Module 7: Solved Problems

1. Deionized water flows through the inner tube of 30-mm diameter in a thin-walled concentric tube heat exchanger of 0.19-m length. Hot process water at 95°C flows in the annulus formed with the outer tube of 60-mm diameter. The deionized water is to be heated from 40° to 60°C at a flow rate of 5 kg/s. The thermo physical properties of the fluids are:

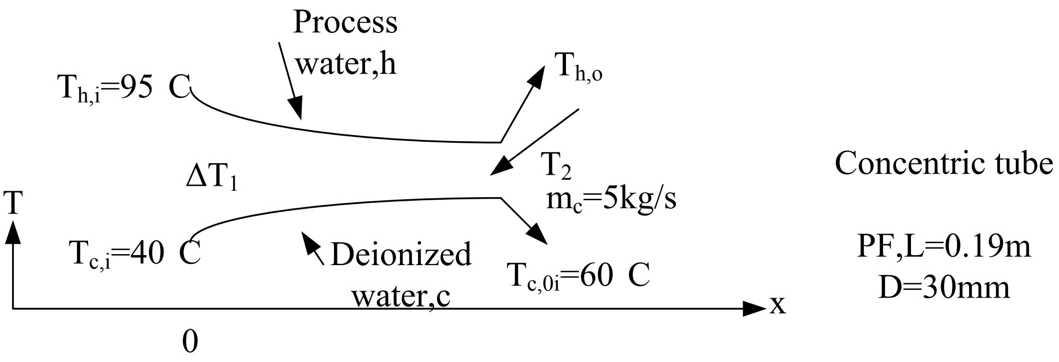
	DEIONIZED WATER	**PROCESS WATER**
kg/m3)	**982.3**	**967.1**
c_p(J/kg.K	**4181**	**4197**
k(W/m.K	**0.643**	**0.673**
N.s/m^2	**548**	**324**
pr	**3.56**	**2.02**

(a) Considering a parallel-flow configuration of the heat exchanger, determine the minimum flow rate required for the hot process water.
(b) Determine the overall heat transfer coefficient required for the conditions of part a.
(c) Considering a counter flow configuration, determine the minimum flow rate required for the hot process water. What is the effectiveness of the exchanger for this situation?

$\rho($

$\mu(\qquad)*10^6$

Schematic:



Assumptions: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes.

Analysis: (a) from overall energy balances,

$$q = (\dot{m}c)_h(T_{h,i} - T_{h,o}) = (\dot{m}c)_h(T_{c,o} - T_{c,i})$$

For a fixed term $T_{h,i}$, $(\dot{m})_h$ will be a minimum when $T_{h,o}$ is a minimum. With the parallel flow configuration, this requires that $T_{h,o}=T_{c,o}=60°C$. Hence,

$$\dot{m}h,\min = \frac{(mc)_c(\dot{T}_{c,o} - T_{c,i})}{c_h(T_{h,i} - T_{h,o})} = \frac{5kg/s \times 4181J/kg.K(60-40)^{\circ}C}{4197J/kg.K(95-60)°C} = 2.85kg/s$$

(b)From the rate equation and the log mean temperature relation,

$$q = UA\Delta T_{lm,PF} \qquad \Delta T_{lm,PF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}$$

And since $\Delta T_2=0$, $\Delta T_{lm}=0$ so that UA=∞. Since A=πDL is finite, U must be extremely large. Hence, the heating cannot be accomplished with this arrangement.

(c) With the CF arrangements $\dot{m}_h$ will be a minimum when T_{ho} is a minimum. This requires that $T_{h,o}$ is a minimum. This requires that $T_{h,o}$ is a minimum. This requires that $T_{h,o}=T_{c,i}=40°C$. Hence, from the overall energy balance,

