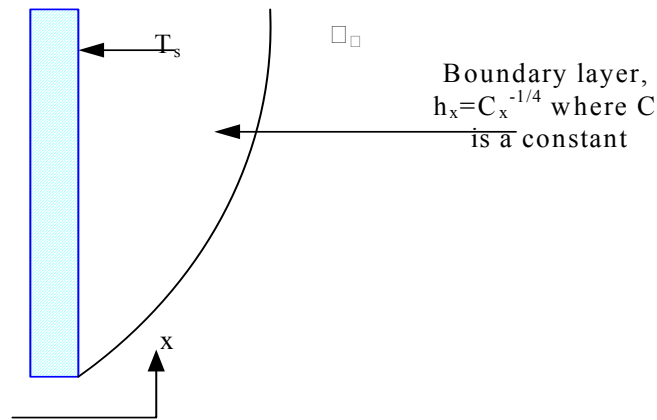


MODULE 6: Worked-out Problems

Problem 1:

For laminar free convection from a heated vertical surface, the local convection coefficient may be expressed as $h_x = Cx^{-1/4}$, where h_x is the coefficient at a distance x from the leading edge of the surface and the quantity C , which depends on the fluid properties, is independent of x . Obtain an expression for the ratio \bar{h}_x/h_x , where \bar{h}_x is the average coefficient between the leading edge ($x=0$) and the x location. Sketch the variation of h_x and \bar{h}_x with x .

Schematic:

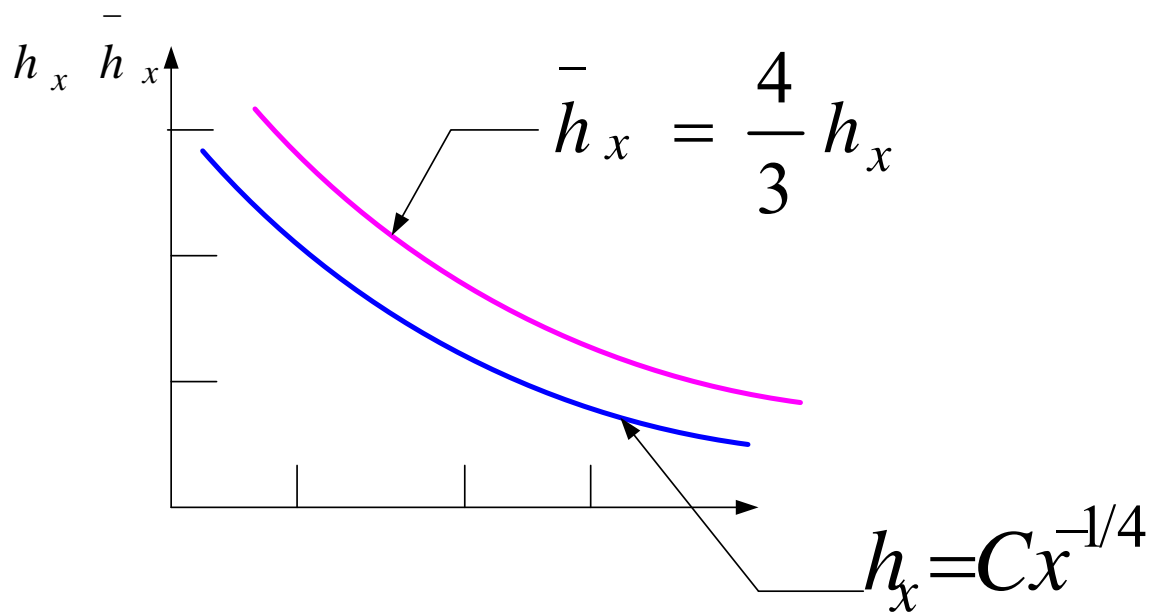


Analysis: It follows that average coefficient from 0 to x is given by

$$\bar{h}_x = \frac{1}{x} \int_0^x h_x dx = \frac{C}{x} \int_0^x x^{-1/4} dx$$
$$\bar{h}_x = \frac{4C}{3} x^{3/4} = \frac{4}{3} Cx^{-1/4} = \frac{4}{3} h_x$$

Hence $\frac{\bar{h}_x}{h_x} = \frac{4}{3}$

The variation with distance of the local and average convection coefficient is shown in the sketch.



Comments: note that $\bar{h}_x/h_x = 4/3$, independent of x . hence the average coefficients for an entire plate of length L is $\bar{h}_L = 4/3h_L$, where h_L is the local coefficient at $x=L$. note also that the average exceeds the local. Why?

Problem 2:

Experiments to determine the local convection heat transfer coefficient for uniform flow normal to heated circular disk have yielded a radial Nusselt number distribution of the form

$0.5C$

$$\text{Nu}_D = \frac{h(r)D}{k} = \left[1 + \left(a \frac{r}{r_0} \right)^n \right]$$

Where n and a are positive. The Nusselt number at the stagnation point is correlated in terms of the Reynolds number ($\text{Re}_D = VD/\nu$) and Prandtl

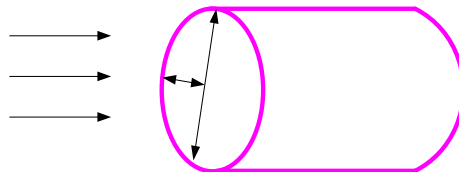
$$\text{Nu}_0 = \frac{h(r=0)D}{k} = 0.814 \text{Re}_D^{1/2} \text{Pr}^{0.36}$$

Obtain an expression for the average Nusselt number, $\bar{Nu}_D = \bar{h}D/k$, corresponding to heat transfer from an isothermal disk. Typically boundary layer development from a stagnation point yields a decaying convection coefficient with increasing distance from the stagnation point. Provide a plausible for why the opposite trend is observed for the disk.

Known: Radial distribution of local convection coefficient for flow normal to a circular disk.

Find: Expression for average Nusselt number.

Schematic:



Assumptions: Constant properties.

Analysis: The average convection coefficient is

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{\pi r_o^2} \int_0^{r_o} \frac{k}{D} \text{Nu}_0 [1 + a(r/r_o)^n] 2\pi r dr$$

$$\bar{h} = \frac{k \text{Nu}_0}{r_o^3} \left[\frac{r^2}{2} + \frac{a r^{n+2}}{(n+2)r_o^n} \right]_0^{r_o}$$

Where Nu_0 is the Nusselt number at the stagnation point ($r=0$).hence,

$$\bar{\text{Nu}}D = \frac{\bar{h}D}{k} = 2\text{Nu}_0 \left[\frac{(r/r_o)^2}{2} + \frac{a}{(n+2)} \left(\frac{r}{r_o} \right)^{n+2} \right]_0^{r_o}$$

$$\bar{\text{Nu}}D = \text{Nu}_0 [1 + 2a/(n+2)]$$

$$\bar{\text{Nu}}D = [1 + 2a/(n+2)] 0.841 \text{Re}_D^{1/2} \text{Pr}^{0.36}$$

Comments: The increase in $h(r)$ with r may be explained in terms of the sharp turn, which the boundary layer flow must take around the edge of the disk. The boundary layer accelerates and its thickness decreases as it makes the turn, causing the local convection coefficient to increase.

Problem 3:

In a flow over a surface, velocity and temperature profiles are of the forms

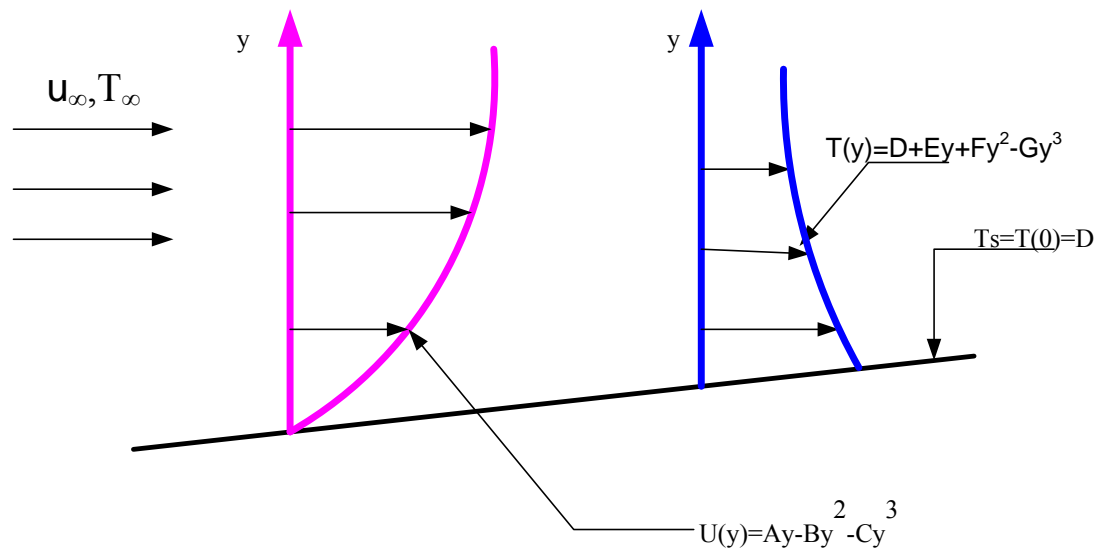
$$u(y)=Ay+By^2-Cy^3 \quad \text{and} \quad T(y)=D+Ey+Fy^2-Gy^3$$

Where the coefficients A through G are constants. Obtain expressions for friction coefficients C_f and the convection coefficient h in terms of u_∞ , T_∞ and appropriate profile coefficients and fluid properties.

Known: form of the velocity and temperature profiles for flow over a surface.

Find: expressions for the friction and convection coefficients.

Schematic:



Analysis: The shear stress at the wall is

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu [A + 2By - 3Cy^2]_{y=0} = A\mu.$$

Hence, the friction coefficient has the form,

$$C_f = \frac{\tau_s}{\rho u_\infty^2 / 2} = \frac{2A\mu}{\rho u_\infty^2}$$

$$C_f = \frac{2Av}{u_\infty^2}$$

The convection coefficient is

$$h = \frac{-k_f (\partial T / \partial y)_{y=0}}{T_s - T_\infty} = \frac{-k_f [E + 2Fy - 3Gy^2]_{y=0}}{D - T_\infty}$$

$$h = \frac{-k_f E}{D - T_\infty}$$

Comments: It is a simple matter to obtain the important surface parameters from knowledge of the corresponding boundary layers profile. However is rarely simple matter to determine the form of the profile.

Problem 4:

In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as

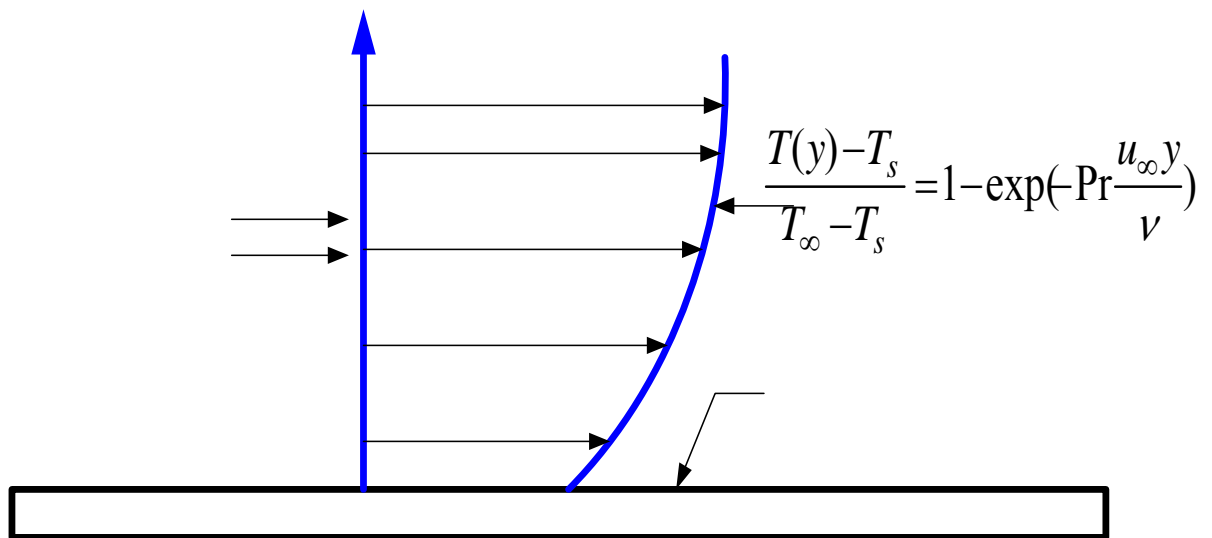
$$\frac{T - T_s}{T_\infty - T_s} = 1 - \exp\left(-\text{Pr} \frac{u_\infty y}{\nu}\right)$$

Where y is the distance normal to the surface and the Prandtl number, $\text{Pr} = c_p \mu / k = 0.7$, is a dimensionless fluid property. If $T_\infty = 400\text{K}$, $T_s = 300\text{K}$, and $u_\infty / \nu = 5000\text{m}^{-1}$, what is the surface heat flux?

