{i\to j} = J_i \cdot A_i \cdot F_{i\to j}$$



likewise,

$$q_{j\to i} = J_j \cdot A_j \cdot F_{j\to j}$$

The net heat transfer is then:

$$q_{j\to i}\,(\text{net}) = J_i \cdot A_i \cdot F_{i\to j} - J_j \cdot A_j \cdot F_{j\to j}$$

From reciprocity we note that $F_{1\to 2} \cdot A_1 = F_{2\to 1} \cdot A_2$ so that

$$q_{j\to i}\,(\text{net}) = J_i \cdot A_i \cdot F_{i\to j} - J_j \cdot A_i \cdot F_{i\to j} = A_i \cdot F_{i\to j} \cdot (J_i - J_j)$$

Net Energy Leaving a Surface

The net energy leaving a surface will be the difference between the energy leaving a surface and the energy received by a surface:

$$q_{1\rightarrow} = [\varepsilon \cdot E_b - \alpha \cdot G] \cdot A_1$$

Combine this relationship with the definition of Radiosity to eliminate G.

$$J \equiv \varepsilon \cdot E_b + \rho \cdot G \Rightarrow G = [J - \varepsilon \cdot E_b]/\rho$$

$$q_{1\rightarrow} = \{\varepsilon \cdot E_b - \alpha \cdot [J - \varepsilon \cdot E_b]/\rho\} \cdot A_1$$

Assume opaque surfaces so that $\alpha + \rho = 1 \Rightarrow \rho = 1 - \alpha$, and substitute for ρ.

$$q_{1\rightarrow} = \{\varepsilon \cdot E_b - \alpha \cdot [J - \varepsilon \cdot E_b]/(1 - \alpha)\} \cdot A_1$$

Put the equation over a common denominator:

$$q_{1\rightarrow} = \left[\frac{(1-\alpha)\cdot\varepsilon\cdot E_b - \alpha\cdot J + \alpha\cdot\varepsilon\cdot E_b}{1-\alpha}\right]\cdot A_1 = \left[\frac{\varepsilon\cdot E_b - \alpha\cdot J}{1-\alpha}\right]\cdot A_1$$

assume that $\alpha = \varepsilon$

$$q_{1\rightarrow} = \left[\frac{\varepsilon\cdot E_b - \varepsilon\cdot J}{1-\varepsilon}\right]\cdot A_1 = \left[\frac{\varepsilon\cdot A_1}{1-\varepsilon}\right]\cdot\left(E_b - J\right)$$

Electrical Analogy for Radiation

We may develop an electrical analogy for radiation, similar to that produced for conduction. The two analogies should not be mixed: they have different dimensions on the potential differences, resistance and current flows.

	Equivalent Current	Equivalent Resistance	Potential Difference
Ohms Law	I	R	ΔV
Net Energy Leaving Surface	$q_{1?}$	$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right]$	E_b - J
Net Exchange Between Surfaces	$q_{i?\,j}$	$\frac{1}{A_1 \cdot F_{1\rightarrow 2}}$	$J_1 - J_2$

- **Insulated surfaces.** In steady state heat transfer, a surface cannot receive net energy if it is insulated. Because the energy cannot be stored by a surface in steady state, all energy must be re-radiated back into the enclosure. *Insulated surfaces are often termed as re-radiating surfaces.*
- **Black surfaces:** A black, or ideal surface, will have no surface resistance:

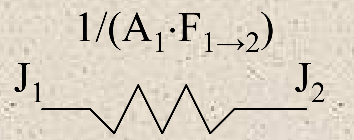
$$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right] = \left[\frac{1-1}{1 \cdot A}\right] = 0$$

In this case the nodal Radiosity and emissive power will be equal.