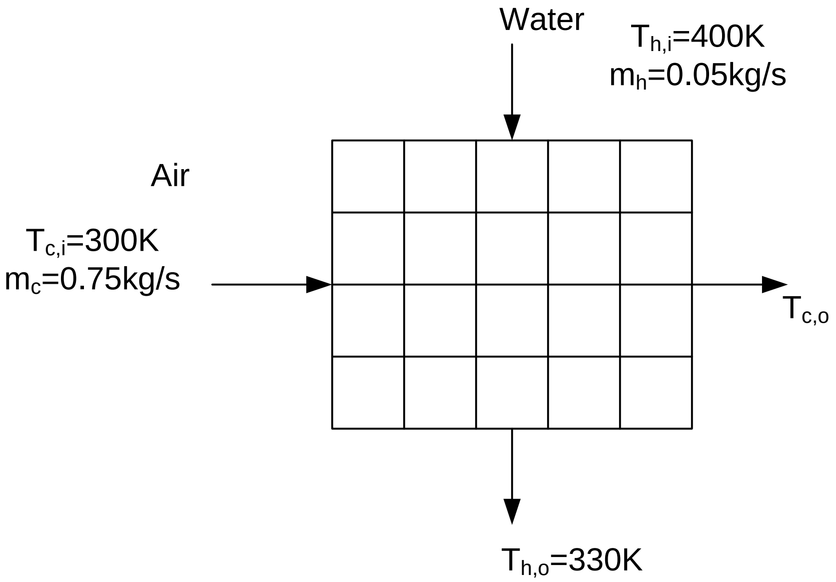
$$\dot{m} = \frac{5kg/s \times 4181J/kg.K(60-40)K}{4197J/kg.K(95-40)K} = 1.81kg/s$$

For this condition, $C_{min}=C_h$ which is cooled from $T_{h,i}$ to $T_{c,i}$, hence $\varepsilon=1$

Comments: For the counter flow arrangement, the heat exchanger must be infinitely long.

2. Water with a flow rate of 0.05kg/s enters an automobile radiator at 400K and leaves at 330 K. The water is cooled by air in cross flow which enters at 0.75kg/s and leaves at 300K. If the overall heat transfer coefficient is 200W/m^2.K, what is the required heat transfer surface area?

   Schematic:



Assumptions: (1) Negligible heat loss to surroundings and kinetic and potential energy changes, (2) Constant properties.

Analysis: The required heat transfer rate is

$$q = (\dot{m}c)_h(T_{h,i} - T_{h,o}) = 0.05kg/s(4209J/kg.K)70K = 14{,}732W$$

Using the ε-NTU method,

$$C_{\min} = C_h = 210.45 W / K$$

$$C_{\max} = C_c = 755.25 W / K,$$

$$hence, C_{\min} / C_{\max} (T_{h,i} - T_{c,i}) = 210.45 W / K (100 K) = 21{,}045 W$$

and

$$\varepsilon = q / q_{\max} = 14{,}732 W / 21{,}045 W = 0.700$$

From figure, NTU≈1.5, hence

$$A = NTU(C_{\min} / U) = 1.5 \times 210.45 W / K (200 W / m^2.K) = 1.58 m^2$$

Comments: (1) the air outlet temperature is

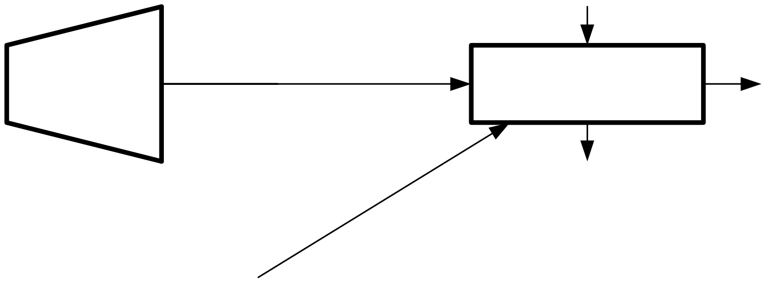
$$T_{c,o} = T_{c,i} + q / C_c = 300 K + (14{,}732 W / 755.25 W / K) = 319.5 K$$

(2) Using the LMTD approach, ΔT_{lm}=51.2 K, R=0.279 and P=0.7. Hence from fig F≈0.95 and

$$A = q / FU\Delta T_{lm} = (14{,}732 W) / [0.95 (200 W / m^2.K) 51.2 K] = 1.51 m^2.$$

3. Saturated steam leaves a steam turbine at a flow rate of 1.5kg/s and a pressure of 0.51 bars. The vapor is to be completely condensed to saturated liquid in a shell-and –tube heat exchanger which uses water as the coolant. The water enters the thin-walled tubes at 17°C and leaves at 57°C. If the overall heat transfer coefficient of 200W/m^2.K, determine the required heat exchanger surface area and the water flow rate. After extended operation, fouling causes the overall heat transfer coefficient to decrease to 100W/m^2.K. For the same water inlet temperature and flow rate, what is the new vapor flow rate required for complete condensation?

Schematic:



Assumptions: (1) Negligible heat loss to surroundings, (2) Negligible wall conduction resistance.