Known: Boundary layer temperature distribution

Find: Surface heat flux.

Schematic:



Properties:

Air ($T_s = 300\text{K}$): $k = 0.0263 \text{ W/m.k}$

Analysis:

Applying the Fourier's law at $y=0$, the heat flux is

$$q_s'' = -k \frac{\partial T}{\partial y} \Big|_{y=0} = -k(T_\infty - T_s) \left[\text{Pr} \frac{u_\infty}{\nu} \right] \exp \left[-\text{Pr} \frac{u_\infty y}{\nu} \right] \Big|_{y=0}$$

$$q_s'' = -k(T_\infty - T_s) \text{Pr} \frac{u_\infty}{\nu}$$

$$q_s'' = -0.02063 \text{w/m.K}(100\text{K}) 0.7 \times 5000 \text{ 1/m}$$

Comments: (1) Negligible flux implies convection heat transited surface (2) Note use of k at T_s to evaluate from q_s'' Fourier's law.

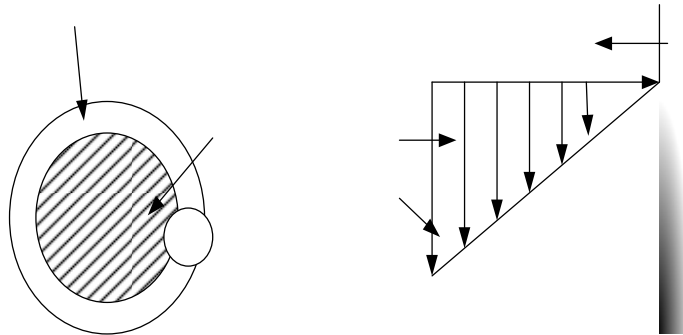
Problem 5:

Consider a lightly loaded journal bearing using oil having the constant properties $\mu=10^{-2}$ kg/s-m and $k=0.15$ W/m. K. if the journal and the bearing are each mentioned at a temperature of 400C, what is the maximum temperature in the oil when the journal is rotating at 10m/s?

Known: Oil properties, journal and bearing temperature, and journal speed for lightly loaded journal bearing.

Find: Maximum oil temperature.

Schematic:



Assumptions: (1) steady-state conditions, (2) Incompressible fluid with constant properties, (3) Clearances is much less than journal radius and flow is Couette.

Analysis: The temperature distribution corresponds to the result obtained in the text example on Couette flow.

$$T(y) = T_o + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right]$$

Bearing

The position of maximum temperature is obtained from

$$\frac{dT}{dy} = 0 = \frac{\mu}{2k} U^2 \left[\frac{1}{L} - \left(\frac{2y}{L^2} \right) \right]$$

Oil
 $\mu=10^{-2}$ kg/s.m

$$y=L/2.$$

Or,

The temperature is a maximum at this point since $d^2T/dy^2 < 0$. hence

$$T_{m,ax} = T(l/2) = T_o + \frac{\mu}{2k} U^2 \left[\frac{1}{2} - \frac{1^2}{4} \right] = T_o + \frac{\mu}{8k} U^2$$

$$T_{max} = 40^\circ \text{C} + \frac{10^{-2} \text{kg/s.m}(10\text{m/s})^2}{8 \times 0.15\text{W/m.K}}$$

$$T_{max} = 40.83^\circ \text{C}$$

Comments: Note that T_{max} increases with increasing μ and U , decreases with increasing k , and is independent of L .

Problem 6:

Consider two large (infinite) parallel plates, 5mm apart. One plate is stationary, while the other plate is moving at a speed of 200m/s. both plates are maintained at 27°C. Consider two cases, one for which the plates are separated by water and the other for which the plates are separated by air.

For each of the two fluids, which is the force per unit surface area required to maintain the above condition? What is the corresponding requirement?

What is the viscous dissipation associated with each of the two fluids?

What is the maximum temperature in each of the two fluids?

Known: conditions associated with the Couette flow of air or water.

Find: (a) Force and power requirements per unit surface area, (2) viscous dissipation,(3) maximum fluid temperature.

Schematic:



Assumptions: (1) Fully developed Couette flow, (2) Incompressible fluid with constant properties.

Properties: Air (300K); $\mu=184.6 \cdot 10^{-7} \text{ N}\cdot\text{s}/\text{m}^2$, $k=26.3 \cdot 10^{-3} \text{ W}/\text{m}\cdot\text{K}$; water (300K): $\mu=855 \cdot 10^{-6} \text{ N}\cdot\text{s}/\text{m}^2$, $k=0.613 \text{ W}/\text{m}\cdot\text{K}$

Analysis: (a) the force per unit area is associated with the shear stress. Hence, with the linear velocity profile for Couette flow $\tau = \mu (du/dy) = \mu (U/L)$.

$$\text{Air : } \tau_{\text{air}} = 184.6 \times 10^{-7} \text{ N.s/m}^2 \times \frac{200 \text{ m/s}}{0.005 \text{ m}} = 0.738 \text{ N/m}^2$$

$$\text{Water : } \tau_{\text{water}} = 855 \times 10^{-6} \text{ N.s/m}^2 \times \frac{200 \text{ m/s}}{0.005 \text{ m}} = 34.2 \text{ N/m}^2$$

With the required power given by $P/A = \tau U$

$$\text{Air : } (P/A)_{\text{air}} = (0.738 \text{ N/m}^2) \times 200 \text{ m/s} = 147.6 \text{ W/m}^2$$

$$\text{Water : } (P/A)_{\text{water}} = (34.2 \text{ N/m}^2) \times 200 \text{ m/s} = 6840 \text{ W/m}^2$$

(b) The viscous dissipation is $\mu\phi = \mu(du/dy)^2 = \mu(U/L)^2$. hence

$$\text{Air : } (\mu\mu\phi)_{\text{air}} = 184.6 \times 10^{-7} \text{ N.s/m}^2 \times \left[\frac{200 \text{ m/s}}{0.005 \text{ m}} \right]^2 = 2.95 \times 10^4 \text{ W/m}^3$$

$$\text{Water : } (\mu\mu\phi)_{\text{water}} = 855 \times 10^{-6} \text{ N.s/m}^2 \times \left[\frac{200 \text{ m/s}}{0.005 \text{ m}} \right]^2 = 1.37 \times 10^6 \text{ W/m}^3$$

The location of the maximum temperature corresponds to $y_{\text{max}} = L/2$. Hence $T_{\text{max}} = T_0 + \mu U^2 / 8k$ and

$$\text{Air : } (T_{\text{max}})_{\text{air}} = 27^\circ\text{C} + \frac{184.6 \times 10^{-7} \text{ N.s/m}^2 \times (200 \text{ m/s})^2}{8 \times 0.0263 \text{ W/m.K}} = 30.5^\circ\text{C}$$

$$\text{Water : } (T_{\text{max}})_{\text{water}} = 27^\circ\text{C} + \frac{855 \times 10^{-6} \text{ N.s/m}^2 \times (200 \text{ m/s})^2}{80.613 \text{ W/m.K}} = 34.0^\circ\text{C}$$

Comments: (1) the viscous dissipation associated with the entire fluid layer, $\mu\phi(LA)$, must equal the power, P .

(2) Although $(\mu\phi)_{\text{water}} \gg (\mu\phi)_{\text{air}}$, $k_{\text{water}} \gg k_{\text{air}}$. Hence, $T_{\text{max,water}} \approx T_{\text{max,air}}$.

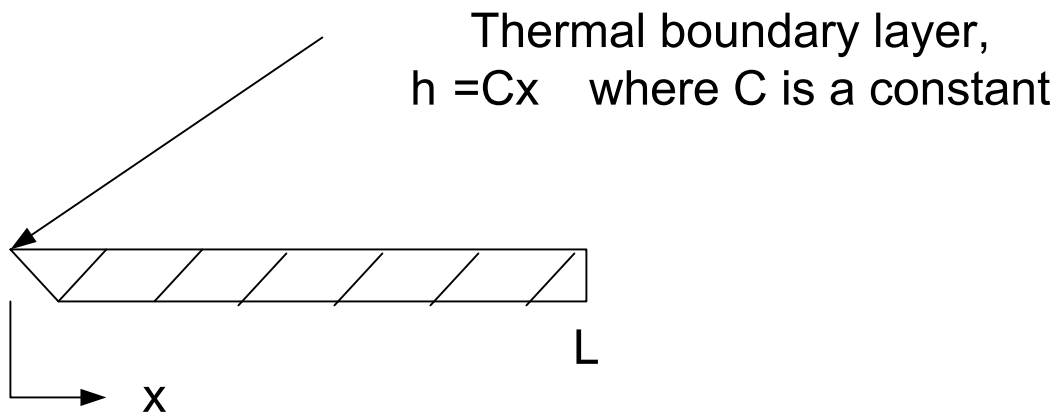
Problem 7:

A flat plate that is 0.2m by 0.2 m on a side is orientated parallel to an atmospheric air stream having a velocity of 40m/s. the air is at a temperature of $T_\infty=20^\circ\text{C}$, while the plate is maintained at $T_s=120^\circ\text{C}$. The air flows over the top and bottom surfaces of the plate, and measurement of the drag force reveals a value of 0.075N. What is the rate of heat transfer from both sides of the plate to the air?

Known: Variation of h_x with x for flow over a flat plate.

Find: Ratio of average Nusselt number for the entire plate to the local Nusselt number at $x=L$.

Schematic:



Analysis: The expressions for the local and average Nusselt number are

$$\text{Nu}_L = \frac{h_L L}{k} = \frac{(CL^{-1/2})L}{k} = \frac{CL^{1/2}}{k}$$

$$\text{Nu}_L = \frac{\bar{h}_L L}{k}$$

where

$$\bar{h}_L = \frac{1}{L} \int_0^L h_x dx = \frac{C}{L} \int_0^L x^{-1/2} dx = \frac{2C}{L} L^{1/2} = 2CL^{-1/2}$$

Hence

$$\bar{\text{Nu}}_L = \frac{2CL^{-1/2}(L)}{k} = \frac{2CL^{1/2}}{k}$$

and

$$\frac{\bar{\text{Nu}}_L}{\text{Nu}_L} = 2.$$

Comments: note the manner in which $\bar{\text{Nu}}_L$ is defined in terms of \bar{h}_L . Also note that

$$\bar{\text{Nu}}_L \neq \frac{1}{L} \int_0^L \text{Nu}_x dx$$

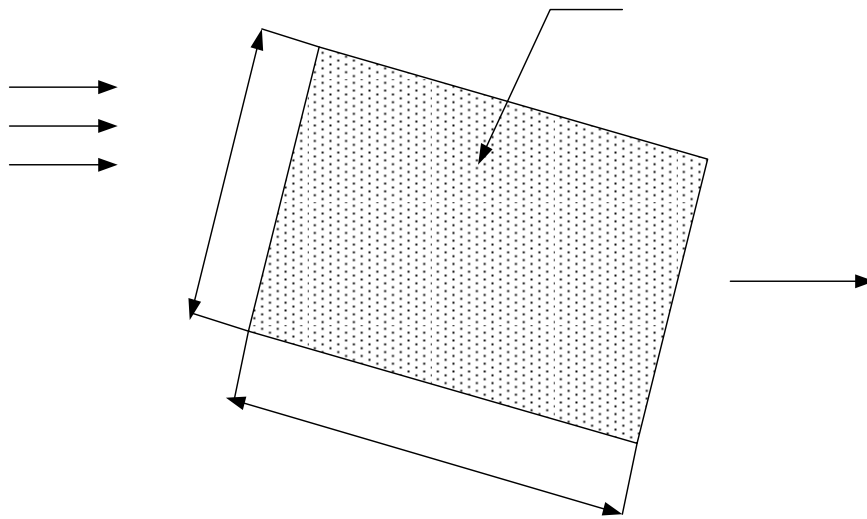
Problem 8:

For flow over a flat plate of length L , the local heat transfer coefficient h_x is known to vary as $x^{-1/2}$, where x is the distance from the leading edge of the plate. What is the ratio of the average Nusslet number for the entire plate to the local Nusslet number at $x=L$ (Nu_L)?