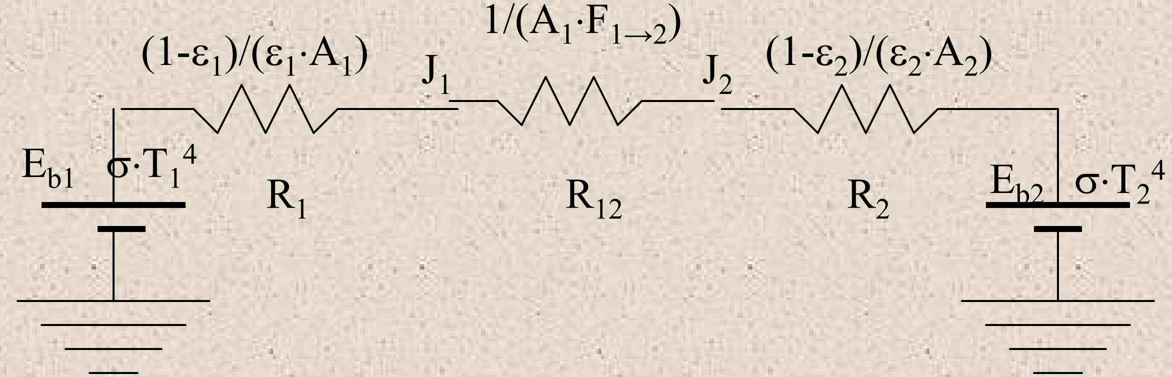
- Large surfaces: Surfaces having a large surface area will behave as black surfaces, irrespective of the actual surface properties:

$$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right] = \left[\frac{1-\varepsilon}{\varepsilon \cdot \infty}\right] = 0$$

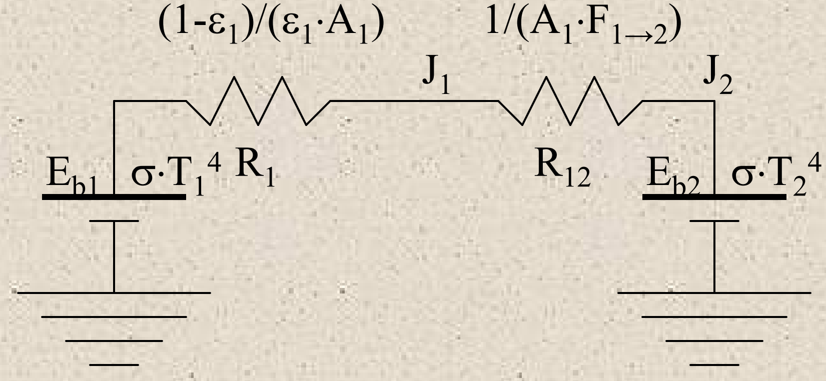
Consider the case of an object, 1, placed inside a large enclosure, 2. The system will consist of two objects, so we proceed to construct a circuit with two radiosity nodes.



Now we ground both Radiosity nodes through a surface resistance.



Since A_2 is large, $R_2 = 0$. The view factor, $F_{1\rightarrow 2} = 1$



Sum the series resistances:

$$R_{Series} = (1-\varepsilon_1)/(\varepsilon_1 \cdot A_1) + 1/A_1 = 1/(\varepsilon_1 \cdot A_1)$$

Ohm's law:

$$i = \Delta V/R$$

or by analogy:

$$q = \Delta E_b/R_{Series} = \varepsilon_1 \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

Returning for a moment to the coal grate furnace, let us assume that we know (a) the total heat being produced by the coal bed, (b) the temperatures of the water walls and (c) the temperature of the super heater sections.

Apply Kirchoff’s law about node 1, for the coal bed:

$$q_1 + q_{2\to1} + q_{3\to1} = q_1 + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

Similarly, for node 2:

$$q_2 + q_{1\to2} + q_{3\to2} = \frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$$

And for node 3:

$$q_3 + q_{1\to3} + q_{2\to3} = \frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0$$

The three equations must be solved simultaneously. Since they are each linear in J, matrix methods may be used:

$$\begin{bmatrix} -\frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{12}} & \frac{1}{R_{13}} \\ \frac{1}{R_{12}} & -\frac{1}{R_2} - \frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{23}} \\ \frac{1}{R_{13}} & \frac{1}{R_{23}} & -\frac{1}{R_3} - \frac{1}{R_{13}} - \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} -q_1 \\ -\frac{E_{b2}}{R_2} \\ -\frac{E_{b3}}{R_3} \end{bmatrix}$$

Surface 1: Find the coal bed temperature, given the heat flow:

$$q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{\sigma \cdot T_1^4 - J_1}{R_1} \Rightarrow T_1 = \left[\frac{q_1 \cdot R_1 + J_1}{\sigma}\right]^{0.25}$$

Surface 2: Find the water wall heat input, given the water wall temperature:

$$q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{\sigma \cdot T_2^4 - J_2}{R_2}$$

Surface 3: (Similar to surface 2) Find the water wall heat input, given the water wall temperature:

$$q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{\sigma \cdot T_3^4 - J_3}{R_3}$$