Properties: Table for
sat.Water:

$$(\bar{T}_c = 310K): \ c_{p,c} = 4178 J/kg.K; (p = 0.51\,\text{bars}): T_{sat} = 355K, h_{fg} = 2304 kJ/kg.$$

Analysis: (a) The required heat transfer rate is

$$q = \dot{m}_h h_{fg} = 1.5 kg/s(2.304 \times 10^6 J/kg) = 3.46 \times 10^6 W$$

And the corresponding heat capacity rate for the water is

$$C_c = C_{\min} = q/(T_{c,o} - T_{c,i}) = 3.48 \times 10^6 W/40K = 86{,}400 W/K$$

$$hence, \varepsilon = q/(C_{\min}[T_{h,i} - T_{c,i}]) = 3.46 \times 10^6 W/86{,}400 W/K(65K) = 0.62$$

$$\text{since} \quad \mathrm{C_{min}/C_{max}} = 0,$$

$$\mathrm{NTU} = -\ln(1 - \varepsilon) = -\ln(1 - 0.62) = 0.97$$

And

$$A = NTU(C_{\min}/U) = 0.97(86{,}400 W/K/2000 W/m2.K) = 41.9 m2$$

$$\dot{m}_c = C_c / c_{p,c} = 86{,}400 W/K/4178 J/kg.K = 20.7 kg/s$$

(b) using the final overall heat transfer coefficient, find

Since $\mathrm{C_{min}/C_{max}} = 0,$

$$\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-0.485) = 0.384$$

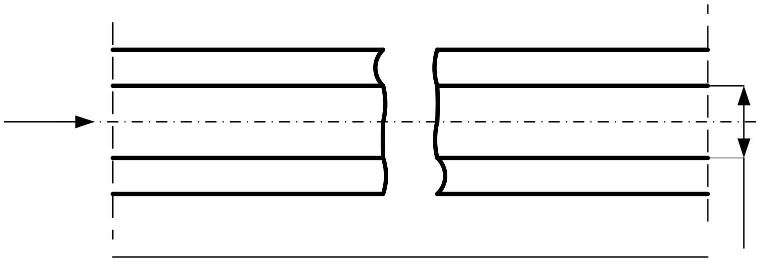
$$\text{hence, } \mathrm{q} = \varepsilon \mathrm{C_{min}}(T_{h,i} - T_{c,i}) = 0.384(886{,}400 W/K)65K = 2.16106 W$$

$$\dot{m}_h = q/h_{fg} = 2.16 \times 10^6 W/2.304 \times 10^6 J/kg = 0.936 kg/s$$

Comments: The significant reduction (38%) in $\dot{m}_h$ represents a significant loss in turbine power. Periodic cleaning of condenser surfaces should be employed to minimize the adverse effects of fouling.

4. Water at 225 kg/h is to be heated from 35 to 95°C by means of a concentric tube heat exchanger. Oil at 225kg/h and 210°C, with a specific heat of 2095 J/kg.K, is to be used as the hot fluid. If the overall heat transfer coefficient based on the outer diameter of the inner tube if 550W/m^2.K, determine the length of the exchanger if the outer diameter is 100mm.

Schematic:



Assumptions: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

Properties: Table for Water:

$$(\bar{T}_c = (35+95)^\circ C/2 = 338K) : c_{p,c} = 4188 J/kg.K$$

Oil

Analysis: From rate equation with $A_o = \pi D_o L$, $L = q/U_o D_o \Delta T \lambda_m$

The heat rate, q, can be evaluated from an energy balance on the cold fluid,

$\dot{m}_h = 225kg/h$

$C_h = 2095J/kg.K$

$T_{h,i} = 210°C$

$$q = \dot{m}_c c_c (T_{c,0} - T_{c,i}) = \frac{225kg/h}{3600s/h} \times 4188J/kg.K(95-35)K = 15,705W$$

In order to evaluate $\Delta T \lambda_m$, we need to know whether the exchanger is operating in CF or PF. From an energy balance on the hot fluid, find

$$T_{h,o} = T_{h,i} - q/\dot{m}_h c_h = 210^\circ C - 15,705W / \frac{225kg/h}{3600s/h} \times 2095 \frac{J}{kg.K} = 90.1°C$$

Since $T_{h,o}<T_{c,o}$ it follows that HXer operation must be CF. From eq. for log mean temperature difference,