Known: Drag force and air flow conditions associated with a flat plate.

Find: Rate of heat transfer from the plate.

Schematic:



Assumptions: (1) Chilton-Colburn analogy is applicable.

Properties: Air(70°C,1atm): $\rho=1.018\text{kg/m}^3$, $c_p=1009\text{J/kg.K}$, $pr=0.70$, $\nu=20.22*10^{-6}\text{m}^2/\text{s}$.

Analysis: the rate of heat transfer from the plate is

$$u_\infty=40\text{m/s}$$
$$T=20^\circ\text{C}$$

$$q = 2\bar{h}(L)^2(T_s - T_\infty)$$

Where \bar{h} may be obtained from the Chilton-Colburn analogy,

$$\frac{\bar{j}_H}{2} = \frac{\bar{C}_f}{2} = \bar{S} t \text{Pr}^{2/3} = \frac{\bar{h}}{\rho u_\infty c_p} \text{Pr}^{2/3}$$

$$\frac{\bar{C}_f}{2} = \frac{1}{2} \frac{\tau_s}{\rho u_\infty^2 / 2} = \frac{1}{2} \frac{(0.075 \text{N/2})(0.2 \text{m})^2}{1.018 \text{kg/m}^3 (40 \text{m/s})^2 / 2} = 5.76 \times 10^{-4}$$

hence,

$$\bar{h} = \frac{\bar{C}_f}{2} \rho u_\infty c_p \text{Pr}^{-2/3}$$

$$\bar{h} = 5.76 \times 10^{-4} (1.018 \text{kg/m}^3) 40 \text{m/s} (1009 \text{J/kg.K}) (0.70)^{-2/3}$$

$$\bar{h} = 30 \text{W/m}^2 \cdot \text{K}$$

The heat rate is

$$q = 2(30 \text{W/m}^2 \cdot \text{K})(0.3 \text{m})^2 (120 - 20)^\circ \text{C}$$

$$q = 240 \text{W}$$

Comments: Although the flow is laminar over the entire surface ($\text{Re}_L = u_\infty L / \nu = 40 \text{m/s} \times 0.2 \text{m} / 20.22 \times 10^{-6} \text{m}^2/\text{s} = 4.0 \times 10^5$), the pressure gradient is zero and the Chilton-Colburn analogy is applicable to average, as well as local, surface conditions. Note that the only contribution to the drag force is made by the surface shear stress.

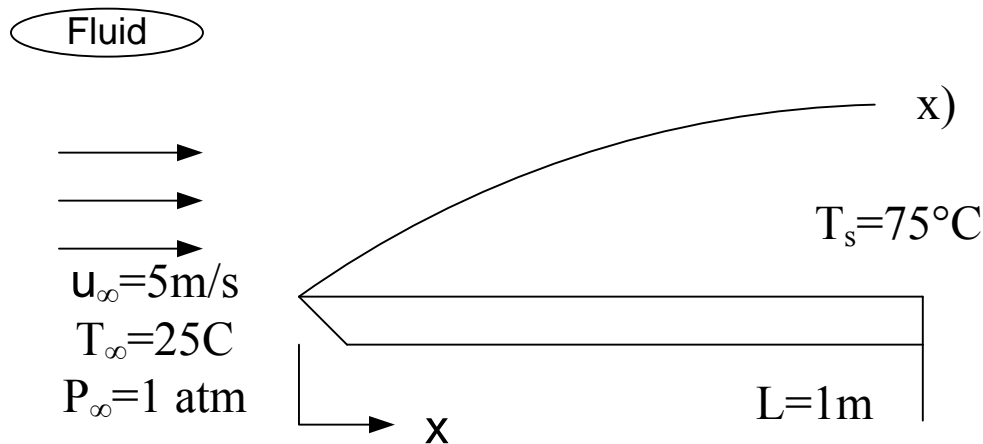
Problem 9:

Consider atmospheric air at 25°C in parallel flow at 5m/s over both surfaces of 1-m-long flat plate maintained at 75°C. Determine the boundary layer thickness, the surface shear stress, and the heat flux at the trailing edge. Determine the drag force on the plate and the total heat transfer from the plate, each per unit width of the plate.

Known: Temperature, pressure, and velocity of atmospheric air in parallel flow over a Plate of prescribed length and temperature.

Find: (a) Boundary layer thickness, surface shear stress and heat flux at trailing edges, (b) drag force and total heat transfer flux per unit width of plate.

Schematic:



Assumptions: (1) Critical Reynolds number is 5×10^5 , (2) flow over top and bottom surfaces

Properties: ($T_f=323\text{ K}$, 1 atm) Air: $\rho=1.085\text{ kg/m}^3$, $\nu=18.2 \times 10^{-6}\text{ m}^2/\text{s}$, $k=0.028\text{ W/m.K}$, $pr=0.707$

Analysis: (a) calculate the Reynolds number to know the nature of flow

$$\text{Re}_L = \frac{u_\infty L}{\nu} = \frac{5\text{ m/s} \times 1\text{ m}}{18.2 \times 10^{-6}\text{ m}^2/\text{s}} = 2.75 \times 10^5$$

Hence the flow is laminar, and at $x=L$

$$\delta = 5L\text{Re}_L^{-1/2} = 5 \times 1\text{m}/(2.75 \times 10^5)^{1/2} = 9.5\text{mm}$$

$$\tau_{s,L} = (\rho\mu^2/2)0.664\text{Re}_L^{-1/2} = \frac{1.085 \text{ kg}}{2 \text{ m}^3} (5\text{m/s})^2 0.664/(2.75 \times 10^5)^{1/2}$$

$$\tau_{s,L} = 0.0172\text{kg/m.s}^2 = 0.0172\text{N/m}^2$$

Using the correct correlation,

$$\text{Nu}_L = \frac{hL_L}{k} = 0.332\text{Re}_L^{1/2} \text{Pr}^{1/3} = 0.332(2.75105)^{1/2} (0.707)^{1/3} = 155.1$$

hence,

$$h_L = 155.1(0.028\text{W/m.K})/1\text{m} = 4.34\text{W/m}^2 \cdot \text{K}$$

$$q''_s(L) = hL(T_s - T_\infty) = 4.34\text{W/m}^2 \cdot \text{K}(75^\circ\text{C} - 25^\circ\text{C}) = 217\text{W/m}^2$$

(b) The drag force per unit area plate width is $D' = 2L\bar{\tau}_{s,L}$ where the factor of two is included to account for both sides of the plate. Hence with

$$\bar{\tau}_{s,L} = (\rho\mu^2_\infty/2)1.328\text{Re}_L^{-1/2} = (1.085\text{kg/m}^3/2)(5\text{m/s})^2 1.328(2.75 \times 10^5)^{-1/2}$$

The drag is

$$\bar{\tau}_{s,L} = 0.0343\text{N/m}^2$$

$$D' = 2(1\text{m})0.0343\text{N/m}^2 = 0.0686\text{N/m}$$

$$\text{also with } \bar{h}_L = 2h_L = 8.68\text{W/m}^2 \cdot \text{K}$$

$$q'' = 2L\bar{h}_L(T_s - T_\infty) = 2(1\text{m})8.68\text{W/m}^2 \cdot \text{K}(75 - 25)^\circ\text{C} = 868\text{W/m}^2$$

Problem 10:

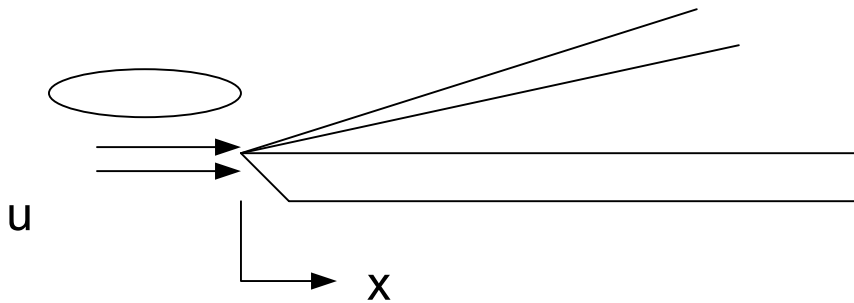
Engine oil at 100°C and a velocity of 0.1m/s flows over both surfaces of a 1-m-long flat plate maintained at 20°C. Determine

- The velocity and thermal boundary thickness at the trailing edge.
- The local heat flux and surface shear stress at the trailing edge.
- The total drag force and heat transfer per unit area width of the plate.

Known: Temperature and velocity of engine oil Temperature and length of flat plate.

Find: (a) velocity and thermal boundary thickness at the trailing edge, (b) Heat flux and surface shear stress at the trailing edge, (c) total drag force and heat transfer per unit plate width.

Schematic:



Assumptions: engine oil ($T_f=33\text{K}$): $\rho=864\text{kg/m}^3$, $\nu=86.1 \times 10^{-6}\text{m}^2/\text{s}$, $k=0.140\text{W/m /K}$, $\text{Pr}=1081$.

Analysis: (a) calculate the Reynolds number to know the nature of flow

$$\text{Re}L = \frac{u_{\infty} L}{\nu} = \frac{0.1\text{m/s} \times 1\text{m}}{86.1 \times 10^{-6}\text{m}^2/\text{s}} = 1161$$

Hence the flow is laminar at $x=L$, and

$$\delta = 5L\text{Re}_L^{-1/2} = 5(1\text{m})(1161)^{-1/2} = 0.147\text{m}$$

$$\delta_t = \delta\text{Pr}^{-1/3} = 0.147(1081)^{-1/3} = 0.0143\text{m}$$

(b) The local convection coefficient and local flux at $x=L$ are

Engine oil

$= 0.1\text{m/s}$

$$h_L = \frac{k}{L} 0.3325 L \text{Re}_L^{-1/2} \text{Pr}^{1/3} = \frac{0.140 \text{W/m.K}}{1 \text{m}} 0.332(1161)(1081)^{1/3} = 16.25 \text{W/m}^2 \cdot \text{K}$$

$$q'' = h_L(T_s - T_\infty) = 16.25 \text{W/m}^2 \cdot \text{K}(20 - 100)^\circ\text{C} = -1300 \text{W/m}^2$$

Also the local shear stress is

$$\bar{\tau}_{sL} = (\rho \mu^2 / 2) 0.664 \text{Re}_L^{-1/2} = \frac{864 \text{kg/m}^3}{2} (0.1 \text{m/s})^2 0.664 (1161)^{-1/2}$$

$$\bar{\tau}_{sL} = 0.0842 \text{kg/m.s}^2 = 0.0842 \text{N/m}^2$$

(c) With the drag force per unit width given by $D' = 2L\bar{\tau}_{sL}$ where the factor of 2 is included to account for both sides of the plate, it follows that

$$\bar{\tau}_{sL} = (\rho \mu^2 / 2) 1.328 \text{Re}_L^{-1/2} = 2(1 \text{m}) 864 \text{kg/m}^3 (0.1 \text{m/s})^2 1.328 (1161)^{-1/2} = 0.673 \text{N/m}$$

with $\bar{h}_L = 2h_L = 32.5 \text{W/m}^2 \cdot \text{K}$, it also follows that

$$q = 2L\bar{h}_L(T_s - T_\infty) = 2(1 \text{m}) 32.5 \text{W/m}^2 \cdot \text{K}(20 - 100)^\circ\text{C} = -5200 \text{W/m}$$

Comments: Note effect of Pr on (δ/δ_t) .

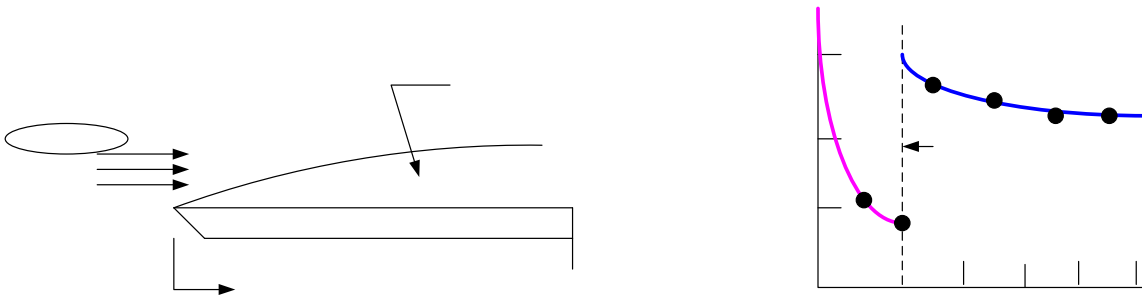
Problem 11:

Consider water at 27°C in parallel flow over an isothermal, 1-m-long, flat plate with a velocity of 2m/s. Plot the variation of the local heat transfer coefficient with distance along the plate. What is the value of the average coefficient?

