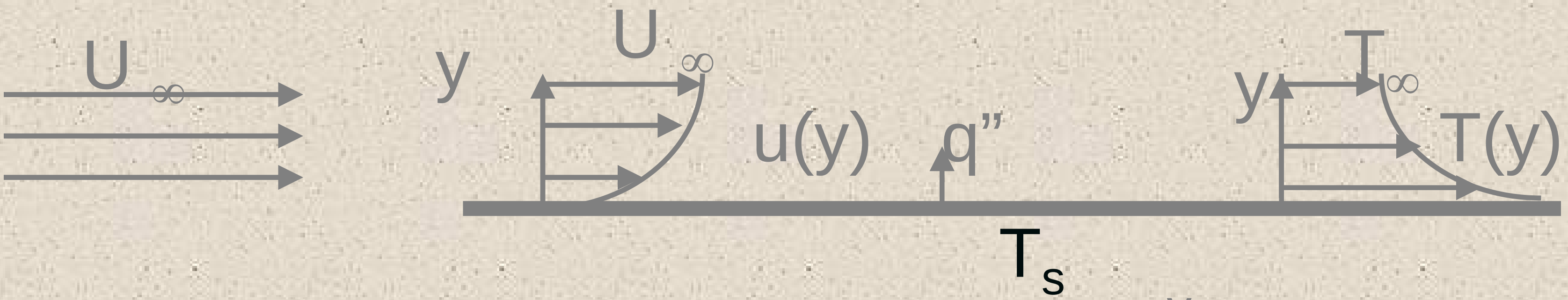
MODULE 6

Convection Heat Transfer

Convection

- Heat transfer in the presence of a fluid motion on a solid surface
- Various mechanisms at play in the fluid:
  - *advection* $\rightarrow$ physical transport of the fluid
  - *diffusion* $\rightarrow$ conduction in the fluid
  - *generation* $\rightarrow$ due to fluid friction
- But fluid directly in contact with the wall does not move relative to it; hence direct heat transport to the fluid is by conduction in the fluid only.



$$q''_{conv} = -k_f \left.\frac{\partial T}{\partial y}\right)_{y=0} = h\left(T_s - T_\infty\right)$$

But $\left.\frac{\partial T}{\partial y}\right)_{y=0}$ depends on the whole fluid motion, and both fluid flow and heat transfer equations are needed

Convection

Convection

Free or natural convection (induced by buoyancy forces)

forced convection (driven externally)

May occur with phase change (boiling, condensation)

Heat transfer rate $q = h(T_s - T_\infty)$ W

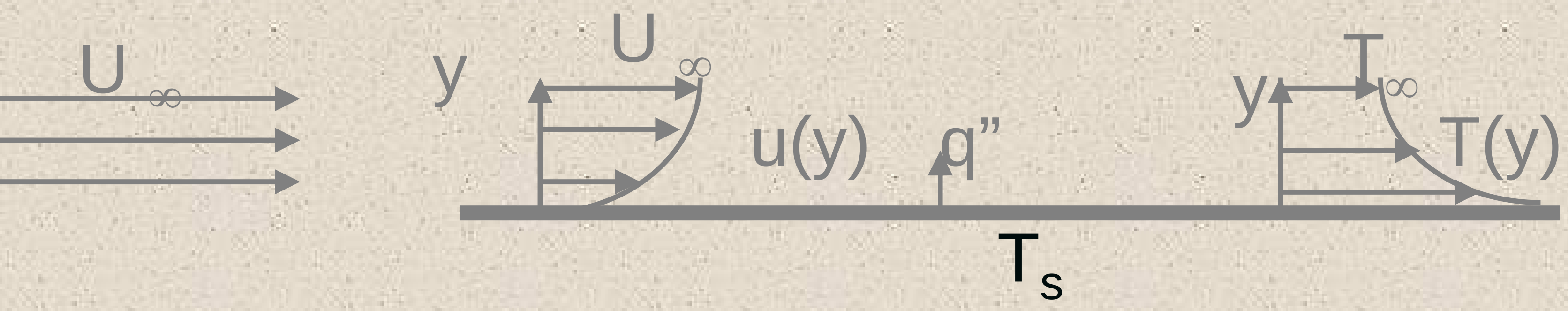
h=heat transfer coefficient (W /m^{2}K)

(*h* is not a property. It depends on geometry ,nature of flow, thermodynamics properties etc.)

Typical values of h (W/m^{2}K)

Free convection:	gases:	2 - 25
	liquid:	50 - 100
Forced convection:	gases:	25 - 250
	liquid:	50 - 20,000
Boiling/Condensation:		2500 -100,000

Convection rate equation



Main purpose of convective heat transfer analysis is to determine:

- flow field
- temperature field in fluid
- heat transfer coefficient, h

q’’=heat flux = $h(T_s - T_\infty)$

q’’ = $-k(\partial T/ \partial y)_{y=0}$

Hence, $h = [-k(\partial T/ \partial y)_{y=0}] / (T_s - T_\infty)$

The expression shows that in order to determine h, we must first determine the temperature distribution in the thin fluid layer that coats the wall.

Classes of convective flows:

- extremely diverse
- several parameters involved (fluid properties, geometry, nature of flow, phases etc)
- systematic approach required
- classify flows into certain types, based on certain parameters
- identify parameters governing the flow, and group them into meaningful **non-dimensional numbers**
- need to understand the physics behind each phenomenon

Common classifications:

A. *Based on geometry:*

External flow / Internal flow

B. *Based on driving mechanism*

Natural convection / forced convection / mixed convection

C. *Based on number of phases*

Single phase / multiple phase

D. *Based on nature of flow*

Laminar / turbulent

How to solve a convection problem ?

- Solve governing equations along with boundary conditions
- Governing equations include

  1. conservation of mass
  2. conservation of momentum
  3. conservation of energy

- In **Conduction** problems, only (3) is needed to be solved. Hence, only ***few parameters*** are involved
- In **Convection**, all the governing equations need to be solved.

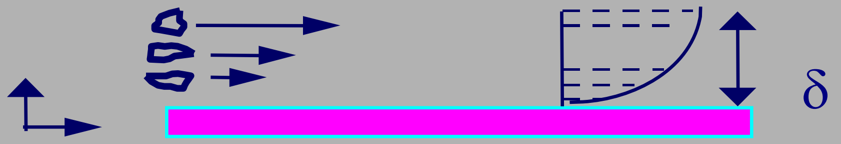
  $\Rightarrow$ **large number of parameters** can be involved

Forced convection: **Non-dimensional groupings**

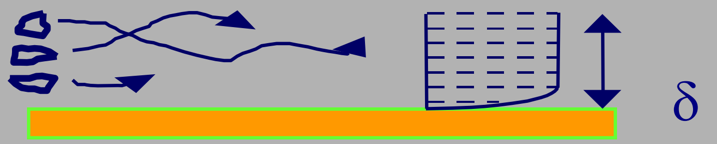
- **Nusselt No**. Nu = hx / k = (convection heat transfer strength)/ (conduction heat transfer strength)
- **Prandtl No.** Pr = ν/α = (momentum diffusivity)/ (thermal diffusivity)
- **Reynolds No**. Re = U x / ν = (inertia force)/(viscous force)

Viscous force provides the dampening effect for disturbances in the fluid. If dampening is strong enough $\Rightarrow$ **laminar flow**

Otherwise, instability $\Rightarrow$ **turbulent flow** $\Rightarrow$ **critical Reynolds number**



Laminar

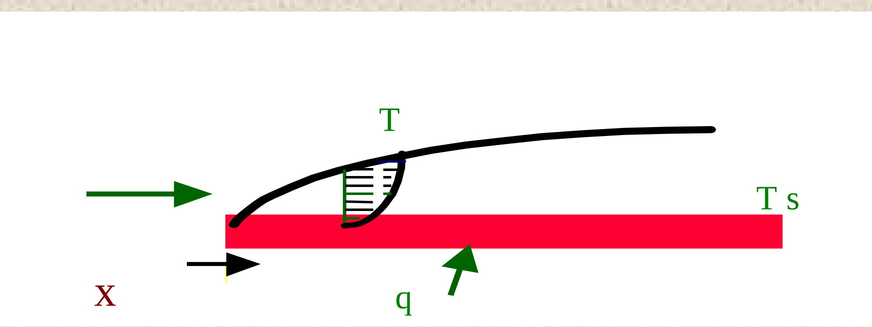


Turbulent

FORCED CONVECTION:

external flow (over flat plate)

An internal flow is surrounded by solid boundaries that can restrict the development of its boundary layer, for example, a pipe flow. An external flow, on the other hand, are flows over bodies immersed in an unbounded fluid so that the flow boundary layer can grow freely in one direction. Examples include the flows over airfoils, ship hulls, turbine blades, etc.



