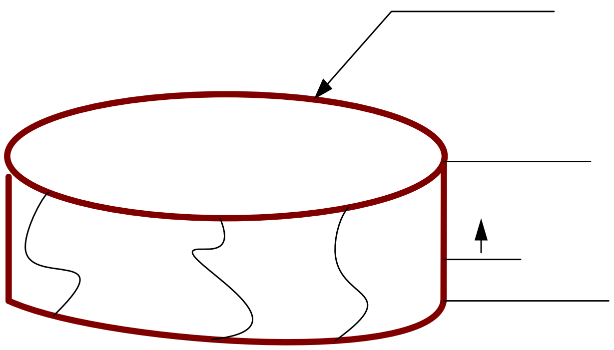
Module 5: Worked out problems

Problem 1:
A microwave oven operates on the principle that application of a high frequency field causes water molecules in food to resonate. This leads to a uniform generation of thermal energy within the food material. Consider heating of a food material by microwave, as shown in the figure below, from refrigeration temperatures to 90º in 30 s. Sketch temperature distributions at specific times during heating and cooling.

Known: Microwave and radiant heating conditions for a slab of beef.

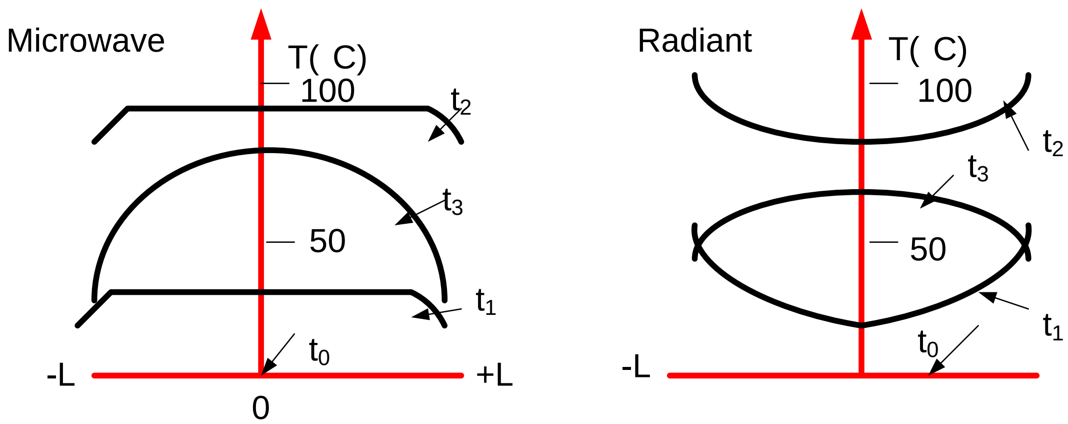
Find: Sketch temperature distributions at specific times during heating and cooling.

Schematic:



Assumptions: (1) one-dimensional conduction in x, (2) uniform internal heat generation for microwave, (3) uniform surface heating for radiant oven, (4) heat loss from surface of meat to surroundings is negligible during the heat process, (5) symmetry about mid plane.

Analysis:



Comments:

(1) With uniform generation and negligible surface heat loss, the temperature distribution remains nearly uniform during microwave heating. During the subsequent surface cooling, the maximum temperature is at the mid plane.

(2) The interior of the meat is heated by conduction from the hotter surfaces during radiant heating, and the lowest temperature is at the mid plane. The situation is reversed shortly after cooling begins, and the maximum temperature is at the mid plane.

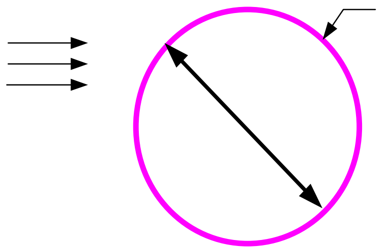
Problem 2:

The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature- time history of a sphere fabricated from pure copper. The sphere which is 12.7 mm in diameter is at 66º C before it is inserted into an air stream having a temperature of 27ºC. A thermocouple on the outer surface of the sphere indicates 55ºC, 69 s after the sphere is inserted into an air stream. Assume, and then justify, that the sphere behaves as a space-wise isothermal object and calculate the heat transfer coefficient.