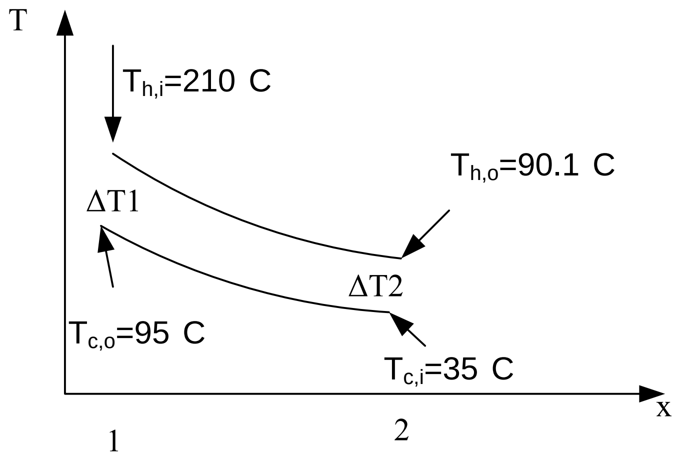
$$\Lambda T_{\lambda m,CF} = \frac{\Delta T_1 - \Delta T_2}{\lambda n(\Delta T_1 / \Delta T_2)} = \frac{(210-95)-(90.1-35)}{\lambda n(115/55.1)} °C = 81.5°C$$

Substituting numerical values, the HXer length is

$$L = 15{,}705W / 550W / m^2.K\pi(0.10m) \times 81.4K = 1.12m$$

Comments: The ε–NTU method could also be used. It would be necessary to perform the hot fluid energy balance to determining CF operation existed. The capacity rate is $C_{min}/C_{max}=0.50$. From eq. for effectiveness, and from with q evaluated from an energy balance on the hot fluid,

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{210-90.1}{210-35} = 0.69$$



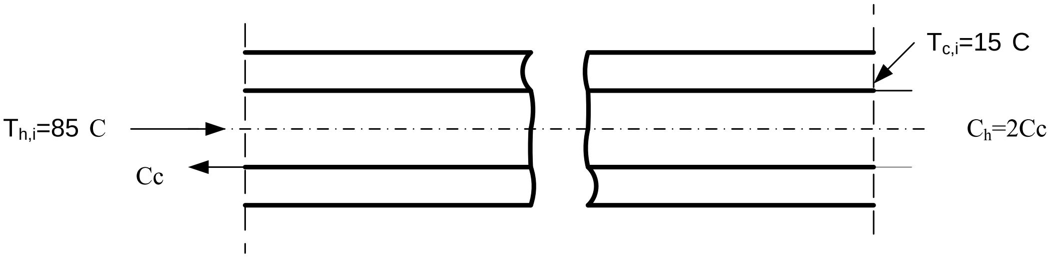
From fig, find NTU≈1.5 giving

$$L = NTU.C_{\min} / U_o \pi D_o \approx 1.5 \times 130.94 \frac{W}{K} 550 \frac{W}{m^2.K} .\pi(0.10m) \approx 1.14m$$

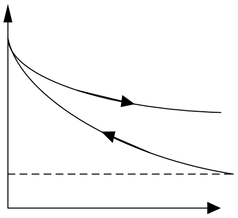
Note the good agreement by both methods.

5. Consider a very long, concentric tube heat exchanger having hot and cold water inlet temperatures of 85 and 15°C. The flow rate of the hot water is twice that of the cold water. Assuming equivalent hot and cold water specifies heats; determine the hot water outlet temperature for the following modes of operation (a) Counter flow, (b) Parallel flow.

Schematic:



Assumptions: (1) equivalent hot and cold water specific heats, (2) Negligible Kinetic and potential energy changes, (3) No eat loss to surroundings.



Analysis: the heat rate for a concentric tube Heat exchanger with very large surface area Operating in the counter flow mode is ○

$$q = q_{\max} = C_{\min}(T_{h,i} - T_{c,i})$$

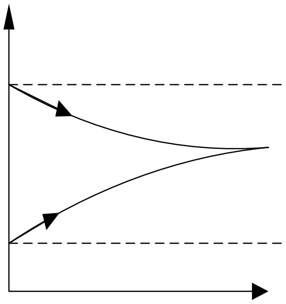
Combining the above relation and rearranging, find

$$T_{h,o} = -\frac{C_{\min}}{C_h}(T_{h,i} - T_{c,i}) + T_{h,i} = -\frac{C_c}{C_h}(T_{h,i} - T_{c,i}) + T_{h,i}$$