- Fluid particle adjacent to the solid surface is at rest
- These particles act to retard the motion of adjoining layers
- ⇒ **boundary layer effect**

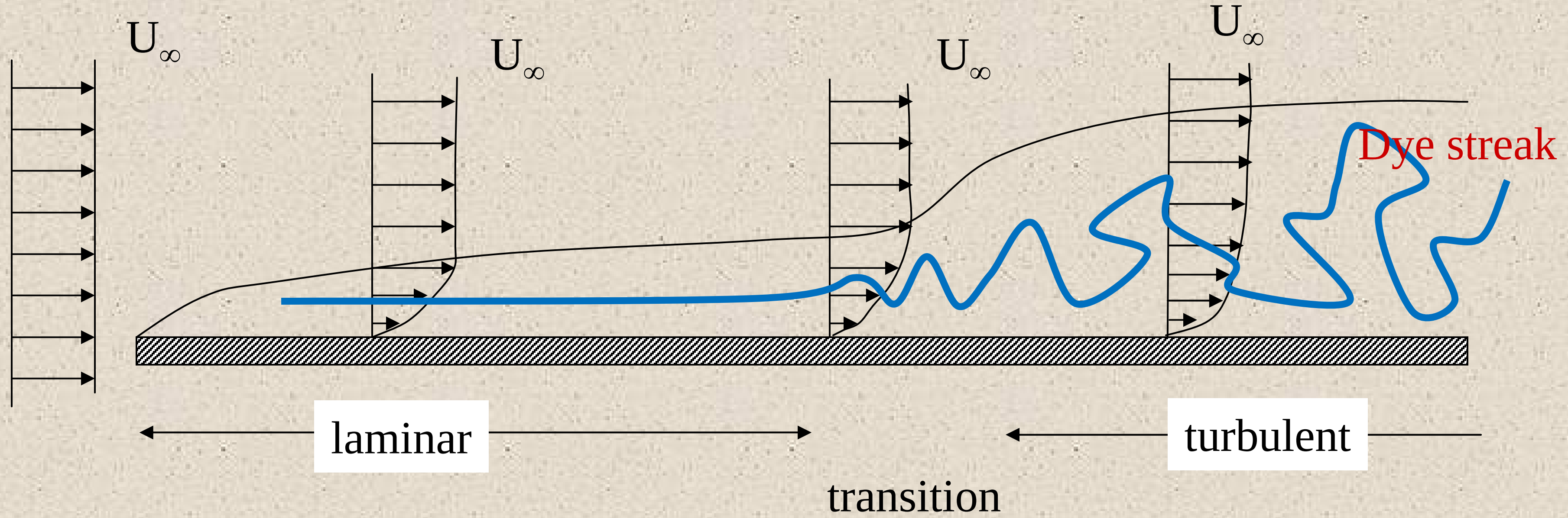
Momentum balance: inertia forces, pressure gradient, viscous forces, body forces

Energy balance: convective flux, diffusive flux, heat generation, energy storage

h=f(Fluid, Vel ,Distance,Temp)

Hydrodynamic boundary layer

One of the most important concepts in understanding the external flows is the boundary layer development. For simplicity, we are going to analyze a boundary layer flow over a flat plate with no curvature and no external pressure variation.



Boundary layer definition

➢Boundary layer thickness (δ): defined as the distance away from the surface where the local velocity reaches to 99% of the free-stream velocity, that is $u(y=\delta)=0.99U_\infty$. Somewhat an easy to understand but arbitrary definition.

➢Boundary layer is usually very thin: δ/x usually $<< 1$.

Hydrodynamic and Thermal boundary layers

As we have seen earlier,the hydrodynamic boundary layer is a region of a fluid flow, near a solid surface, where the flow patterns are directly influenced by viscous drag from the surface wall.

$$0<u<U, \quad 0<y<\delta$$

The Thermal Boundary Layer is a region of a fluid flow, near a solid surface, where the fluid temperatures are directly influenced by heating or cooling from the surface wall.

$$0<t<T, \quad 0<y<\delta_t$$

The two boundary layers may be expected to have similar characteristics but do not normally coincide. Liquid metals tend to conduct heat from the wall easily and temperature changes are observed well outside the dynamic boundary layer. Other materials tend to show velocity changes well outside the thermal layer.

Effects of Prandtl number, Pr

Pr >>1

$\nu >> \alpha$

e.g., oils

Pr = 1

$\nu = \alpha$

e.g., air and gases have Pr ~ 1

(0.7 - 0.9)

$$\frac{u}{U_\infty} \text{ similar to } \frac{T - T_W}{T_\infty - T_W}$$

(Reynold's analogy)

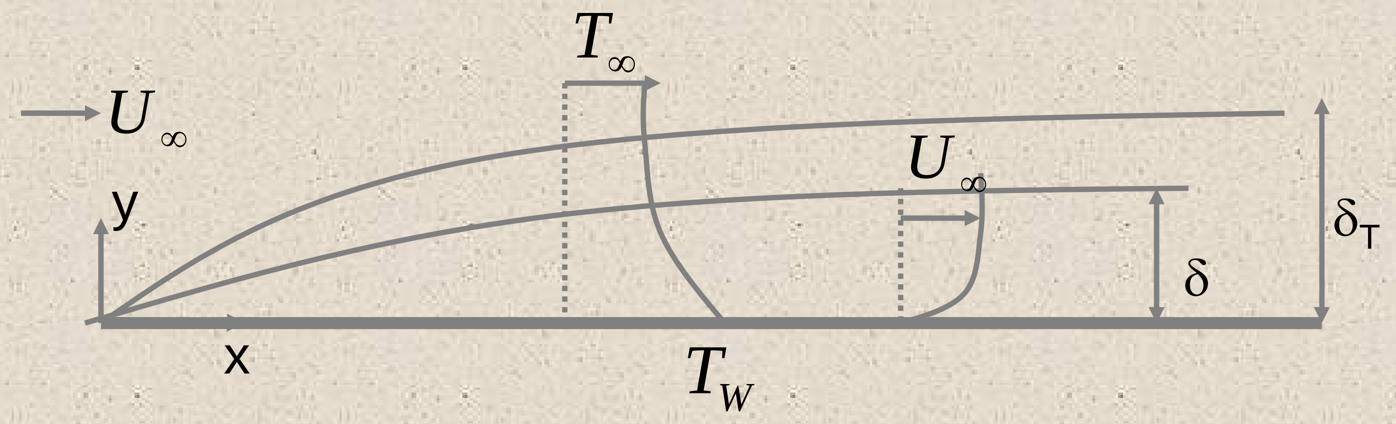
Pr <<1

$\nu << \alpha$

e.g., liquid metals

Boundary layer equations (laminar flow)