Known: The temperature-time history of a pure copper sphere in air stream.

Find: The heat transfer coefficient between and the air stream

Schematic:



Assumptions: (1) temperature of sphere is spatially uniform, (2) negligible radiation exchange, (3) constant properties.

Properties: From table of properties, pure copper (333K): =8933 kg/m^3, c_p=389 J/kg.K, k=389W/m.K

Analysis: the time temperature history is given by

$$\frac{\theta(t)}{\theta_i} = \exp\left(-\frac{t}{R_t C_t}\right)$$

Where $R_t = \dfrac{1}{hA_s}$ $\quad A_s = \pi D^2$

$C_t = \rho V c_p$ $\quad V = \dfrac{\pi D^3}{6}$

$\theta = T - T_\infty$

T_∞=27°C

Recognize that when t = 69 s

$$\frac{\theta(t)}{\theta_i} = \frac{(55-27)^\circ C}{(66-27)^\circ C} = 0.718 = \exp\left(-\frac{t}{\tau_t}\right) = \exp\left(-\frac{69s}{\tau_t}\right)$$

And noting that $\tau_t = R_t C_t$ find

$\tau_t = 208s$

Hence,

$$h = \frac{\rho V c_p}{A_s \tau_t} = \frac{8933 kg/m^3 (\pi 0.0127^3 m^3 / 6) 389 J/kg.K}{\pi 0.0127^2 m^2 \times 208 s}$$

$$h = 35.3\, W/m^2.K$$

Comments: Note that with $L_c = D_0 / 6$

$$Bi = \frac{hL_c}{k} = 35.3 W/m^2.K \times \frac{0.0127}{6} m / 398\, W/m.K = 1.88 \times 10^{-4}$$

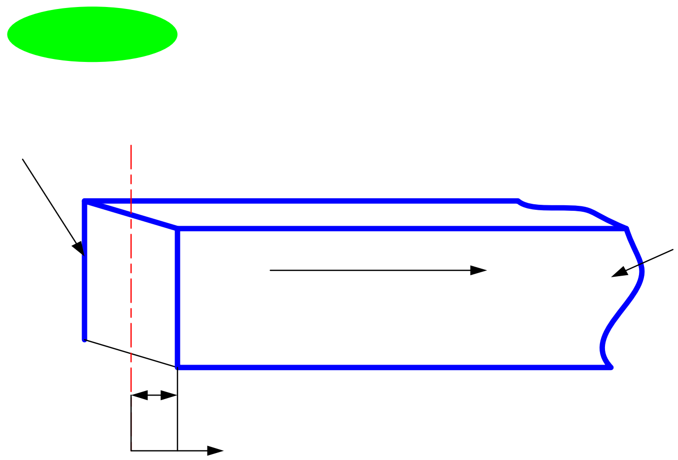
Hence Bi<0.1 and the spatially isothermal assumption is reasonable.

Problem 3:
A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and enclosed alternating layers of the storage material and the flow passage. Each layer of the storage material is aluminium slab of width=0.05m which is at an initial temperatures of 25ºC. consider the conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and convection coefficient assumed to have constant values of T=600ºC and h=100W/m^2.K throughout the channel how long will it take to achieve 75% of the maximum possible energy storage? What is the temperature of the aluminium at this time?

Known: Configuration, initial temperature and charging conditions of a thermal energy storage unit.

Find: Time required achieving 75% of maximum possible energy storage. Temperature of storage medium at this time.

Schematic:



Assumptions: (1) one-dimensional conduction, (2) constant properties, (3) negligible heat exchange with surroundings.

Properties: From any table of properties: Aluminum, pure (T 600K=327C): k=231W/m.K, c= 1033 J/kg.K, =2702kg/m3.