Known: velocity and temperature of air in parallel flow over a flat plate of prescribed length.

Find: (a) variation of local convection coefficient with distance along the plate, (b) Average convection coefficient.

Schematic:



Assumptions: (1) Critical Reynolds number is 5×10^5 .

Properties: Water (300K): $\rho = 997 \text{ kg/m}^3$, $\mu = 855 \times 10^{-6} \text{ N}\cdot\text{s/m}^2$, $\nu = \mu/\rho = 0.858 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.613 \text{ W/m}\cdot\text{K}$, $\text{Pr} = 5.83$

Analysis: (a) With

$$\text{Re}_L = \frac{u_\infty L}{\nu} = \frac{2 \text{ m/s} \times 1 \text{ m}}{0.858 \times 10^{-6} \text{ m}^2/\text{s}} = 2.33 \times 10^5$$

Boundary layer conditions are mixed and

$$T_f = 300 \text{ K}$$

Water

$$x_c = L(\text{Re}_{x,c}/\text{Re}_L) = 1\text{m}(5 \times 10^5 / 2.33 \times 10^6) = 0.215\text{m}$$

$$\text{for } x \leq 0.215\text{m}, h_x = \frac{k}{x} 0.332 \text{Re}_x^{1/2} \text{Pr}^{1/3} = 0.332k \left(\frac{u_\infty}{\nu} \right)^{1/2} \text{Pr}^{1/3} x^{-1/2}$$

x(m)	0.1	0.215
hx(W/m².K)	1768	1206

$$\text{for } x > 0.215\text{m}, h_x = \frac{k}{x} 0.0296 \text{Re}_x^{4/5} \text{Pr}^{1/3} = 0.0296k \left(\frac{u_\infty}{\nu} \right)^{4/5} \text{Pr}^{1/3} x^{-0.2}$$

x(m)	0.0215	0.4	0.6	0.8	1.0
h_x (W/m².K)	5514	4871	4491	4240	4055

The Spatial variation of the local convection coefficient is shown above

(b) The average coefficient is

$$\bar{h}_L = \frac{k}{L} (0.037 \text{Re}_L^{4/5} - 871) \text{Pr}^{1/3} = \frac{0.613 \text{W/m.K}}{1\text{m}} [0.0379(2.33 \times 10^6)^{4/5} - 871(5.83)^{1/3}]$$

$$\bar{h}_L = 4106 \text{W/m}^2 \cdot \text{K}$$

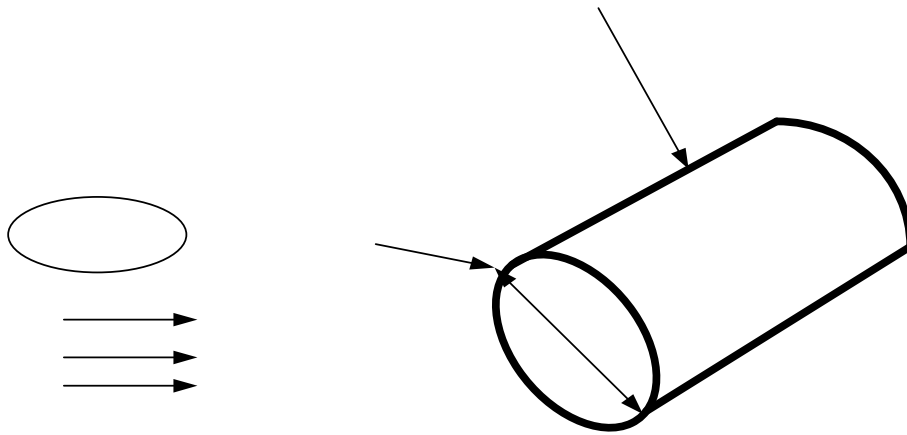
Problem 12:

A circular cylinder of 25-mm diameter is initially at 150C and is quenched by immersion in a 80C oil bath, which moves at a velocity of 2m.s in cross flow over the cylinder. What is the initial rate of heat loss unit length of the cylinder?

Known: Diameter and initial temperature of a circular cylinder submerged in an oil bath of prescribed temperature and velocity.

Find: initial rate of heat loss unit per length.

Schematic:



Assumptions: (1) Steady-state conditions, (2) uniform surface temperature.

Properties: Engine oil ($T_\infty=353\text{K}$): $\nu=38.1 \cdot 10^{-6} \text{m}^2/\text{s}$, $k=0.138 \text{W/m} \cdot \text{K}$, $\text{Pr}_\infty=501$;
($T_s=423\text{K}$): $\text{Pr}_s=98$.

Analysis: The initial heat loss per unit length is

$$D=25\text{mm}$$

$$q = \bar{h} \pi D (T_s - T_\infty)$$

where \bar{h} may be computed from the Zhukauskas relation. when

$$Re_D = \frac{VD}{\nu} = \frac{2\text{m/s}(0.025\text{m})}{38.1 \times 10^{-6} \text{m}^2/\text{s}} = 1312$$

Find $C = 0.26$ and $m = 0.6$ from table 7.4 . hence

$$\bar{h} = \frac{k}{D} C Re_D^m Pr^n (Pr_\infty/Pr_s)^{1/4} = \frac{0.138\text{W/m.K}}{0.025\text{m}} 0.26(1312)^{0.6} (501)^{0.37} \left(\frac{501}{98}\right)^{1/4}$$

$$\bar{h} = 1600\text{W.m}^2 .\text{K}$$

$$q' = 1600\text{W/m}^2 .\text{K}(\pi)0.025\text{m}(150 - 80)^\circ\text{C} = 8.8\text{W/m}$$

Comments: Evaluating properties at the film temperature, $T_f=388\text{K}$ ($\nu=14.0 \times 10^{-6}\text{m}^2/\text{s}$, $k=0.135\text{W/m. K}$, $Pr=196$), find $Re_D=3517$.

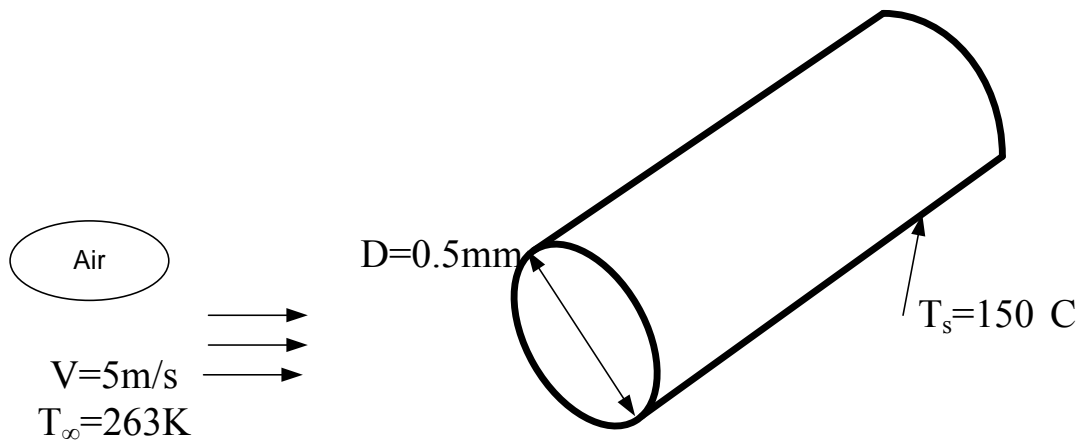
Problem 13:

An uninsulated steam pipe is used to transport high-temperature steam from one building to one another. The pipe is 0.5-m diameter, has a surface temperature of 150C, and is exposed to ambient air at -10C. the air moves in cross flow over the pipe with a velocity of 5m.s. What is the heat loss per unit length of pipe?

Known: Diameter and surface temperature of uninsulated steam pipe. Velocity and temperature of air in cross flow.

Find: Heat loss per unit length.

Schematic:



Assumptions: (1) steady-state conditions, (2) uniform surface temperature

Properties: Air (T=263K, 1atm): $\nu=12.6 \times 10^{-6} \text{ m}^2/\text{s}$, $k=0.0233 \text{ W/m} \cdot \text{K}$, $\text{Pr}=0.72$; (T_s=423K, 1atm); $\text{Pr}_s=0.649$.

Analysis: the heat loss per unit length is

$$q = \bar{h} \pi D (T_s - T_\infty)$$

where \bar{h} may be computed from the Zhukauskas relation. when

$$\text{Re}_D = \frac{VD}{\nu} = \frac{2\text{m/s}(0.025\text{m})}{38.1 \times 10^{-6} \text{m}^2/\text{s}} = 1312$$

Find $C = 0.26$ and $m = 0.6$ from table 7.4 . hence

$$\bar{h} = \frac{k}{D} C \text{Re}_D^m \text{Pr}^n (\text{Pr}_\infty / \text{Pr}_s)^{1/4} = \frac{0.138\text{W/m.K}}{0.025\text{m}} 0.26(1312)^{0.6} (501)^{0.37} \left(\frac{501}{98}\right)^{1/4}$$

$$\bar{h} = 1600\text{W.m}^2 .\text{K}$$

Hence the heat rate is

$$q' = 16.3\text{W/m}^2 .\text{K}(\pi)0.5\text{m}(150 - (-10))^\circ\text{C} = 4100\text{W/m}$$

Comments: Note that $q' \propto D^m$, in which case the heat loss increases significantly with increasing D.

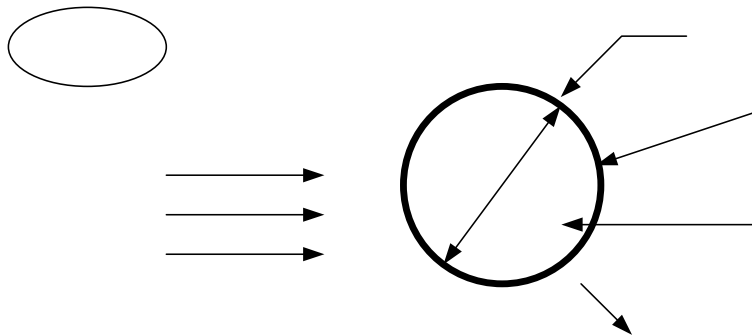
Problem 14:

Atmospheric air at 25°C and velocity of 0.5m/s flows over a 50-W incandescent bulb whose surface temperature is at 140°C. The bulb may be approximated as a sphere of 50-mm diameter. What is the rate of heat loss by convection to the air?

Known: Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

Find: Heat loss by convection.

Schematic:



Assumptions: (1) steady-state conditions, (2) uniform surface temperature.

Properties: Air ($T_f=25^\circ\text{C}$, 1atm): $\nu=15.71 \times 10^{-6} \text{m}^2/\text{s}$, $k=0.0261 \text{W/m} \cdot \text{K}$, $\text{Pr}=0.71$, $\mu=183.6 \times 10^{-7} \text{N}\cdot\text{s}/\text{m}^2$; Air ($T_s=140^\circ\text{C}$, 1atm): $\mu=235.5 \times 10^{-7} \text{N}\cdot\text{s}/\text{m}^2$

Analysis:

$$q = \bar{h} \pi D (T_s - T_\infty)$$

Air

where \bar{h} may be computed from the Whitaker relation. when

$$\bar{h} = \frac{k}{D} [2 + (0.4\text{Re}_D^{1/2} + 0.06\text{Re}_D^{2/3})\text{Pr}^{0.4} (\mu/\mu_s)^{1/4}]$$

Where

$$\text{Re}_D = \frac{VD}{\nu} = \frac{0.5\text{m/s} \times 0.05\text{m}}{15.71 \times 10^{-6} \text{m}^2/\text{s}} = 1591$$

hence

$$\bar{h} = \frac{0.0261\text{W/m.K}}{0.05\text{m}} \left\{ 2 + [0.4(1591)^{1/2} + 0.06(1591)^{2/3}] (0.71)^{0.4} \left(\frac{183.6}{235.5} \right)^{1/4} \right\}$$

$$\bar{h} = 11.4\text{W/m}^2 \cdot \text{K}$$

and the heat rate is

$$q = 11.4 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \pi (0.05\text{m})^2 (140 - 25)^\circ\text{C} = 10.3\text{W}$$

Comments: (1) The low value of \bar{h} suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3W

(2) The surface of the bulb also dissipates heat to the surroundings by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

(3) The Correlation has been used its range of application $(\mu/\mu_s) < 1$.