- Simpler than general equations because boundary layer is thin



- Equations for 2D, laminar, steady boundary layer flow

$$\text{Conservation of mass:}\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\text{Conservation of x-momentum:}\quad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)$$

$$\text{Conservation of energy:}\quad u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right)$$

- Note: for a flat plate, U_∞ is constant , hence $\frac{dU_\infty}{dx}=0$

Exact solutions: Blasius

Boundary layer thickness $\frac{\delta}{x} = \frac{4.99}{\sqrt{\mathrm{Re}_x}}$

Skin friction coefficient $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$

$$\left(\mathrm{Re}_x = \frac{U_\infty x}{\nu}, \quad \tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \right)$$

Average drag coefficient $C_D = \frac{1}{L}\int_0^L C_f\, dx = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \quad \left(\mathrm{Re}_L = \frac{U_\infty L}{\nu} \right)$

Local Nusselt number $Nu_x = 0.339\, \mathrm{Re}_x^{1/2}\, \mathrm{Pr}^{1/3}$

Average Nusselt number $\overline{N}u = 0.678\, \mathrm{Re}_L^{1/2}\, \mathrm{Pr}^{1/3}$

Heat transfer coefficient

- Local heat transfer coefficient:

$$h_x = \frac{Nu_x k}{x} = \frac{0.339\, k\, \mathrm{Re}_x^{1/2}\, \mathrm{Pr}^{1/3}}{x}$$

- Average heat transfer coefficient:

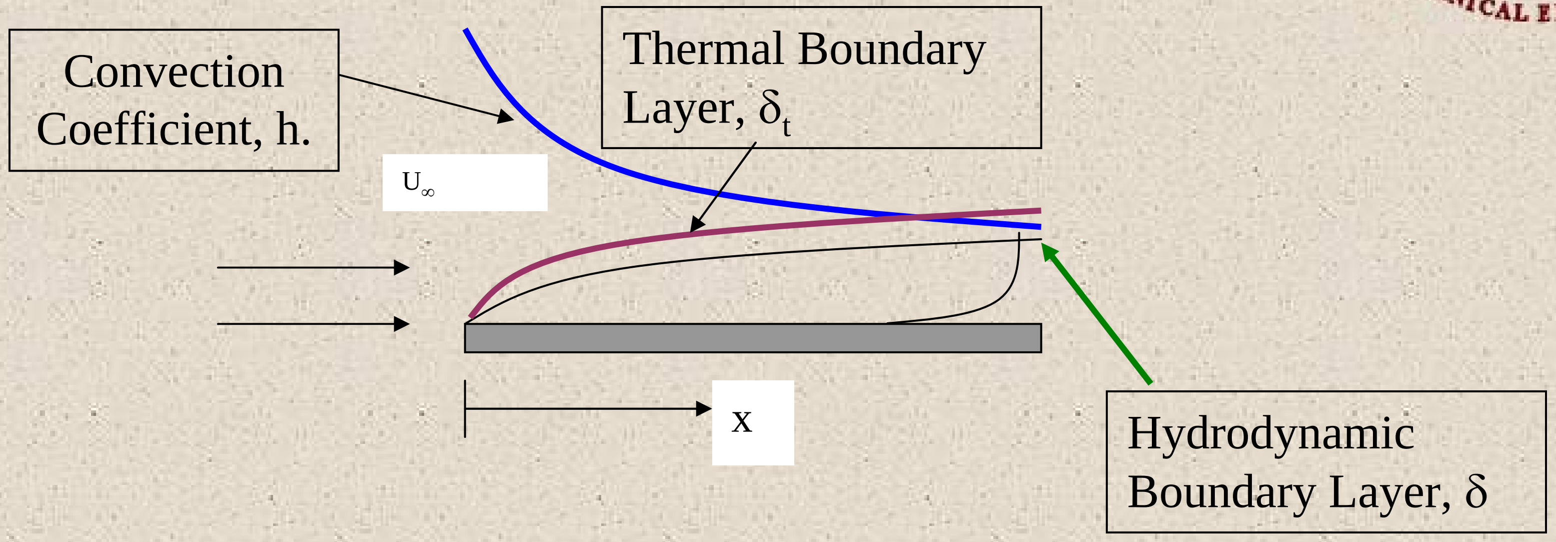
$$\bar{h} = \frac{\overline{Nu}\ k}{L} = \frac{0.678\, k\, \mathrm{Re}_L^{1/2}\, \mathrm{Pr}^{1/3}}{L}$$

- Recall: $q_w = \bar{h} A \left(T_w - T_\infty\right)$, heat flow rate from wall

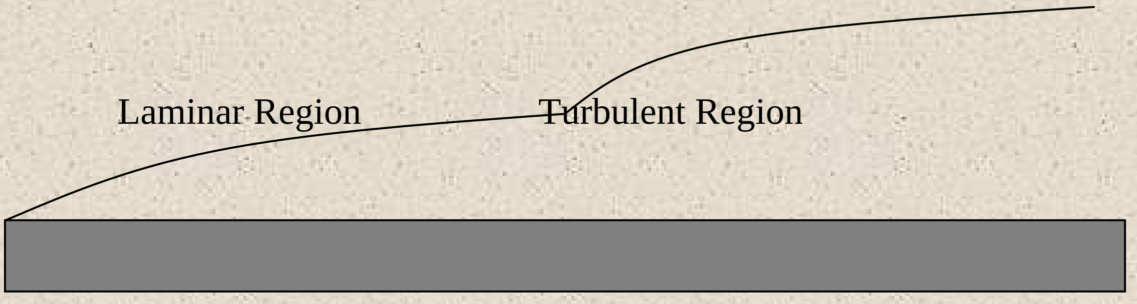
- Film temperature, T_{film}

For heated or cooled surfaces, the thermophysical properties within the boundary layer should be selected based on the average temperature of the wall and the free stream; $T_{film} = \frac{1}{2}\left(T_w + T_\infty\right)$

Heat transfer coefficient



Laminar and turbulent b.l.



Turbulent boundary layer

* Re_x increases with x. Beyond a critical value of Reynolds number ($\mathrm{Re}_x = \mathrm{Re}_{xc}$), the flow becomes transitional and eventually turbulent.

$$\mathrm{Re}_{xc} = \frac{U_\infty x_c}{\nu} \quad (\text{For flow over flat plate, } x_c \approx 5\times10^5)$$

* Turbulent b.l. equations are similar to laminar ones, but infinitely more difficult to solve.

* We will mainly use correlations based on experimental data :

$$C_f = 0.059\,\mathrm{Re}_x^{-0.2} \qquad (\mathrm{Re}_x > 5\times10^5)$$

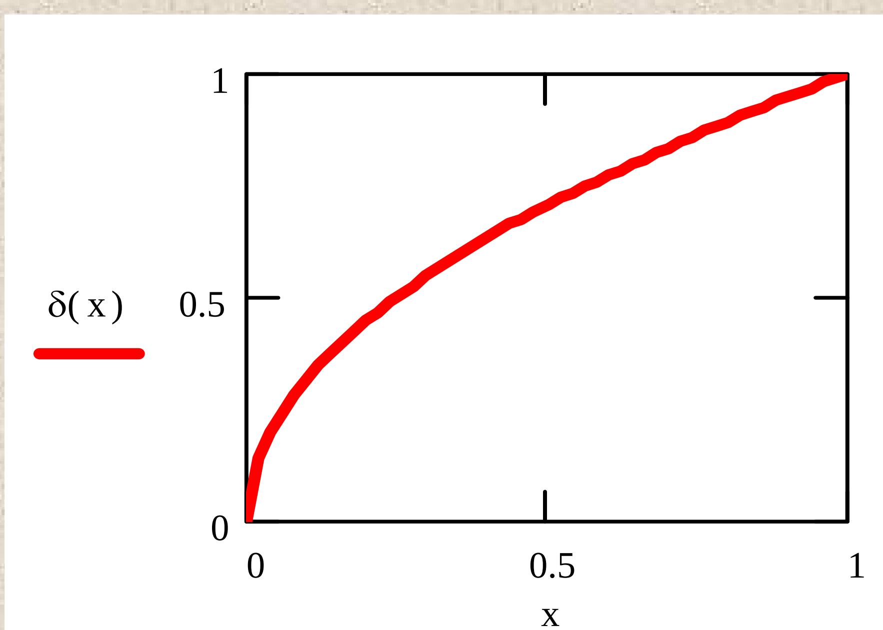
$$C_D = 0.072\,\mathrm{Re}_L - \frac{1}{\mathrm{Re}_L}\left(0.072\,\mathrm{Re}_{xc}^{0.8} - 1.328\,\mathrm{Re}_{xc}^{0.5}\right)$$