Analysis: recognizing the characteristic length is the half thickness, find

$$Bi = \frac{hL}{k} = \frac{100\,W/m^2.K \times 0.025m}{231\,W/m.K} = 0.011$$

Hot gas

Hence, the lumped capacitance method may be used.

$$Q = (\rho Vc)\theta_i[1-\exp(-t/\tau_i)] = -\Delta E_{st}$$
$$-\Delta E_{st,\max} = (\rho Vc)\theta_i$$

Dividing eq. (1) and (2), the condition sought is for

$\Delta E_{st} / \Delta E_{st,\max} = 1 - \exp(-t / \tau_{th}) = 0.75$

Solving for τ_{th} and substituting numerical values, find

$$\tau_{th} = \frac{\rho Vc}{hA_s} = \frac{\rho Lc}{h} = \frac{2702kg / m^3 \times 0.025m \times 1033J / kg.K}{100W / m^2.K} = 698s$$

Hence, the time required is

-exp (-t/698s) =-0.25 or t=968s.

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-t / \tau_{th})$$

$$T = T_\infty + (T_i - T_\infty)\exp(-t / \tau_{th}) = 600°C - (575°C)\exp(-968 / 698)$$

T=456°C

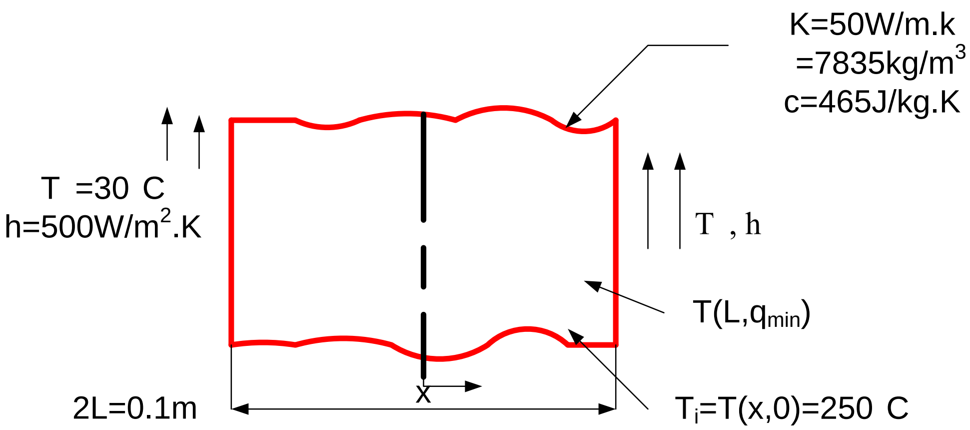
Comments: for the prescribed temperatures, the property temperatures dependence is significant and some error is incurred by assuming constant properties. However, selecting at 600K was reasonable for this estimate.

Problem 4:
A one-dimensional plane wall with a thickness of 0.1 m initially at a uniform temperature of 250C is suddenly immersed in an oil bath at 30°C. assuming the convection heat transfer coefficient for the wall in the bath is 500 W/m^2.K. Calculate the surface temperature of the wall 9 min after immersion. The properties of the wall are k=50 W/m.K, ρ=7835 kg/m^3, and c=465 J/kg.K.

Known: plane wall, initially at a uniform temperature, is suddenly immersed in an oil bath and subjected to a convection cooling process.

Find: Surface temperature of the wall nine minutes after immersion, T (L, 9 min).

Schematic:



Assumptions: The Biot number for the plane wall is

$$Bi = \frac{hL_c}{k} = \frac{500\,W/m^2.K \times 0.05\,m}{50\,W/m.K} = 0.50$$

Since Bi>0.1, lumped capacitance analysis is not appropriate.

$$Fo = \frac{\alpha t}{L^2} = \frac{(k/\rho c)_t}{L^2} = \frac{50W/m.K/7835\,kg/m^3 \times 465\,J/kg.K \times (9\times 60)s}{(0.05m)^2} = 2.96$$

And Bi^{-1}=1/0.50 = 2, find

$$\frac{\theta_0}{\theta_i} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} \approx 0.3$$