Problem 15:

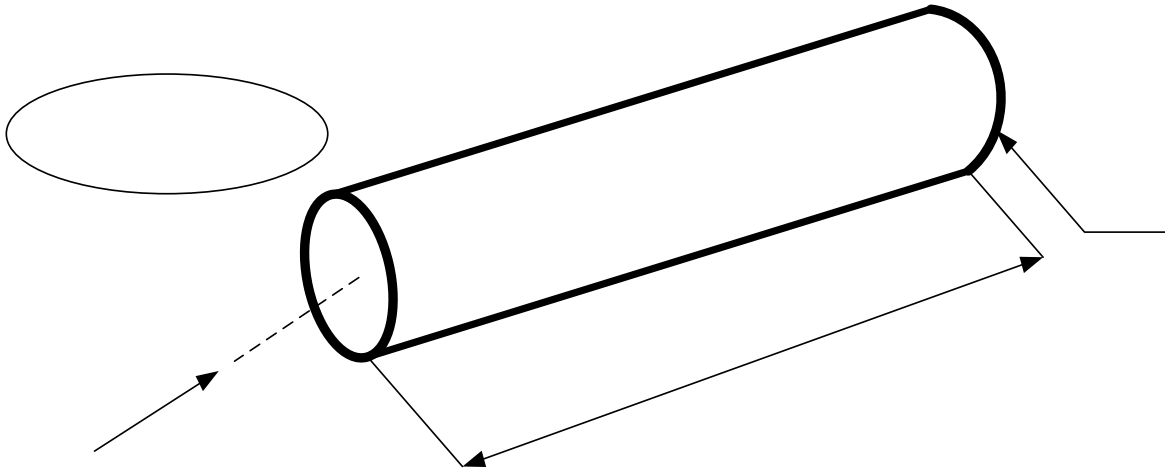
Water at 27° C flows with a mean velocity of 1m/s through a 1-km-long cast iron pipe at 0.25 m inside diameter.

- (a) Determine the pressure drop over the pipe length and the corresponding pump power requirement, if the pipe surface is clean.
- (b) If the pipe surface roughness is increased by 25% because of contamination, what is the new pressure drop and pump power requirement.

Known: Temperature and velocity of water in a cast iron pipe of prescribed dimensions.

Find: pressure drops and power requirement for (a) a clean surface and (b) a surface with a 25% larger roughness.

Schematic:



Assumptions: (1) Steady, fully developed flow.

Properties: Water (300K): $\rho=1000 \text{ kg/m}^3$, $\mu=855 \cdot 10^{-6} \text{ N.s/m}^2$.

Analysis: (a) from eq.8.22, the pressure drop is

$$\Delta p = f \frac{\rho u_m^2}{2D} L$$

$e = 2.6 \times 10^{-4}$ m for clean cast iron; hence $e/D = 1.04 \times 10^{-3}$. With

$$Re_D = \frac{u_m D}{\nu} = \frac{1 \text{ m/s} \times 0.25 \text{ m}}{855 \times 10^{-6} \text{ N.s/m}^2 / 1000 \text{ kg/m}^3} = 2.92 \times 10^5$$

find from fig 8.3 that $f \approx 0.021$. hence ,

$$\Delta P = 0.021 \frac{(1000 \text{ kg/m}^3)(1 \text{ m/s})^2}{2(0.25 \text{ m})} 1000 \text{ m} = 4.2 \times 10^4 \text{ kg/s}^2 \cdot \text{m}$$

$$\Delta P = 4.2 \times 10^4 \text{ N/m}^2 = 0.42 \text{ bar}$$

The pump power requirement is

$$P = \Delta p \cdot \dot{V} = \Delta p (\pi D^2 / 4) u_m$$

$$P = 4.2 \times 10^4 \text{ N/m}^2 (\pi \times 0.25^2 / 4) \text{ m}^2 \times 1 \text{ m/s} = 2.06 \text{ kW}$$

(b) Increasing ε by 25% it follows that $\varepsilon = 3.25 \times 10^{-4}$ m and $e/D = 0.0013$. With Re_D unchanged, from fig 8.3, it follows that $f \approx 0.0225$. Hence

$$\Delta P_2 = (f_2/f_1) \Delta P_1 = 0.45 \text{ bar}$$

$$P_2 = (f_2/f_1) P_1 = 2.21 \text{ kW}$$

Comments: (1) Note that $L/D = 4000 \gg (x_{fd,h}/D) \approx 10$ for turbulent flow and the assumption of fully developed conditions is justified.

(2) Surface fouling results in increased surface and increases operating costs through increasing pump power requirements.

Problem 16:

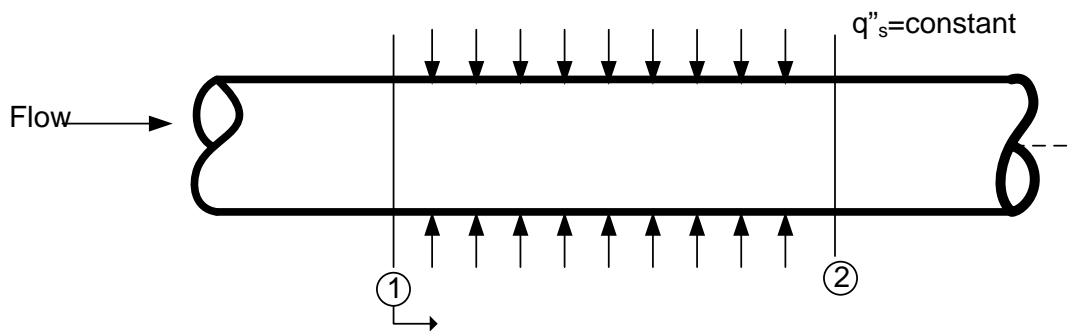
Consider flow in a circular tube. Within the test section length (between 1 and 2) a constant heat flux q''_s is maintained.

- (a) For the two cases identified, sketch, qualitatively, the surface temperature $T_s(x)$ and the fluid mean temperature $T_m(x)$ as a function of distance along the test section x . In case A flow is hydrodynamically and thermally fully developed. In case B flow is not developed.
- (b) Assuming that the surface flux q''_s and the inlet mean temperature $T_{m,1}$ are identical for both cases, will the exit mean temperature $T_{m,2}$ for case A be greater than, equal to, or less than $T_{m,2}$ for case B? Briefly explain why?

Known: internal flow with constant surface heat flux, q''_s .

Find: (a) Qualitative temperature distributions (x), under developing and fully developed flow, (b) exit mean temperature for both situations.

Schematic:



Assumptions: (a) Steady-state conditions, (b) constant properties, (c) incompressible flow.

Analysis: Based upon the analysis, the constant surface heat flux conditions,

$$\frac{dT_m}{dx} = \text{constant}$$

Hence, regardless of whether the hydrodynamic or thermal boundary layer is fully developed, it follows that

$T_m(x)$ is linear

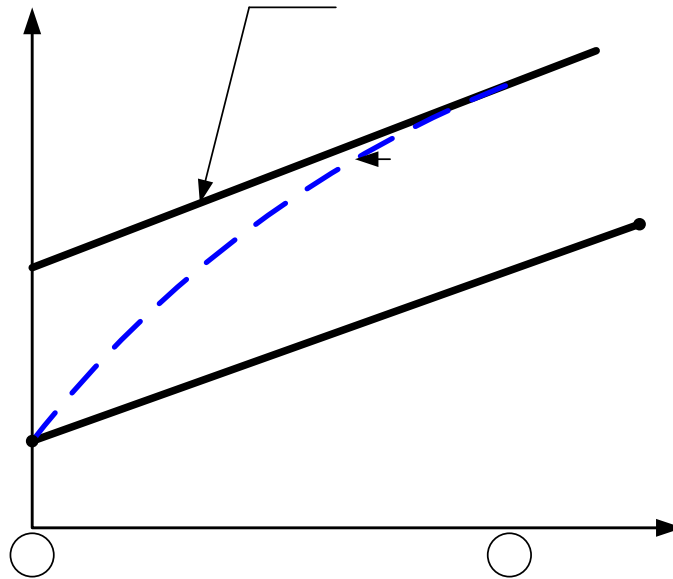
$T_{m,2}$ will be the same for all flow conditions.

The surface heat flux can be written as

$$q_s'' = h[T_s - T_m(x)]$$

Under fully developed flow and thermal conditions, $h=h_{fd}$ is a constant.

When flow is developing $h > h_{fd}$. Hence, the temperature distributions appear as below.



Fully

Problem 17:

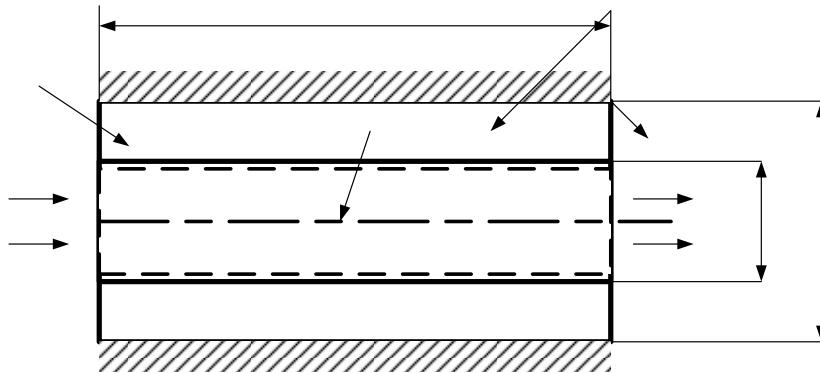
A thick-walled, stainless steel (AISI 316) pipe of inside and outside diameter $D_i=20\text{mm}$ and $D_o=40\text{mm}$ is heated electrically to provide a uniform heat generation rate of $\dot{q} = 10^6 \text{ W / m}^3$. The outer surface of the pipe is insulated while water flows through the pipe at a rate of $\dot{m} = 0.1 \text{ kg / s}$

- (a) If the water inlet temperature is $T_{m,i}=20^\circ\text{C}$ and the desired outlet temperature is $T_{m,o}=40^\circ\text{C}$, what is the required pipe length
- (b) What are the location and value of the maximum pipe temperature?

Known: Inner and outer diameter of a steel pipe insulated on the outside and experiencing uniform heat generation. Flow rate and inlet temperature of water flowing through the pipe.

Find: (a) pipe length required to achieve desired outlet temperature, (b) location and value of maximum pipe temperature.

Schematic:



Assumptions: (1) steady-state conditions, (2) constant properties, (3) negligible kinetic energy, potential energy and flow work changes, (4) one-dimensional radial conduction in pipe wall, (5) outer surface is adiabatic.