$$Nu_x = 0.029\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

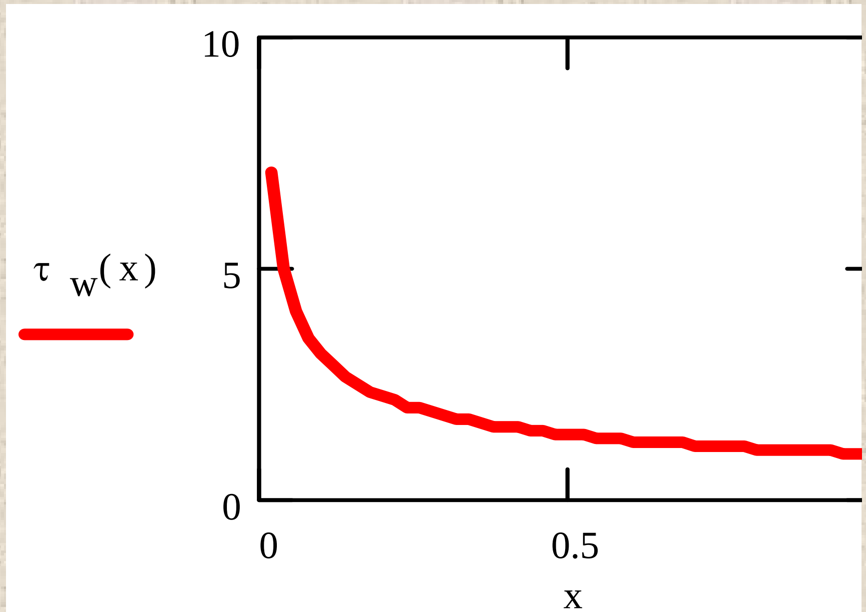
$$\overline{Nu} = 0.036\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} - \mathrm{Pr}^{1/3}\left(0.036\,\mathrm{Re}_{xc}^{0.8} - 0.664\,\mathrm{Re}_{xc}^{0.5}\right)$$

* Calculate heat transfer coefficient in usual way : $h = \frac{Nu\ k}{x}$ etc.

Laminar Boundary Layer Development



- Boundary layer growth: $\delta \propto \sqrt{x}$
- Initial growth is fast
- Growth rate $d\delta/dx \propto 1/\sqrt{x}$, decreasing downstream.



- Wall shear stress: $\tau_w \propto 1/\sqrt{x}$
- As the boundary layer grows, the wall shear stress decreases as the velocity gradient at the wall becomes less steep.

Example

Determine the boundary layer thickness, the wall shear stress of a laminar water flow over a flat plate. The free stream velocity is 1 m/s, the kinematic viscosity of the water is 10^{-6} m^2/s. The density of the water is 1,000 kg/m^3. The transition Reynolds number $Re=Ux/\nu=5\times10^5$. Determine the distance downstream of the leading edge when the boundary transitions to turbulent. Determine the total frictional drag produced by the laminar and turbulent portions of the plate which is 1 m long. If the free stream and plate temperatures are 100 °C and 25 °C, respectively, determine the heat transfer rate from the plate.

$$\delta(x) = 5\sqrt{\frac{\nu x}{U_\infty}} = 5\times10^{-3}\sqrt{x} \quad (m).$$

Therefore, for a 1m long plate, the boundary layer grows by 0.005(m), or 5 mm, a very thin layer.

The wall shear stress, $\tau_w = \dfrac{0.332\rho U_\infty^2}{\sqrt{\mathrm{Re}_x}} = 0.332U_\infty\sqrt{\dfrac{\rho\mu U_\infty}{x}} = \dfrac{0.0105}{\sqrt{x}}(Pa)$

The transition Reynolds number: $\mathrm{Re} = \dfrac{U_\infty x_{tr}}{\nu} = 5\times10^5, \quad x_{tr} = 0.5(m)$

Example (cont..)

The total frictional drag is equal to the integration of the wall shear stress:

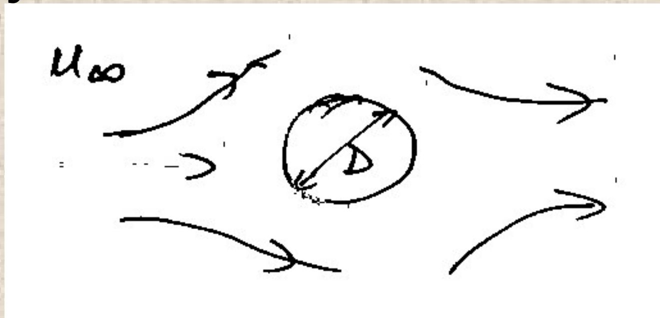
$$F_D = \int_0^{x_{tr}} \tau_w (1) dx = \int_0^{x_{tr}} 0.332 U_\infty \sqrt{\frac{\rho \mu U_\infty}{x}} dx = \frac{0.664 \rho U_\infty^2}{\sqrt{\mathrm{Re}_{x_{tr}}}} = 0.939(N)$$

Define skin friction coefficient: C_f

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_\infty^2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$ for a laminar boundary layer.

Forced convection over exterior bodies

- Much more complicated.
- Some boundary layer may exist, but it is likely to be curved and U∞ will not be constant.
- Boundary layer may also separate from the wall.
- Correlations based on experimental data can be used for flow and heat transfer calculations
- Reynolds number should now be based on a characteristic diameter.



$$\mathrm{Re}_D = \frac{\rho U_\infty D}{\mu}$$

- If body is not circular, the equivalent diameter Dh is used

$$D_h = \frac{4 \times \text{Area}}{\text{Perimeter}}$$

$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U_\infty^2 A_{normal}} \quad ; \quad \overline{N}_u = \frac{\overline{h} D}{k} \quad ; \quad \overline{h} = \frac{\overline{N}_u k}{D}$$

Flow over circular cylinders

$$\overline{\mathrm{Nu}} = C\,\mathrm{Re}_D^m \frac{\mathrm{Pr}^{.62}}{\mathrm{Pr}_s^{.25}}$$