We know that Bi^{-1}=1/0.50 ≈ 2 and for X/L=1, find

$$\frac{\theta(1,t)}{\theta_0} \approx 0.8$$

By combining equation, $\theta(1,t)$=0.8(θ_0) = 0.8(0.3 θ_i) =0.24 θ_i

Recalling that $\theta = T(L,t) - T_\infty$ and $\theta_i = T_i - T_\infty$, it follows that

$$T(L,t) = T_\infty + 0.24\,(T_i - T_\infty) = 30°C + 0.24\,(250-30)°C = 83°C$$

Comments: (1) note that figure provides a relationship between the temperature at any x/L and the centerline temperature as a function of only the Biot number. Fig applies to the centerline temperature which is a function of the Biot number and the Fourier number. The centerline temperature at t=9min follows from equation with

$$T(0,t) - T_\infty = 0.3(T_i - T_\infty) = 0.3(250-30)°C = 66°C$$

(2) Since F_0>=0.2, the approximate analytical solution for θ^* is valid. From table with Bi=0.50, and ζ_1 =0.6533 rad and C_1=1.0701. Substituting numerical values into equations

θ^*=0.303 and $\theta^*(1, F_O)$ =0.240

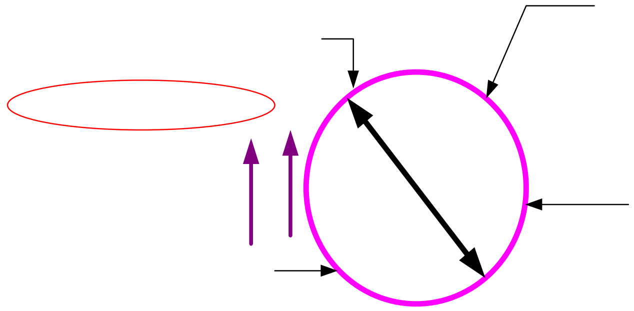
From this value, find T (L, 9 min) =83°C which is identical to graphical result.

Problem 5:
A long cylinder of 30mm diameter, initially at a uniform temperature of 1000K, is suddenly quenched in a large, constant-temperature oil bath at 350K. The cylinder properties are k=1.7W/m.K, c=1600 J/kg.K, and ρ=400 kg/m^3, while the convection coefficient is 50W/m^2.K. Calculate the time required for the surface cylinder to reach 500K.

Known: A long cylinder, initially at a uniform temperature, is suddenly quenched in large oil bath.

Find: time required for the surface to reach 500K.

Schematic:



Assumptions: (1) one dimensional radial conduction, (2) constant properties

Analysis: check whether lumped capacitance methods are applicable.

$$BI_c = \frac{hL_c}{k} = \frac{h(r_0/2)}{k} = \frac{50W/m^2.K(0.015m/2)}{1.7W/m.K} = 0.221$$

Since BI_c>0.1, method is not suited. Using the approximate series solutions for the infinite cylinder,

$$\theta^*(r^*, Fo) = C_1 \exp(-\varsigma_1^2 Fo) \times J_o(\varsigma_1^2 r^*)$$

D=30mm

Solving for F_o and letting =1, find

Bath

$$F_o = -\frac{1}{\varsigma_1^2} \ln\left[\frac{\theta^*}{C_1 J_o(\varsigma_1^2)}\right]$$

where $\theta^*(1, F_0) = \frac{T(r_o, t_{o)} - T\infty}{T_i - T_\infty} = \frac{(500-350)K}{(1000-350)K} = 0.231$

From table, Bi=0.441, find ς_1 =0.8882 rad and C_1=1.1019. From table find J_o (ς_1^2) =0.8121.
Substituting numerical values into equation,

T_∞=350K

h=50W/m^2.K

$T(r_o,t)$=500K

$$F_o = -\frac{1}{(0.8882)^2} \ln[0.231/1.1019 \times 0.8121] = 1.72$$

From the definition of the Fourier number, $F_o = \frac{\alpha t}{r_o^2} = F_o . r_o^2 \frac{\rho c}{k}$

$$t = 1.72\ (0.015m)^2 \times 400\ kg/m^3 \times 1600\ J/kg.K / 1.7W/m.K = 145\ s$$

Comments: (1) Note that F_o>=0.2, so approximate series solution is appropriate.