Properties: Stainless steel 316 ($T \approx 400\text{K}$): $k=15\text{W/m.k}$; water ($T_m = 303\text{K}$); $c_p=4178\text{J/kg.K}$, $k=0.617\text{W/m.K}$, $\rho=803 \cdot 10^{-6}\text{N.s/m}^2$, $Pr=5.45$

Analysis: (a) performing an energy balance for a control volume about the inner tube, it follows that

$$\dot{m} c_p (T_{m,o} - T_{m,i}) = \dot{q} = \dot{q}(\pi/4)(D_o^2 - D_i^2)L$$

$$L = \frac{\dot{m} c_p (T_{m,o} - T_{m,i})}{\dot{q}(\pi/4)(D_o^2 - D_i^2)L} = \frac{(0.1\text{kg/s})4178(\text{J/kg.K})20^\circ\text{C}}{10^6 \text{ W/m}^3 (\pi/4)[(0.4\text{m}^2 - (0.02)^2]}$$

$$L = 8.87\text{m}$$

(b) The maximum wall temperature exists at the pipe exit ($x=L$) and the insulated surface ($r=r_o$). The radial temperature distribution in the wall is of the form

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \lambda n r + C_2$$

considering the boundary conditions ;

$$r = r_o; \quad \left. \frac{dT}{dr} \right)_{r=r_o} = 0 = -\frac{\dot{q}}{2k}r_o + \frac{C_1}{r_o} \quad C_1 = \frac{\dot{q}r_o^2}{2k}$$

$$r = r_i : T(r_i) = T_s = \frac{-\dot{q}}{4k}r_i^2 + \frac{\dot{q}r_o^2}{2k} \lambda n r_i + C_2 \quad C_2 = \frac{\dot{q}}{4k}r_i^2 - \frac{\dot{q}r_o^2}{2k} \lambda n r_i + T_s$$

The temperature distribution and the maximum wall temperature ($r=r_o$) are

$$T(r) = \frac{\dot{q}}{4k}(r^2 - r_i^2) + \frac{\dot{q}r_o^2}{2k}\lambda n \frac{r}{r_i} + T_s$$

$$T_{w,max} = T(r_o) = -\frac{\dot{q}}{4k}(r_o^2 - r_i^2) + \frac{\dot{q}r_o^2}{2k}\lambda n \frac{r_o}{r_i} + T_s$$

where T_s , the inner surface temperature of the wall at exit, it follows from

$$q_s'' = \frac{\dot{q}(\pi/4)(D_o^2 - D_i^2)L}{\pi D_i L} = \frac{\dot{q}(D_o^2 - D_i^2)}{4D_i} = h(T_s - T_{m,o})$$

Where h is the local convection coefficient at the exit. With

$$Re_D = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4 \times 0.1 \text{ kg/s}}{\pi (0.02 \text{ m}) 803 \times 10^{-6} \text{ N.s/m}^2} = 7928$$

The flow is turbulent and, with $(L/D_i) = (8.87 \text{ m}/0.02 \text{ m}) = 444 \gg (x_{fd}/D) \approx 10$, it is also fully developed. Hence, from the Dittus-Boelter correlation,

$$h = \frac{k}{D_i} (0.023 Re_D^{4/5} Pr^{0.4}) = \frac{0.617 \text{ W/m.K}}{0.02 \text{ m}} 0.023 (7928)^{4/5} 5.45^{0.4} = 1840 \text{ W/m}^2 \cdot \text{K}$$

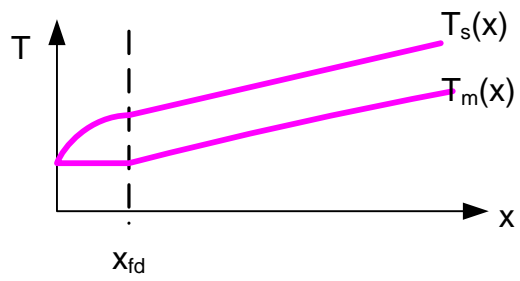
Hence the inner surface temperature of the wall at the exit is

$$T_s = \frac{\dot{q}(D_o^2 - D_i^2)}{4D_i} + T_{m,o} = \frac{10^6 \text{ W/m}^3 [(0.04 \text{ m})^2 - (0.02 \text{ m})^2]}{4 \times 180 \text{ W/m}^2 \cdot \text{K} (0.02 \text{ m})} + 40^\circ \text{C} = 48.2^\circ \text{C}$$

and

$$T_{w,max} = -\frac{10^6 \text{ W/m}^3}{415 \text{ W/m.K}} [(0.02)^2 - (0.01)^2] + \frac{10^6 \text{ W/m}^3 (0.02)^2}{215 \text{ W/m.K}} \lambda n \frac{0.02}{0.01} + 48.2^\circ \text{C} = 52.4^\circ \text{C}$$

Comments: The physical situation corresponds to a uniform surface heat flux, and T_m increases linearly with x . in the fully developed region, T_s also increases linearly with x .



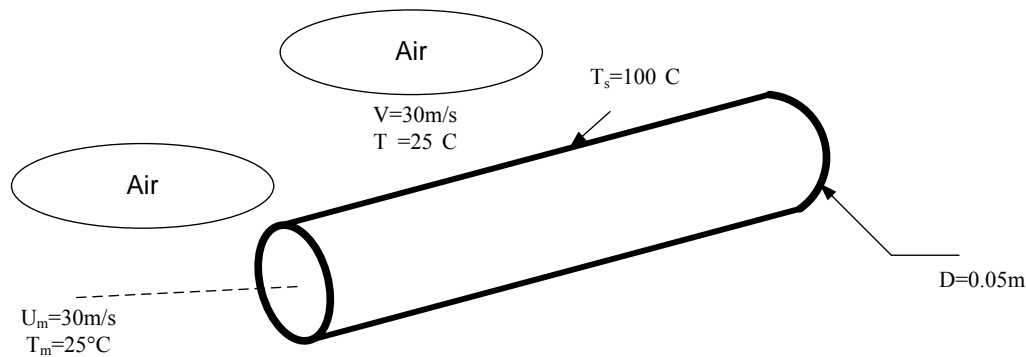
Problem 18:

The surface of a 50-mm diameter, thin walled tube is maintained thin walled tube is maintained at 100°C. In one case air is cross flow very the tube with a temperature of 25°C and a velocity of 30m/s. In another case air is in fully developed flow through the tube with a temperature of 25°C and a mean velocity of 30m/s. compare the heat flux from the tube to the air for the two cases.

Known: surface temperature and diameter of a tube. Velocity and temperature of air in cross flow. Velocity and temperature of air in fully developed internal flow.

Find: convection heat flux associated with the external and internal flows.

Schematic:



Assumptions: (1) steady-state conditions, (2) uniform cylinder surface temperature, (3) fully developed internal flow

Properties: Air (298K): $\nu = 15.71 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.0261 \text{ W/m.K}$, $\text{Pr} = 0.71$

Analysis: for the external and internal flow

$$\text{Re}_D = \frac{VD}{\nu} = \frac{u_m D}{\nu} = \frac{30\text{ m/s} \times 0.05\text{ m}}{15.71 \times 10^{-6} \text{ m}^2/\text{s}} = 9.55 \times 10^4$$

From the Zhukauskas relation for the external flow, with $C = 0.26$ and $m = 0.6$

$$\bar{N}u_D = C \text{Re}_D^m \text{Pr}(\text{Pr}/\text{Pr}_s)^{1/4} = 0.26(9.55 \times 10^4)^{0.6} (0.71)^{0.37} (1)^{1/4} = 223$$

Hence, the convection coefficient and heat rate are

$$\bar{h} = \frac{k}{D} \bar{N}u_D = \frac{0.0261 \text{ W / m.K}}{0.05 \text{ m}} \times 223 = 116.4 \text{ W / m}^2 \cdot \text{K}$$

$$q'' = h(T_s - T_\infty) = 116.4 \text{ W / m}^2 \cdot \text{K} (100 - 25)^\circ \text{C} = 8.73 \times 10^3 \text{ W / m}^2$$

Using the Dittus-Boelter correlation, for the internal flow, which is Turbulent,

$$\bar{N}u_D = 0.023 \text{Re}_D^{4/5} \text{Pr}^{0.4} = 0.023(9.55 \times 10^4)^{4/5} (0.71)^{0.4} = 193$$

$$\bar{h} = \frac{k}{D} \bar{N}u_D = \frac{0.0261 \text{ W / m.K}}{0.05} \times 193 = 101 \text{ W / m}^2 \cdot \text{K}$$

and the heat flux is

$$q'' = h(T_s - T_m) = 101 \text{ W / m}^2 \cdot \text{K} (100 - 25)^\circ \text{C} = 7.58 \times 10^3 \text{ W / m}^2$$

Comments: Convection effects associated with the two flow conditions are comparable.

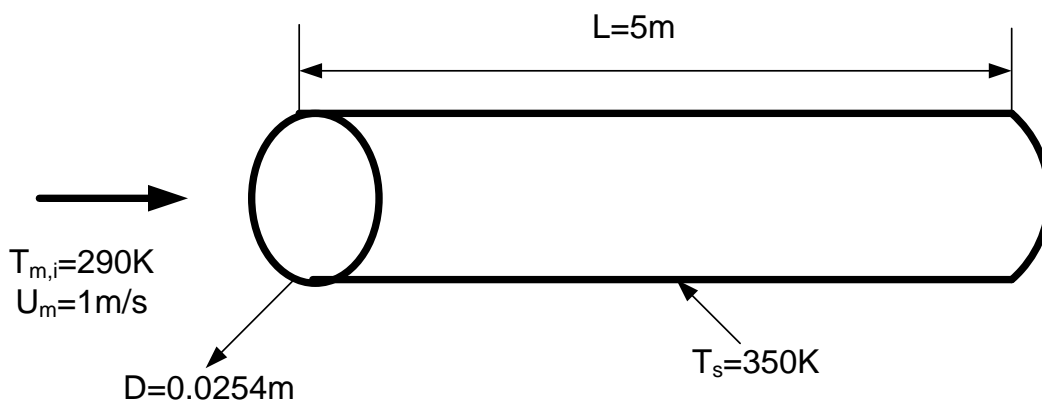
Problem 19:

Cooling water flows through the 25.4 mm diameter thin walled tubes of a stream condenser at 1 m/s, and a surface temperature of 350K is maintained by the condensing steam. If the water inlet temperature is 290 K and the tubes are 5 m long, what is the water outlet temperature? Water properties may be evaluated at an assumed average temperature of 300K

Known: Diameter, length and surface temperature of condenser tubes. Water velocity and inlet temperature.

Find: Water outlet temperature.

Schematic:



Assumptions: (1) Negligible tube wall conduction resistance, (2) Negligible kinetic energy, potential energy and flow work changes.

Properties: Water (300K): $\rho=997\text{kg/m}^3$, $c_p=4179\text{J/kg}\cdot\text{K}$, $\mu=855\cdot 10^{-6}\text{kg/s}\cdot\text{m}$, $k=0.613\text{W/m}\cdot\text{K}$, $\text{Pr}=5.83$

Analysis:

$$T_{m,o} = T_s - (T_s - T_{m,i}) \exp[-(\pi D L \dot{m} c_p) \bar{h}]$$

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{997 \text{ kg/m}^3 (1 \text{ m/s}) 0.0254 \text{ m}}{855 \times 10^{-6} \text{ kg/s.m}} = 29,618$$

The flow is turbulent. Since $L/D=197$, it is reasonable to assume fully developed flow throughout the tube. Hence

$$\bar{h} = NuD(k/D) = 176(0.613 \text{ W/m.K}/0.0254 \text{ m}) = 4248 \text{ W/m}^2 \cdot \text{K}$$

With

$$\dot{m} = \rho u_m (\pi \pi^2/4) = (\pi \pi/4) 997 \text{ k/m}^3 (1 \text{ m/s})(0.0254 \text{ m})^2 = 0.505 \text{ kg/s}$$

$$T_{m,o} = 350 \text{ K} - (60 \text{ K}) \exp \left[\frac{\pi(0.0254 \text{ m}) 5 \text{ m} (4248 \text{ W/m}^2 \cdot \text{K})}{(0.505 \text{ kg/s})(4179 \text{ J/kg.K})} \right] \approx 323 \text{ K}$$

Comments: The accuracy of the calculations may be improved slightly by reevaluating properties at $T_m = 306.5 \text{ K}$.

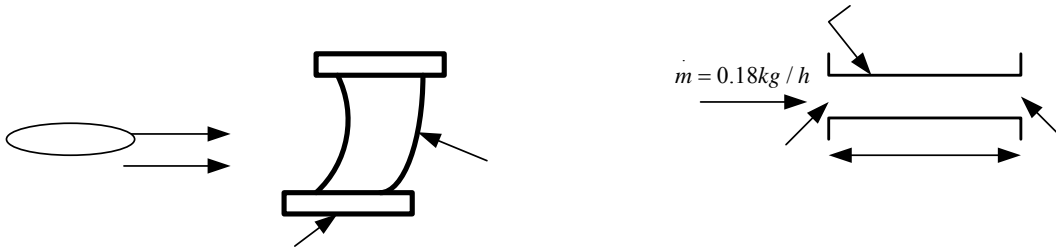
Problem 20:

The air passage for cooling a gas turbine vane can be approximated as a tube of 3-mm diameter and 75-mm length. If the operating temperature of the vane is 650°C, calculate the outlet temperature of the air if it enters the tube at 427°C and 0.18kg/h.

Known: gas turbine vane approximation as a tube of prescribed diameter and length maintained at a known surface temperature. Air inlet temperature and flow rate.

Find: outlet temperature of the air coolant.

Schematic:



Assumptions: (1) Steady-state conditions, (2) negligible Kinetic and potential energy changes.

Properties: Air (assume $\bar{T}_m = 780\text{K}, 1\text{ atm}$): $c_p = 1094\text{J/kg}\cdot\text{K}$, $k = 0.0563\text{ W/m}\cdot\text{K}$, $\mu = 363.7 \cdot 10^{-7}\text{ N}\cdot\text{s/m}^2$, $\text{Pr} = 0.706$; $(T_s = 650^\circ\text{C} = 923\text{K}, 1\text{ atm})$: $\mu = 404.2 \cdot 10^{-7}\text{ N}\cdot\text{s/m}^2$.