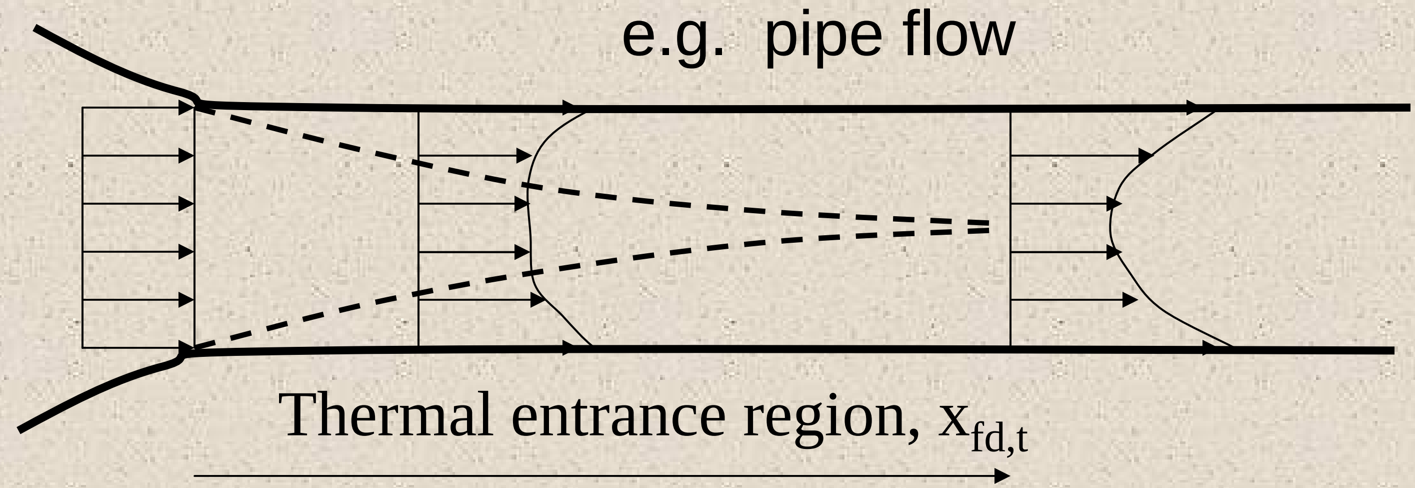
Re_D	C	m
$1 - 40$	0.75	0.4
$40 - 10^3$	0.51	0.5
$10^3 - 2\times10^5$	0.26	0.6
$2\times10^5 - 10^6$	0.08	0.7

All properties at free stream temperature, Pr_s at cylinder surface temperature

FORCED CONVECTION: Internal flow

- Thermal conditions
  - ⇨ Laminar or turbulent
  - ⇨ entrance flow and fully developed thermal condition

e.g. pipe flow

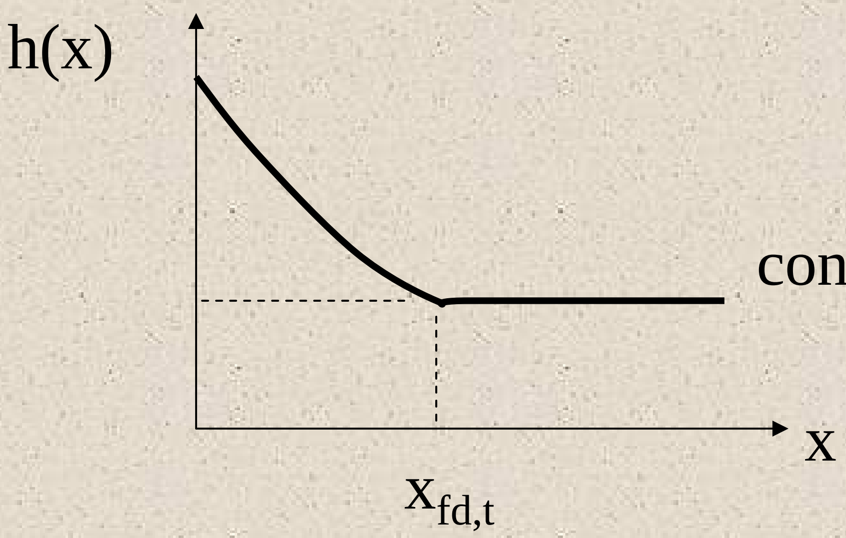


For laminar flows the thermal entrance length is a function of the Reynolds number and the Prandtl number: $x_{fd,t}/D \approx 0.05Re_D Pr$, where the Prandtl number is defined as $Pr = \nu/\alpha$ and α is the thermal diffusitivity.

For turbulent flow, $x_{fd,t} \approx 10D$.

Thermal Conditions

- For a fully developed pipe flow, the convection coefficient does not vary along the pipe length. (provided all thermal and flow properties are constant)



- Newton's law of cooling: $q''_S = hA(T_S-T_m)$

Question: since the temperature inside a pipe flow does not remain constant, we should use a mean temperature T_m , which is defines as follows:

Energy Transfer

Consider the total thermal energy carried by the fluid as

$$\int_A \rho V C_v T dA = \text{(mass flux) (internal energy)}$$

Now imagine this same amount of energy is carried by a body of fluid with the same mass flow rate but at a uniform mean temperature T_m. Therefore T_m can be defined as

$$T_m = \frac{\int_A \rho V C_v T dA}{\dot{m} C_v}$$

Consider T_m as the reference temperature of the fluid so that the total heat transfer between the pipe and the fluid is governed by the Newton's cooling law as: $q_s'' = h(T_s - T_m)$, where h is the local convection coefficient, and T_s is the local surface temperature. Note: usually T_m is not a constant and it varies along the pipe depending on the condition of the heat transfer.

Energy Balance

Example: We would like to design a solar water heater that can heat up the water temperature from 20° C to 50° C at a water flow rate of 0.15 kg/s. The water is flowing through a 0.05 m diameter pipe and is receiving a net solar radiation flux of 200 W/m of pipe length. Determine the total pipe length required to achieve the goal.

Example (cont.)

Questions: (1) How to determine the heat transfer coefficient, h?

There are a total of six parameters involved in this problem: h, V, D, ν, k_f, c_p. The temperature dependence of properties is implicit and is only through the variation of thermal properties. Density ρ is included in the kinematic viscosity, $\nu=\mu/\rho$. According to the Buckingham theorem, it is possible for us to reduce the number of parameters by three. Therefore, the convection coefficient relationship can be reduced to a function of only three variables:

$Nu=hD/k_f$, Nusselt number, $Re=VD/\nu$, Reynolds number, and $Pr=\nu/\alpha$, Prandtl number.

This conclusion is consistent with empirical observation, that is $Nu=f(Re, Pr)$. If we can determine the Reynolds and the Prandtl numbers, we can find the Nusselt number and hence, the heat transfer coefficient, h.

Convection Correlations

⇒ Laminar, fully developed circular pipe flow:

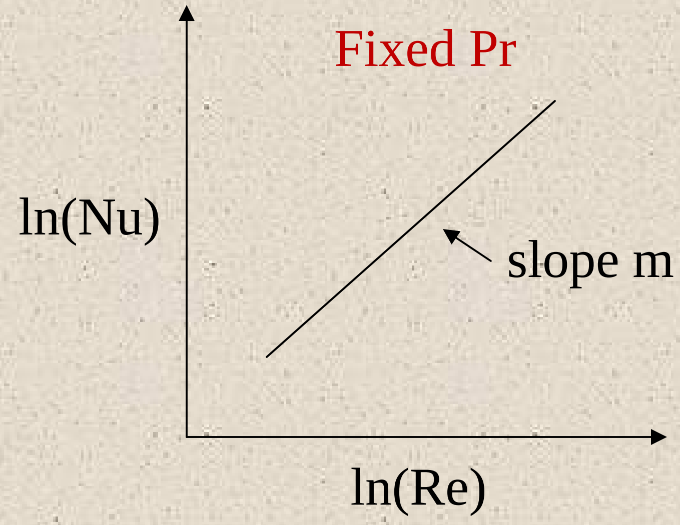
$$Nu_D = \frac{hD}{k_f} = 4.36, \quad \text{when } q_s'' = \text{constant},$$

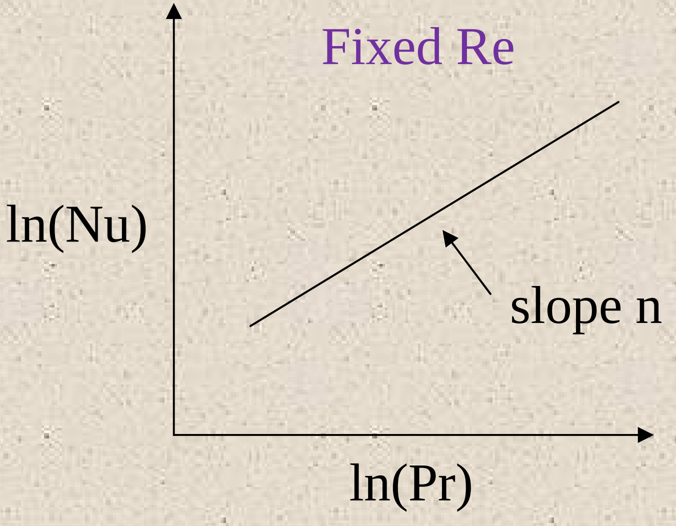
$$Nu_D = 3.66, \quad \text{when } T_s = \text{constant}$$

Note: the therma conductivity should be calculated at T_m.