Substituting numerical values

$$T_{h,o} = -\frac{1}{2}(85-15)°C + 85°C = 50°C$$

For parallel flow operation, the hot and cold outlet temperatures will be equal; that is $T_{c,o}=T_{h,o}$. Hence



$$C_c(T_{c,o} - T_{c,i}) = C_h(T_{h,i} - T_{h,o})$$

Setting $T_{c,o}=T_{h,o}$ and rearranging

$$T_{h,o} = \left[T_{h,i} + \frac{C_c}{C_h}T_{c,i}\right] / \left[1 + \frac{C_c}{C_h}\right]$$

$$T_{h,o} = \left[85 + \frac{1}{2}\times 15\right]°C / \left[1 + \frac{1}{2}\right] = 61.7^{\circ}C$$

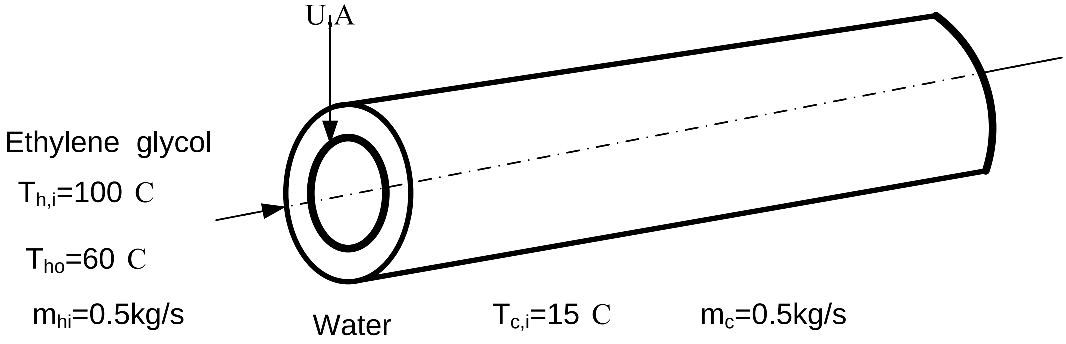
Comments: Note that while $\varepsilon = 1$ for CF operation, for PF operation find $\varepsilon = q/q_{max} = 0.67$.

6. A concentric tube heat exchanger uses water, which is available at 15°C, to cool ethylene glycol from 100 to 60°C. The water and glycol flow rates are each 0.5 kg/s. Determine the maximum possible heat transfer rate and effectiveness of the exchanger. Determine which is preferred, a parallel –flow or counter flow mode of operation?

   Known: Inlet temperatures and flow rate for a concentric tube heat exchanger.

   Find: (a) Maximum possible heat transfer rate and effectiveness, (b) Proffered mode of operation.

   Schematic:



   Assumptions: (1) Steady-state operation, (2) Negligible KE and PE changes, (3) Negligible heat loss to surroundings, (4) Fixed overall heat transfer and coefficient.

   Properties: Table: Ethylene glycol ($\bar{T}_{in} = 80°C$); cp=2650J/kg.K;

   $Water(\bar{T}_m \approx 30°C): c_p = 4178 J/kg.K$

   Analysis: (a) Using the ε-NTU method, find

$$C_{\min} = C_h = \dot{m}_h c_{p,h} = (0.5 kg/s)(2650 J/kg.K) = 1325 W/K$$

$$q_{ma}x = C_{\min}(T_{h.i} - T_{c,i}) = (1325 W/K)(100-15)°C = 1.13 \times 10^5 W$$

$$q = \dot{m}_h c_{p,h}(T_{h.i} - T_{c,i}) = 0.5kg/s(2650J/kg.K)(100-60)°C = 0.53\times10^5 W$$

$$\varepsilon = q/q_{\max} = 0.53\times10^5/1.13\times10^5 = 0.47$$

(b)

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 15°C + \frac{0.53\times10^5}{0.5kg/s\times4178J/kg.K} = 40.4°C$$

Since $T_{c,o} < T_{h,o}$, a parallel flow mode of operation is possible. However, with $(C_{\min}/C_{\max}) = (\dot{m}_h c_{p,h}/\dot{m}_c c_{p,c}) = 0.63$,

From fig $(NTU)_{PF} \approx 0.95$, $(NTU)_{CF} \approx 0.75$

Hence

$(A_{CF}/A_{PF}) = (NTU)_{CF}/(NTU)_{PF} \approx (0.75/0.95) = 0.79$

Because of the reduced size requirement, hence capital investment, the counter flow mode of operation is preferred.