Analysis: For constant wall temperatures heating,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{mc_p}\right)$$

Where $P = \pi D$. for flow in circular passage,

T_s

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.18 \text{ kg/h} (1/3600 \text{ s/h})}{\pi (0.03 \text{ m}) 363.7 \times 10^{-7} \text{ N.s/m}}$$

The flow is laminar, and since $L/D=75\text{mm}/3\text{mm}=25$, the Sieder-Tate correlation including combined entry length fields.

$$\text{Nu}_D = \frac{\bar{h} D}{k} = 1.86 \left(\frac{\text{Re}_D \text{Pr}}{L/D} \right)^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14}$$

$$\bar{h} = \frac{0.0563 \text{ W/m.K}}{0.003 \text{ m}} 1.86 \left(\frac{584 \times 0.706}{25} \right)^{1/3} \left(\frac{363.7 \times 10^{-7}}{404.2 \times 10^{-7}} \right)^{0.14} = 87.5 \text{ W/m}^2 \cdot \text{K}$$

Hence, the air outlet temperature is

$$\frac{650 - T_{m,o}}{(650 - 427)^\circ\text{C}} = \exp\left(- \frac{\pi(0.003\text{m}) \times 0.075\text{m} \times 87.5\text{W/m}^2 \cdot \text{K}}{(0.18/3600)\text{kg/s} \times 1094\text{J/kg.K}} \right)$$

$$T_{m,o} = 578^\circ\text{C}$$

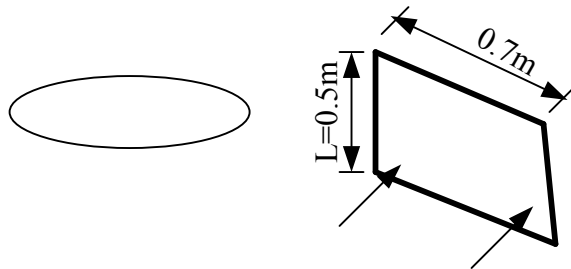
Comments: (1) based upon the calculations for $T_{m,o}=578^\circ\text{C}$, $\bar{T}_m=775\text{K}$ which is in good agreement with our assumption to evaluate the thermo physical properties.

Problem 21

A household oven door of 0.5-m height and 0.7-m width reaches an average surface temperature of 32°C during operation. Estimate the heat loss to the room with ambient air at 22°C. If the door has an emissivity of 1.0 and the surroundings are also at 22°C, comment on the heat loss by free convection relative to that by radiation.

Known: Oven door with average surface temperature of 32°C in a room with ambient temperature at 22°C.

Find: Heat loss to the room. Also, find effect on heat loss if emissivity of door is unity and the surroundings are at 22°C.



Schematic:

Assumptions: (1) Ambient air in quiescent, (2) surface radiation effects are negligible.

Properties: Air ($T_f=300\text{K}$, 1atm): $\nu=15.89 \times 10^{-6} \text{ m}^2/\text{s}$, $k=0.0263 \text{ W/m} \cdot \text{K}$, $\alpha=22.5 \times 10^{-6} \text{ m}^2/\text{s}$, $\text{Pr}=0.707$, $\beta=1/T_f=3.33 \times 10^{-3} \text{ K}^{-1}$

Analysis: the heat rate from the oven door surface by convection to the ambient air is

$$q = \bar{h} A_s (T_s - T_\infty)$$

Air
 $T_\infty = 22^\circ\text{C}$

Where \bar{h} can be estimated from the free convection correlation for a vertical plate,

$$\dot{Nu}L = \frac{\bar{h}L}{k} = \left\{ 0.825 + \frac{0.387Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\}^2$$

The Rayleigh number,

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^2}{\nu\alpha} = \frac{9.8\text{m/s}^2(1/300\text{k})32 - 22\text{K}0.5^3\text{m}^3}{15.89 \times 10^{-6}\text{m}^2/\text{s} \times 22.5 \times 10^{-6}\text{m}^2/\text{s}} = 1.142 \times 10^4$$

Substituting numerical values into equation, find

$$\dot{Nu}L = \frac{\bar{h}L}{k} = \left\{ 0.825 + \frac{0.387(1.142 \times 10^4)^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\}^2 = 63.5$$

$$\bar{h}_L = \frac{k}{L} \dot{Nu}L = \frac{0.0263\text{W/m.K}}{0.5\text{m}} \times 63.5 = 3.34\text{W/m}^2.\text{K}$$

The heat rate using equation is

$$q = 3.34\text{W/m}^2 \cdot k(0.5 \times 0.7)\text{m}^2(32 - 22)\text{K} = 11.7\text{W}$$

Heat loss by radiation, assuming $\epsilon=1$ is

$$q_{\text{rad}} = \epsilon A_s \sigma (T_s^4 - T_{\text{sur}}^4)$$

$$q_{\text{rad}} = 1(0.5 \times 0.7)\text{m}^2 \times 5.6710^{-8}\text{W/m}^2.\text{K}^4 [(273 + 32)^4 - (273 + 22)^2] \text{K}^4 = 21.4\text{W}$$

Note that heat loss by radiation is nearly double that by free convection.

Comments: (1) Note the characteristics length in the Rayleigh number is the height of the vertical plate (door).

Problem 22

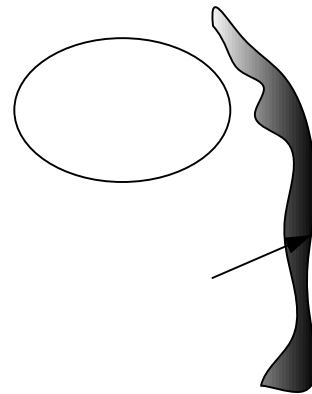
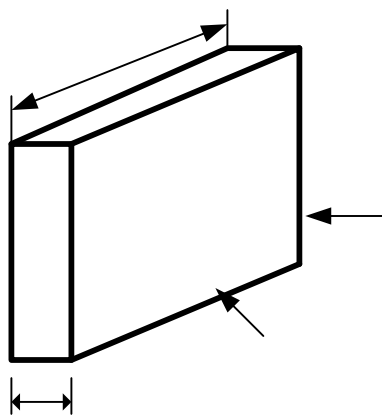
An Aluminum alloy (2024) plate, heated to a uniform temperature of 227°C , is allowed to cool while vertically suspended in a room where the ambient air and surroundings are at 27°C . The plate is 0.3 m^2 with a thickness of 15 mm and an emissivity of 0.25 .

- Develop an expression for the time rate of change of the plate temperature assuming the temperature to be uniform at any time.
- Determine the initial rate of cooling of the plate temperature is 227°C .
- Justify the uniform plate temperature assumption.

Known: Aluminum plate alloy (2024) at uniform temperature of 227°C suspended in a room where the ambient air and the surroundings are at 27°C

Find: (1) expression for the time rate of change of the plate, (2) Initial rate of cooling (K/s) when the plate temperature is 227°C . (3) justify the uniform plate temperature assumption.

Schematic:



Properties: Aluminium alloy 2024 (T=500K): $\rho=2270 \text{ kg/m}^3$,
 $k=186 \text{ W/m.K}$, $c=983 \text{ J/kg.K}$; Air (T_f=400K, 1atm): $v=26.41 \times 10^{-6} \text{ m}^2/\text{s}$,
 $k=0.0338 \text{ W/m.K}$, $\alpha=38.3 \times 10^{-6} \text{ m}^2/\text{s}$, $Pr=0.690$.

Analysis :(a) from an energy balance on the plate considering free

convection and radiation exchange $-\dot{E}_{out} = \dot{E}_{st}$.

$$-\bar{h}_L 2A_s (T_s - T_\infty) - \epsilon 2A_s \sigma (T_s^4 - T_{sur}^4) = \rho A_s \lambda_c \frac{dT}{dt} \text{ or } \frac{dT}{dt} = \frac{-2}{\int \lambda c} [\bar{h}_L (T_s - T_\infty) + \epsilon \sigma (T_s^4 - T_{sur}^4)]$$

Where T_s is the plate temperature assumed to be uniform at any time.

(b) To evaluate (dt/dx), estimate \bar{h}_L . Find first the Rayleigh number

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^2}{\nu\alpha} = \frac{9.8 \text{ m/s}^2 (1/400 \text{ K})(227 - 27) \text{ K} \times 0.3 \text{ m}^3}{26.41 \times 10^{-6} \text{ m}^2/\text{s} \times 38.3 \times 10^{-6} \text{ m}^2/\text{s}} = 1.308 \times 10^8$$

Substituting numerical values, find

$$\bar{Nu}_L = \left\{ 0.68 + \frac{0.670 Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}} \right\}^2 = \left\{ 0.68 + \frac{0.670(1.308 \times 10^8)}{[1 + (0.492/0.690)^{9/16}]^{4/9}} \right\} = 55.5$$

$$\bar{h}_L = \bar{Nu}_L k / L = 55.5 \times 0.0338 \text{ W/m.K} / 0.3 \text{ m} = 6.25 \text{ W/m}^2 \cdot \text{K}$$

$$\frac{dT}{dt} = \frac{-2}{2270 \text{ Kg/m}^3 \times 0.015 \text{ m} \times 983 \text{ J/kg.K}} \times 6.25 \text{ W/m}^2 \cdot \text{K} (227 - 27) \text{ K} + 0.25 (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}) (500^4 - 300^4) \text{ K}$$

$$= 0.099 \text{ K/s}$$

(c) The uniform temperature assumption is justified if the Biot number criterion is satisfied. With $L_c \equiv (V/A_s) = (A_s \lambda / A_s) = \lambda$

and $h_{tot}^- = h_{conv}^- + h_{rad}^-$, $Bi = h_{tot}^- \lambda / \bar{k} \leq 0.1$. Using the linearized radiation coefficient relation find,

$$\bar{h}_{rad} = \epsilon \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2) = 0.25(5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4)(500 + 300)(500^2 + 300^2) \text{K}^3 = 3.86 \text{W/m}^2 \cdot \text{K}$$

Hence $Bi = (6.25 + 3.86) \text{W/m}^2 \cdot \text{K} (0.015 \text{m}) / 186 \text{W/m} \cdot \text{K} = 8.15 \times 10^{-4}$. Since

$Bi \ll 0.1$, the assumption is appropriate.

Problem 23

The ABC Evening News Report in news segment on hypothermia research studies at the University of Minnesota claimed that heat loss from the body is 30 times faster in 10°C water than in air at the same temperature. Is that a realistic statement?

Known: Person, approximated as a cylinder, experiencing heat loss in water or air at 10°C.

Find: Whether heat loss from body in water is 30 times that in air.

Assumptions: (1) Person can be approximated as a vertical cylinder of diameter $D=0.3$ m and length $L=1.8$ m, at 25°C, (2) Loss is only from the lateral surface.

Properties: Air ($\bar{T}=(25+10)^\circ\text{C}/2=290\text{K}$, 1atm); $k=0.0293$ W/m.K, $\nu=19.91 \times 10^{-6} \text{m}^2/\text{s}$, $\alpha=28.4 \times 10^{-6} \text{m}^2/\text{s}$; Water (290K); $k=0.598$ W/m.K; $\nu=\mu v_f=1.081 \times 10^{-6} \text{m}^2/\text{s}$, $\alpha=k v_f/c_p=1.431 \times 10^{-7} \text{m}^2/\text{s}$, $\beta_f=174 \times 10^{-6} \text{K}^{-1}$.