⇒ Fully developed, turbulent pipe flow: Nu = f(Re, Pr),

Nu can be related to Re & Pr experimentally, as shown.





Empirical Correlations

Dittus-Boelter equation: $Nu_D = 0.023 Re^{4/5} Pr^n$,

where n = 0.4 for heating ($T_s > T_m$), n = 0.3 for cooling ($T_s < T_m$).

The range of validity: $0.7 \le Pr \le 160$, $Re_D \ge 10,000$, $L/D \ge 10$.

Note: This equation can be used only for moderate temperature difference with all the properties evaluated at T_m.

Other more accurate correlation equations can be found in other references.
Caution: The ranges of application for these correlations can be quite different.

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \quad \text{(from other reference)}$$

It is valid for $0.5 < Pr < 2000$ and $3000 < Re_D < 5 \times 10^6$.

All properties are calculated at T_m.

Example (cont.)

In our example, we need to first calculate the Reynolds number: water at 35°C, Cp=4.18(kJ/kg.K), μ=7x10^{-4} (N.s/m^2), k_f=0.626 (W/m.K), Pr=4.8.

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{\frac{\dot{m}}{A} D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.15)}{\pi (0.05)(7 \times 10^{-4})} = 5460$$

Re > 4000, it is turbulent pipe flow.

Use the Gnielinski correlation, from the Moody chart, f = 0.036, Pr = 4.8

$$\mathrm{Nu}_D = \frac{(f/8)(\mathrm{Re}_D - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)} = \frac{(0.036/8)(5460-1000)(4.8)}{1 + 12.7(0.036/8)^{1/2}(4.8^{2/3} - 1)} = 37.4$$

$$h = \frac{k_f}{D} Nu_D = \frac{0.626}{0.05}(37.4) = 469(W/m^2.K)$$

Energy Balance

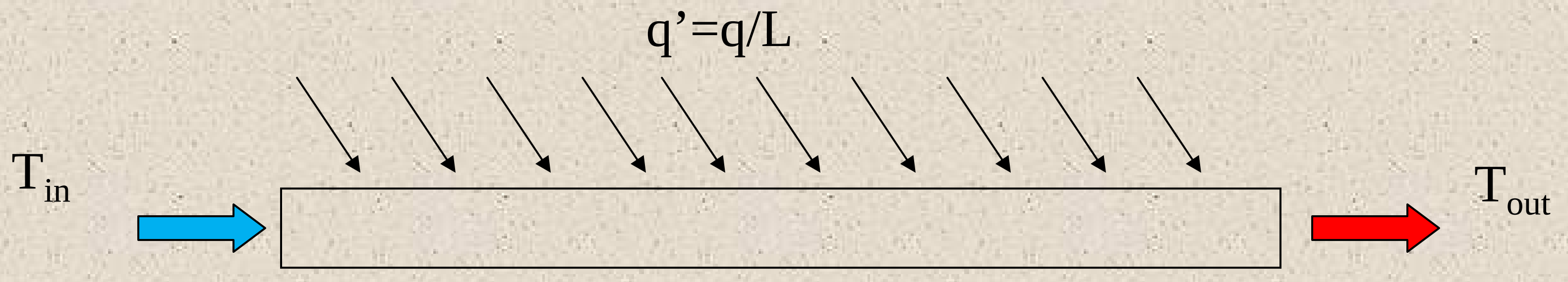
Question (2): How can we determine the required pipe length?

Use energy balance concept: (energy storage) = (energy in) minus (energy out).

energy in = energy received during a steady state operation (assume no loss)

$$q'(L) = \dot{m} C_P (T_{out} - T_{in}),$$

$$L = \frac{\dot{m} C_P (T_{in} - T_{out})}{q'} = \frac{(0.15)(4180)(50-20)}{200} = 94(m)$$



Temperature Distribution

Question (3): Can we determine the water temperature variation along the pipe?

Recognize the fact that the energy balance equation is valid for any pipe length x:

$$q'(x) = \dot{m} C_P (T(x) - T_{in})$$

$$T(x) = T_{in} + \frac{q'}{\dot{m} C_P} x = 20 + \frac{200}{(0.15)(4180)} x = 20 + 0.319 x$$

It is a linear distribution along the pipe

Question (4): How about the surface temperature distribution?

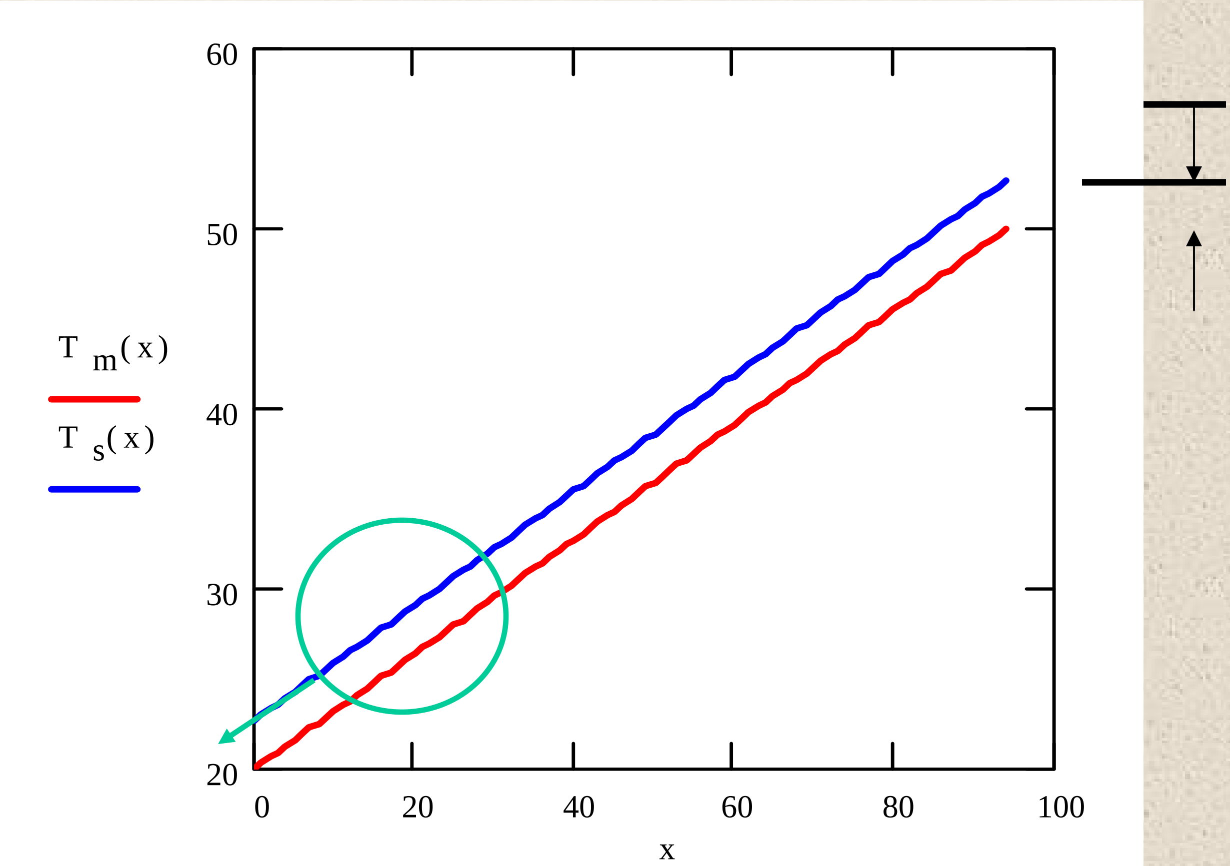
From local Newton's cooling law:

$$\mathrm{q} = \mathrm{hA}(\mathrm{T_s} - T_m) \Rightarrow q' \Delta x = h(\pi D \Delta x)(\mathrm{T_s}(x) - T_m(x))$$

$$T_s(x) = \frac{q'}{\pi D h} + T_m(x) = \frac{200}{\pi (0.05)(469)} + 20 + 0.319 x = 22.7 + 0.319 x \quad (°C)$$