(2) Using the Heisler chart, find F_o as follows. With Bi^{-1}=2.27, find from fir r/r_o=1 that

$$\frac{\theta(r_o,t)}{\theta_o} = \frac{T(r_o,t)-T_\infty}{T(0,t)0-T_\infty} \approx 0.8 \quad \text{or} \qquad T(0,t) = T_\infty + \frac{1}{0.8}[T(r_o,t)-T_\infty] = 537K$$

hence $\frac{\theta_o}{\theta_i} = \frac{(537-350)K}{(1000-350)K} = 0.29$

From fig, with $\frac{\theta_o}{\theta_i}$=0.29 and Bi^{-1}=2.27, find Fo $\approx$ 1.7 and eventually obtain t $\approx$ 144s.

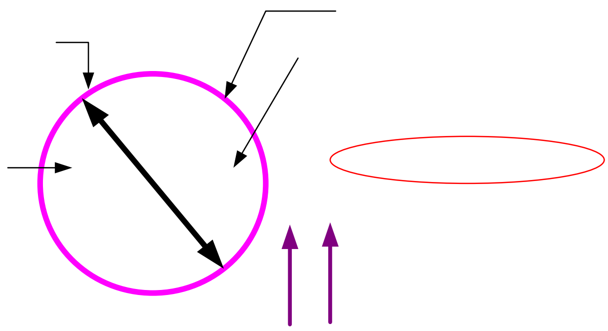
Problem 6:

In heat treating to harden steel ball bearings (c=500 J/kg.K, ρ=7800 kg/m^3, k=50 W/m.K) it is desirable to increase the surface temperature for a short time without significantly warming the interior of the ball. This type of heating can be accomplished by sudden immersion of the ball in a molten salt bath with T_∞=1300 K and h= 5000 W/m^2.K. Assume that any location within the ball whose temperature exceeds 1000 K will be hardened. Estimate the time required to harden the outer millimeter of a ball of diameter 20 mm if its initial temperature is 300 K.

Known: A ball bearing is suddenly immersed in a molten salt bath; heat treatment to harden occurs at locations with T>1000K.

Find: time required to harden outer layer of 1mm.

Schematic:



Assumptions: (1) one-dimensional radial conduction, (2) constant properties, (3) Fo≥0.2.

Analysis: since any location within the ball whose temperature exceeds 1000K will be hardened, the problem is to find the time when the location r=9mm reaches 1000K. Then a 1mm outer layer is hardened. Using the approximate series solution, begin by finding the Biot number.

$$Bi = \frac{hr_o}{k} = \frac{5000\, W/m^2.K(0.020m/2)}{50\, W/m.K} = 1.00$$

Using the appropriate solution form for a sphere solved for F_o , find

$$F_o = -\frac{1}{\varsigma_1^2}\ln\left[\theta^* / C_1 \frac{1}{\varsigma_1 r^*}\sin(\varsigma_1 r^*)\right]$$

D=20mm

ρ=7800kg/m^3

From table, with Bi=1.00, for the sphere find ς_1=1.5708 rad and C_1 =1.2732. with r^* =r/r_o= (9mm/10mm)=0.9, substitute numerical values.

$$F_o = -\frac{1}{(1.5708)^2} \ln\left[\frac{(1000-1300)K}{(300-1300)K} / 1.2732 \frac{1}{1.5708 \times 0.9} \sin(1.5708 \times 0.9 rad) \right] = 0.441$$

From the definition of the Fourier number with α=k/ρc,

$$t = F_o \frac{r^2{}_o}{\alpha} = F_o . r^2 \frac{\rho c}{k} = 0.441 \times \left(\frac{0.020}{2} \right)^2 7800 \frac{kg}{m^3} \times 500 \frac{J}{kg.K} / 50 W / m.K = 3.4 s$$

Comments: (1) note the very short time required to harden the ball. At this time it can be easily shown the center temperature is T(0,3.4s)=871K.