Analysis: in both water (wa) and air(a), the heat loss from the lateral surface of the cylinder approximating the body is

$$q = \bar{h} \pi D L (T_s - T_\infty)$$

Where T_s and T_∞ are the same for both situations. Hence,

$$\frac{q_{wa}}{q_a} = \frac{\bar{h}_{wa}}{\bar{h}_a}$$

Vertical cylinder in air:

$$Ra_L = \frac{g \beta \Delta T^3}{\nu \alpha} = \frac{9.8 \text{m/s}^2 (1/290\text{K})(25-10)k(1.8\text{m})^3}{19.91 \times 10^{-6} \text{m}^2/\text{s} \times 28.410^{-6} \text{m}^2/\text{s}} = 5.228 \times 10^9$$

with $C = 0.1$ and $n = 1/3$,

$$\text{Nu}_L = \frac{\bar{h}_L L}{k} = C \text{Ra}_L^n = 0.1(5.228 \times 10^9)^{1/3} = 173.4 \quad \bar{h}_L = 2.82 \text{W/m}^2 \cdot \text{K}$$

Vertical cylinder in water.

$$\text{Ra}_L = \frac{9.8 \text{m/s}^2 \times 174 \times 10^{-6} \text{K}^{-1} (25 - 10) \text{K} (1.8 \text{m})^3}{1.081 \times 10^{-6} \text{m}^2/\text{s} \times 1.431 \times 10^{-7} \text{m}^2/\text{s}} = 9.643 \times 10^{11}$$

with $C = 0.1$ and $n = 1/3$,

$$\text{Nu}_L = \frac{\bar{h}_L L}{k} = C \text{Ra}_L^n = 0.1(9.643 \times 10^{11})^{1/3} = 978.9 \quad \bar{h}_L = 328 \text{W/m}^2 \cdot \text{K}$$

Hence, from this analysis we find

$$\frac{q_{wa}}{q_a} = \frac{328 \text{W/m}^2 \cdot \text{K}}{2.8 \text{W/m}^2 \cdot \text{K}} = 117 \quad \text{which compares poorly with the claim of 30}$$

Problem 24

In a study of heat losses from buildings, free convection heat transfer from room air at 305 K to the inner surface of a 2.5-m-high wall at 295 K is simulated by performing laboratory experiments using water in a smaller test cell. In the experiments the water and the inner surface of the test cell are maintained at 300 and 290K, respectively. To achieve similarity between conditions in the room and the test cell, what is the required test cell height? If the average Nusselt number for the wall may be correlated exclusively in terms of the Rayleigh number, what is the ratio of the average convection coefficient for the room wall to the average coefficient for the test cell wall?

Known: Air temperature and wall temperature and height for a room. Water temperature and wall temperature for a simulation experiment.

Find: Required test cell height for similarity. Ratio and height convection coefficient for the two cases.

Schematic:



Assumptions: (1) Air and water are quiescent; (2) Flow conditions correspond to free convection boundary layer development on an isothermal vertical plate, (3) constant properties.

Properties: Air ($T_f=300\text{K}$, 1 atm); $k=0.0293\text{ W/m.K}$, $\nu=15.9*10^{-6}\text{m}^2/\text{s}$, $\alpha=22.5*10^{-6}\text{ m}^2/\text{s}$; $\beta=1/T_f=3.33*10^{-3}\text{K}^{-1}$, $k=0.0263\text{W/m.K}$; water ($T_f=295\text{K}$): $\rho=998\text{kg/m}^3$, $\mu=959*10^{-6}\text{N.s/m}^2$, $c_p=4181\text{ J/kg.K}$, $\beta=227.5*10^{-6}\text{K}^{-1}$, $k=0.606\text{ W/m.K}$; Hence $\nu=\mu/\rho=9.61/\text{s}$, $\alpha=k/\rho c_p=1.45*10^{-7}\text{m}^2/\text{s}$.

Analysis: Similarity requires that $Ra_{L,a} = Ra_{L,w}$ where

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

hence,

$$\frac{L_w}{L_a} = \left[\frac{(\alpha\nu)_w \beta_{air}}{(\alpha\nu)_a \beta_{H_2O}} \right]^{1/3} = \left[\frac{9.61 * 1.45 * 10^{-14}}{15.9 * 22.5 * 10^{-12}} \frac{3.33 * 10^{-3}}{0.228 * 10^{-3}} \right]$$

$$L_w = 2.5m(0.179) = 0.45m$$

if $Ra_{L,a} = Ra_{L,w}$, it follows that $Nu_{L,a}^- = Nu_{L,w}^-$. hence

$$\frac{\bar{h}_a}{\bar{h}_w} = \frac{L_w}{L_a} \frac{k_a}{k_w} = \frac{0.45}{2.5} \frac{0.0263}{0.606} = 7.81 * 10^{-3}$$

Comments: Similitude allows us to obtain valuable information for one system by performing experiments for a smaller system and a different fluid.

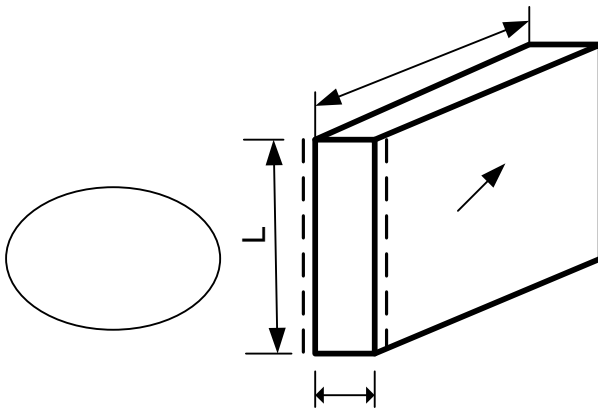
Problem 25

A square plate of pure aluminum, 0.5 m on a side and 16 mm thick, is initially at 300C and is suspended in a large chamber. The walls of the chamber are maintained at 27C, as is the enclosed air. If the surface emissivity of the plate temperature during the cooling process? Is it reasonable to assume a uniform plate temperature during the cooling process?

Known: Initial temperature and dimensions of an aluminum plate. Conditions of the plate surroundings.

Find: (a) initial cooling rate, (2) validity of assuming negligible temperature gradients in the plate during the cooling process.

Schematic:



Assumptions: (1) plate temperature is uniform; (2) chamber air is quiescent, (3) Plate surface is diffuse-gray, (4) Chamber surface is much larger than that of plate, (5) Negligible heat transfer from edges.

Properties: Aluminum (573k); $k=232\text{W/m.k}$, $c_p=1022\text{J/kg.K}$, $\rho=2702\text{ kg/m}^3$:
Air ($T_f=436\text{K}$, 1 atm): $\nu=30.72*10^{-6}\text{m}^2/\text{s}$, $\alpha=44.7*10^{-6}\text{m}^2/\text{s}$, $k=0.0363\text{W/m.K}$,
 $\text{Pr}=0.687$, $\beta=0.00229\text{K}^{-1}$.

Analysis: (a) performing an energy balance on the plate,

$$-q = -2A_s [\bar{h}(T_s - T_\infty) + \varepsilon\sigma(T^4 - T_{sur}^4)] = \dot{E}_{st} = \rho V c_p [dT / dt]$$

$$dT / dt = -2A_s [\bar{h}(T_s - T_\infty) + \varepsilon\sigma(T^4 - T_{sur}^4)] / \rho w c_p$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{9.8\text{m/s}^2 \times 0.00229\text{K} - 1(300 - 27)\text{K}(0.5\text{m})^3}{30.72 * 10^{-6} \text{m}^2/\text{s} \times 44.7 \times 10^{-6} \text{m}^2/\text{s}} = 5.58 \times 10^8$$

$$\bar{h} = \frac{k}{L} \left\{ 0.68 + \frac{0.670 Ra_L^{1/4}}{[1 + (0.492 / Pr)^{9/16}]^{4/9}} \right\} = \frac{0.0363}{0.5} \left\{ 0.68 + \frac{0.670(5.58 \times 10^8)^{1/4}}{[1 + (0.492 / 0.687)^{9/16}]^{4/9}} \right\}$$

$$\bar{h} = 5.8 \text{W} / \text{m}^2 \cdot \text{K}$$

Hence the initial cooling rate is

$$\frac{dT}{dt} = - \frac{2 \{ 5.8 \text{W} / \text{m}^2 \cdot \text{K} (300 - 27) \text{C}^\circ + 0.25 \times 5.67 \times 10^{-8} \text{W} / \text{m}^2 \cdot \text{K}^4 [(573 \text{K})^4 - (300 \text{K})^4] \}}{2702 \text{kg} / \text{m}^3 (0.016 \text{m}) 1022 \text{J} / \text{kg} \cdot \text{K}}$$

$$\frac{dT}{dt} = -0.136 \text{K} / \text{s}$$

(b) To check the Validity of neglecting temperature gradients across the plate thickness, calculate

$$Bi = h_{eff} (w/2) / k \text{ where } h_{eff} = q_{tot}'' / (T_i - T_\infty) = (1583 + 1413) \text{W} / \text{m}^2 / 273 \text{K} = 11.0 \text{W} / \text{m}^2 \cdot \text{K}$$

$$\text{hence } Bi = (11 \text{W} / \text{m}^2 \cdot \text{K}) (0.008 \text{m}) / 232 \text{W} / \text{m} \cdot \text{K} = 3.8 \times 10^{-4}$$

And the assumption is excellent.

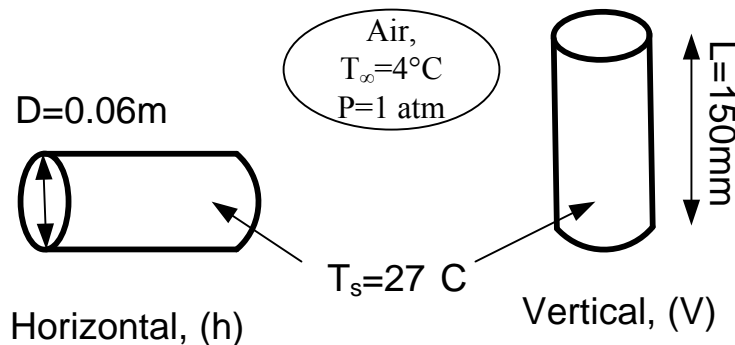
Comments: (1) Longitudinal (x) temperature gradients are likely to be more severe than those associated with the plate thickness due to the variation of h with x. (2) Initially $q''_{conv} \approx q''_{rad}$.

Problem 26

Beer in cans 150 mm long and 60 mm in diameter is initially at 27°C and is to be cooled by placement in a refrigerator compartment at 4°C. In the interest of maximizing the cooling rate, should the cans be laid horizontally or vertically in the compartment? As a first approximation, neglect heat transfer from the ends.

Known: Dimensions and temperature of beer can in refrigerator compartment.

Find: orientation which maximum cooling rate.



Schematic:

Assumptions: (1) End effects are negligible, (2) Compartment air is quiescent, (3) constant properties.

Properties: Air ($T_f=288.5\text{K}$, 1 atm): $\nu=14.87*10^{-6}\text{ m}^2/\text{s}$, $k=0.0254\text{W/m.K}$, $\alpha=21.0*10^{-6}\text{m}^2/\text{s}$, $\text{Pr}=0.71$, $\beta=1/T_f=3.47*10^{-3}\text{K}^{-1}$.

Analysis: The ratio of cooling rates may be expressed as

$$\frac{q_v}{q_h} = \frac{\bar{h}_v}{\bar{h}_h} = \frac{\pi DL (T_s - T_\infty)}{\pi DL (T_s - T_\infty)} = \frac{\bar{h}_v}{\bar{h}_h}$$

For the vertical surface, find

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{9.8\text{m/s}^2 \times 3.47 \times 10^{-3} \text{K}^{-1}(23^\circ\text{C})}{(14.87 \times 10^{-6} \text{m}^2/\text{s})(21 \times 10^{-6} \text{m}^2/\text{s})} L^3 = 2.5 \times 10^9 L^3$$

$$Ra_L = 2.5 \times 10^9 (0.15)^3 = 8.44 \times 10^6$$

Using the correct correlation

$$Nu_L = \left\{ 0.825 + \frac{0.387(8.44 \times 10^6)^{1/6}}{[1 + (0.492/0.71)^{9/16}]^{8/27}} \right\}^2 = 29.7$$

$$\text{hence, } \bar{h}_L = \bar{h}_v = \bar{Nu}_L \frac{k}{L} = 29.7 \frac{0.0254 \text{W} / \text{m.K}}{0.15 \text{m}} = 5.03 \text{W} / \text{m}^2 .\text{K}$$

Using the correct correlation,

$$\bar{Nu}_D = \left\{ 0.60 + \frac{0.387(5.4 \times 10^5)^{1/6}}{[1 + (0.559/0.71)^{9/16}]^{8/27}} \right\}^2 = 12.24$$

$$\bar{h}_D = \bar{h}_h = \bar{Nu}_D \frac{k}{D} = 12.24 \frac{0.0254 \text{W} / \text{m.K}}{0.06 \text{m}} = 5.18 \text{W} / \text{m}^2 .\text{K}$$

$$\text{hence, } \frac{q_v}{q_h} = \frac{5.03}{5.18} = 0.97$$

Comments: in view of the uncertainties associated with equations and the neglect of end effects, the above result is inconclusive. The cooling rates are approximately the same.