At the end of the pipe, $\mathrm{T_s}(x = 94) = 52.7(°C)$

Temperature variation for constant heat flux

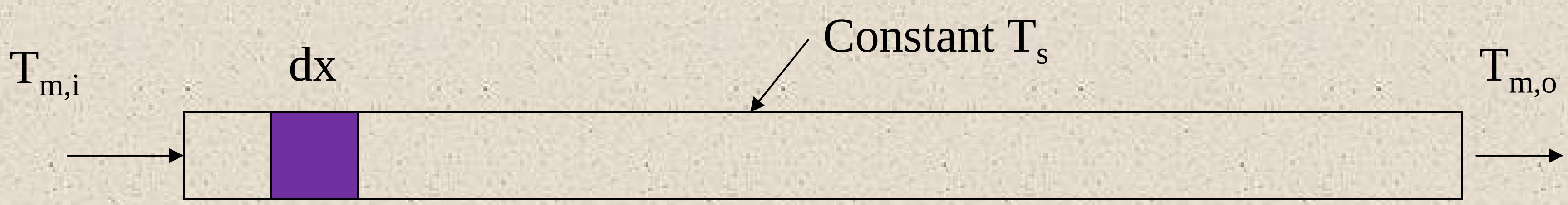


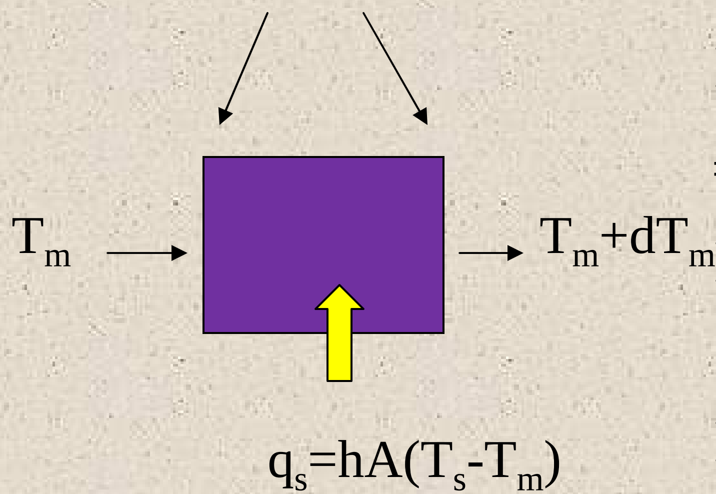
Constant temperature difference due to the constant heat flux.

Note: These distributions are valid only in the fully developed region. In the entrance region, the convection condition should be different. In general, the entrance length x/D≈10 for a turbulent pipe flow and is usually negligible as compared to the total pipe length.

Internal Flow Convection
-constant surface temperature case

Another commonly encountered internal convection condition is when the surface temperature of the pipe is a constant. The temperature distribution in this case is drastically different from that of a constant heat flux case. Consider the following pipe flow configuration:





Energy change $= \dot{m}C_p[(T_m + dT_m) - T_m]$

$= \dot{m}C_p dT_m$

Energy in $= hA(T_s - T_m)$

Energy change = energy in

$\dot{m}C_p dT_m = hA(T_s - T_m)$

Temperature distribution

$$\dot{m}C_p dT_m = hA(T_s - T_m),$$

Note: $q = hA(T_s - T_m)$ is valid locally only, since T_m is not a constant

$$\frac{dT_m}{(T_m - T_s)} = -\frac{hA}{\dot{m}C_P},$$ where A = Pdx, and P is the perimeter of the pipe

Integrate from the inlet to a diatance x downstream:

$$\int_{T_{m,i}}^{T_m(x)} \frac{dT_m}{(T_m - T_s)} = -\int_0^x \frac{hP}{\dot{m}C_P} dx = -\frac{P}{\dot{m}C_P}\int_0^x h dx$$

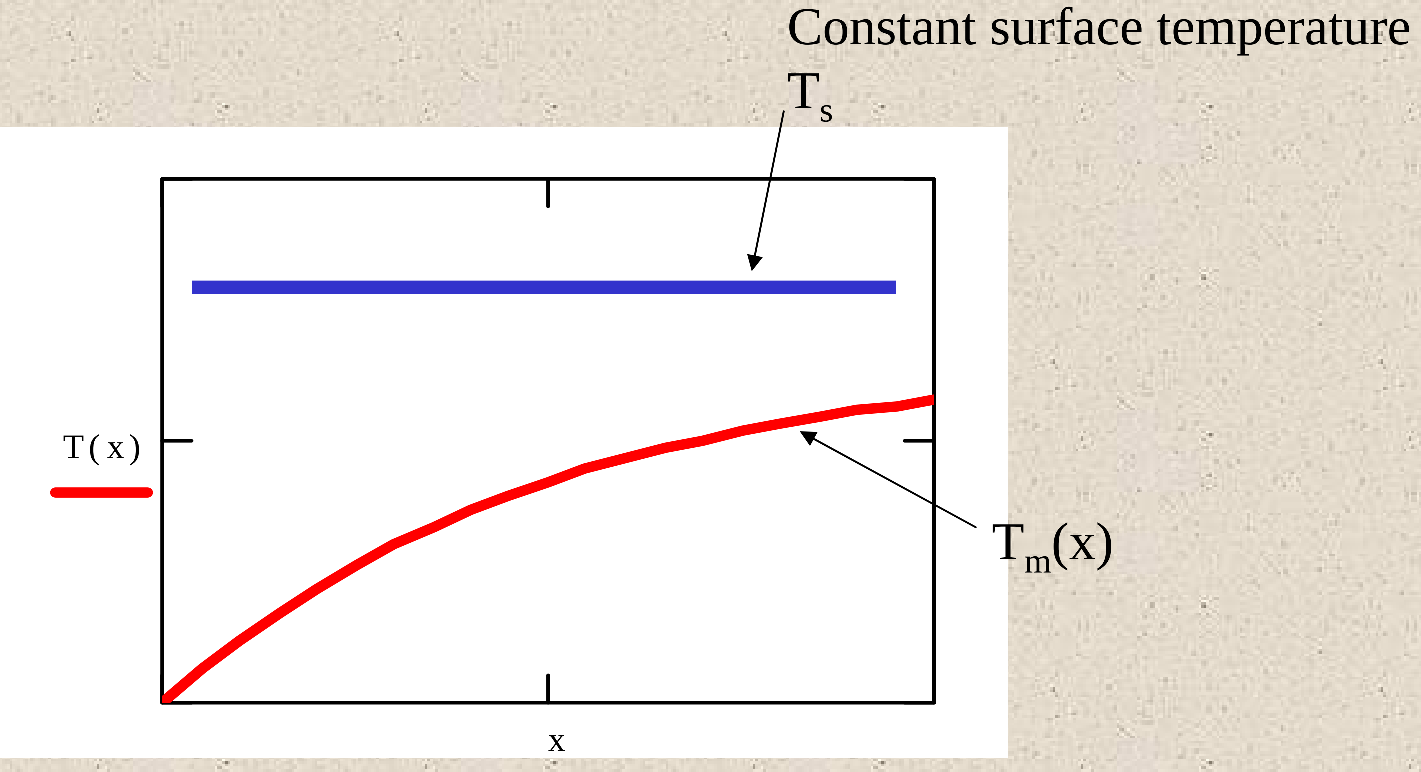
$$\ln(T_m - T_s)\Big|_{T_{m,i}}^{T_m(x)} = -\frac{P\bar{h}}{\dot{m}C_P}x,$$ where L is the total pipe length

and $\bar{h}$ is the averaged convection coefficient of the pipe between 0 & x.