(2) The Heisler charts can also be used. From fig, with Bi-1=1.0 and r/r0=0.9, read θ/θ_o =0.69(±0.03). since

$$\theta = T - T_\infty = 1000 - 1300 = -300K \qquad \theta_i = T_i - T_\infty = -1000K$$

It follows that

$$\frac{\theta}{\theta_i} = 0.30 \qquad \text{since} \frac{\theta}{\theta_i} = \frac{\theta}{\theta_o} . \frac{\theta_o}{\theta_i} \qquad \text{then} \quad \frac{\theta}{\theta_i} = 0.69 \frac{\theta_o}{\theta_i},$$

And then $\frac{\theta_o}{\theta_i} = \frac{0.30}{0.69} = 0.43(\pm 0.02)$

From fig at $\frac{\theta_o}{\theta_i}$ =0.43, Bi^{-1}=1.0, read F_O=0.45(± 0.3) and t=3.5 (± 0.2) s.

Note the use of tolerances assigned as acceptable numbers dependent upon reading the charts to ±5%.

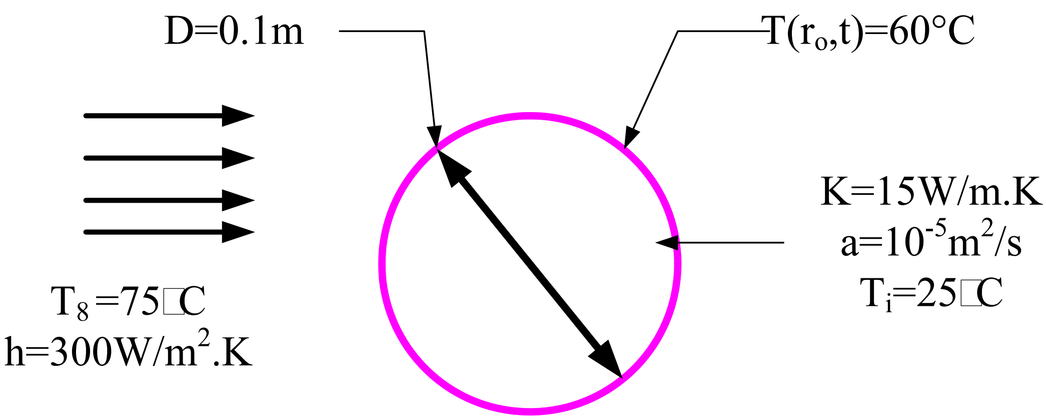
Problem 7:

The convection coefficient for flow over a solid sphere may be determined by submerging the sphere, which is initially at 25°C, into the flow, which is at 75°C and measuring its surface temperature at some time during the transient heating process. The sphere has a diameter of 0.1m, and its thermal conductivity and thermal diffusivity are 15 W/m.K and 10^{-5} m^2/s, respectively. If the convection coefficient is 300W/m^2.K, at what time will a surface temperature of 60°C be recorded?

Known: Initial temperatures and properties of solid sphere. Surface temperatures after immersion in a fluid of prescribed temperatures and convection coefficient.

Find: The process time

Schematic:



Assumptions: (1) one-dimensional, radial conduction, (2) constant properties.

Analysis: the Biot number is

$$Bi = \frac{h(r_0/3)}{k} = \frac{300W/m^2.K(0.05m/3)}{15W/m.K} = 0.333$$

Hence the lumped capacitance methods should be used. From equation

$$\frac{T - T_\infty}{T_i - T_\infty} = C_1 \exp(-\varsigma_1^2 F_o) \frac{\sin(\varsigma_1 r^*)}{\varsigma_1 r^*}$$

At the surface, r^*=1. from table , for Bi=1.0, ς_1=1.5708 rad and C_1=1.2732. hence,

$$\frac{60-75}{25-75} = 0.30 = 1.2732 \exp(-1.5708^2 F_o) \frac{\sin 90°}{1.5708}$$ +Exp (-2.467F_0) =0.370

$$F_o = \frac{\alpha t}{r_0^2} = 0.403 \frac{(0.05m)^2}{10^{-5} m^2 / s}$$

t=100s

Comments:

Use of this technique to determine h from measurement of T (r_o) at a prescribed t requires an iterative solution of the governing equations.