$$\bar{h} = \frac{1}{x}\int_0^x h dx, \qquad \text{or } \int_0^x h dx = \bar{h}x$$

Temperature distribution

$$\frac{T_m(x) - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{P\bar{h}}{\dot{m} C_P} x\right), \quad \text{for constant surface temperature}$$



The difference between the averaged fluid temperature and the surface temperature decreases exponentially further downstream along the pipe.

Log-Mean Temperature Difference

For the entire pipe:

$$\frac{T_{m,o} - T_s}{T_{m,i} - T_s} = \frac{\Delta T_o}{\Delta T_i} = \exp\left(-\frac{\bar{h}(PL)}{\dot{m}C_P}\right) \quad \text{or } \dot{m}C_P = -\frac{\bar{h}A_s}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$q = \dot{m}C_P(T_{m,o} - T_{m,i}) = \dot{m}C_P((T_s - T_{m,i}) - (T_s - T_{m,o}))$$

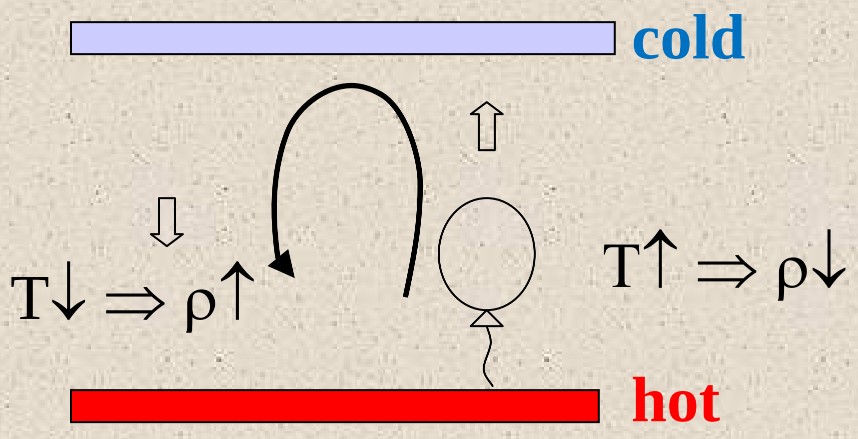
$$= \dot{m}C_P(\Delta T_i - \Delta T_o) = \bar{h}A_s \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)} = \bar{h}A_s \Delta T_{lm}$$

where $\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$ is called the log mean temperature difference.

This relation is valid for the entire pipe.

Free Convection

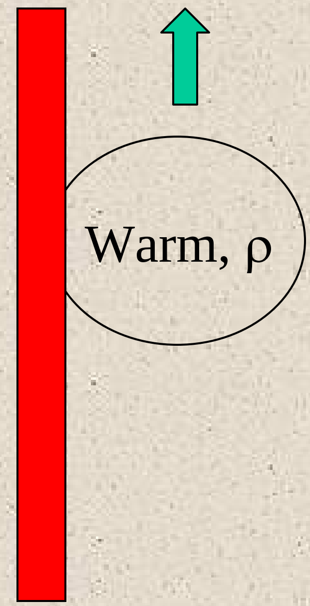
A free convection flow field is a self-sustained flow driven by the presence of a temperature gradient. (As opposed to a forced convection flow where external means are used to provide the flow.) As a result of the temperature difference, the density field is not uniform also. Buoyancy will induce a flow current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is usually much smaller compared to a forced convection heat transfer. It is therefore important only when there is no external flow exists.



Flow is unstable and a circulatory pattern will be induced.

Basic Definitions

Buoyancy effect:



Hot plate

Surrounding fluid, cold, ρ_∞

Net force=$(\rho_\infty$- $\rho)$gV

The density difference is due to the temperature difference and it can be characterized by ther volumetric thermal expansion coefficient, β:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \approx -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} = -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T}$$

$$\Delta\rho \approx \beta\Delta T$$

Grashof Number and Rayleigh Number

Define Grashof number, Gr, as the ratio between the buoyancy force and the viscous force:

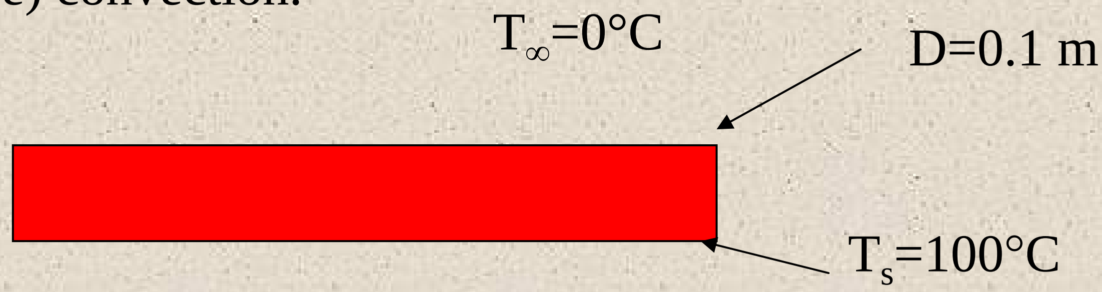
$$Gr = \frac{g\beta\Delta T L^3}{\nu^2} = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2}$$

• Grashof number replaces the Reynolds number in the convection correlation equation. In free convection, buoyancy driven flow sometimes dominates the flow inertia, therefore, the Nusselt number is a function of the Grashof number and the Prandtle number alone. Nu=f(Gr, Pr). Reynolds number will be important if there is an external flow. (combined forced and free convection.

• In many instances, it is better to combine the Grashof number and the Prandtle number to define a new parameter, the Rayleigh number, Ra=GrPr. The most important use of the Rayleigh number is to characterize the laminar to turbulence transition of a free convection boundary layer flow. For example, when $Ra>10^9$, the vertical free convection boundary layer flow over a flat plate becomes turbulent.

Example

Determine the rate of heat loss from a heated pipe as a result of natural (free) convection.



Film temperature(T_f): averaged boundary layer temperature $T_f=1/2(T_s+T_\infty)=50$ °C.
$k_f=0.03$ W/m.K, Pr=0.7, $\nu=2\times10^{-5}$ m^2/s, $\beta=1/T_f=1/(273+50)=0.0031(1/K)$

$$Ra=\frac{g\beta(T_S-T_\infty)L^3}{\nu^2}\Pr=\frac{(9.8)(0.0031)(100-0)(0.1)^3}{(2\times10^{-5})^2}(0.7)=7.6\times10^6.$$

$$Nu_D=\{0.6+\frac{0.387Ra^{1/6}}{[1+(0.559/\Pr)^{9/16}]^{8/27}}\}^2=26.0$$

$$h=\frac{k_f}{D}Nu_D=\frac{0.03}{0.1}(26)=7.8(W/m^2K)$$

$$q=hA(T_S-T_\infty)=(7.8)(\pi)(0.1)(1)(100-0)=244.9(W)$$

Can be significant if the pipe are long.