

Module 1: Worked out problems

Problem 1:

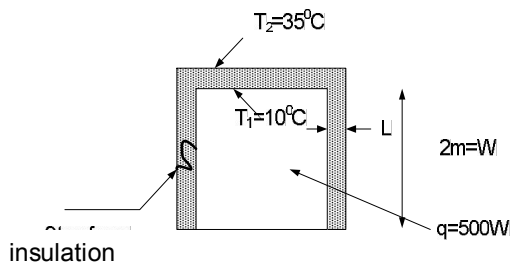
A cold storage consists of a cubical chamber of dimension 2m x 2m x 2m, maintained at 10°C inside temperature. The outside wall temperature is 35°C. The top and side walls are covered by a low conducting insulation with thermal conductivity $k = 0.06 \text{ W/mK}$. There is no heat loss from the bottom. If heat loss through the top and side walls is to be restricted to 500W, what is the minimum thickness of insulation required?

Solution:

Known: Dimensions of the cold storage, inner and outer surfaces temperatures, thermal conductivity of the insulation material.

To find: Thickness of insulation needed to maintain heat loss below 500W.

Schematic:



Assumptions: (1) perfectly insulated bottom, (2) one-dimensional conduction through five walls of areas $A = 4\text{m}^2$, (3) steady-state conditions

Analysis: Using Fourier's law, the heat rate is given by

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{\text{total}}$$

Solving for L and recognizing that $A_{\text{total}} = 5 \cdot \text{W}^2$

$$L = \frac{5k\Delta TW^2}{q}$$

$$L = \frac{5 \cdot 0.06 \text{ W/m.K} \cdot 45^\circ \text{C} \cdot 4 \text{ m}^2}{500 \text{ W}}$$

$$L = 0.108 \text{ m} = 108 \text{ mm}$$

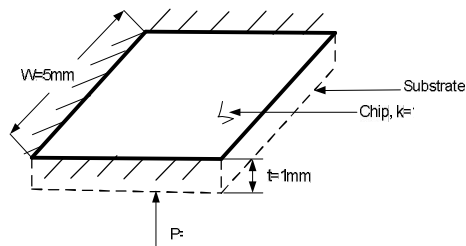
Problem 2:

A square silicon chip is of width $W=5\text{mm}$ on a side and of thickness $t=1\text{mm}$. The chip is mounted in a substrate such that there is no heat loss from its side and back surfaces. The top surface is exposed to a coolant. The thermal conductivity of the chip is 200W/m.K . If 5W are being dissipated by the chip, what is the temperature difference between its back and front surfaces?

Known: Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

Find: temperature drop across the chip

Schematic:



Assumptions: (1) steady-state conditions, (2) constant properties, (3) uniform dissipation, (4) negligible heat loss from back and sides, (5) one-dimensional conduction in chip.

Analysis: All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, Fourier's law,

$$P = q = kA \frac{\Delta T}{t}$$
$$\Delta T = \frac{t.P}{kW^2} = \frac{0.001\text{m} * 5\text{W}}{200\text{W/m.K} (0.005\text{m})^2}$$
$$\Delta T = 1.0^\circ\text{C}$$

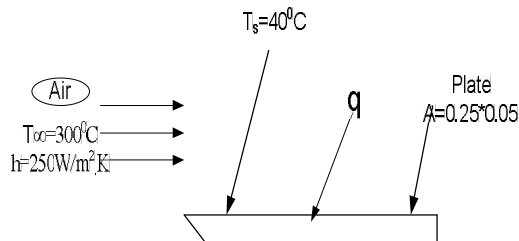
Comments: for fixed P , the temperature drop across the chip decreases with increasing k and W , as well as with decreasing t .

Problem 3:

Air flows over a rectangular plate having dimensions 0.5 m x 0.25 m. The free stream temperature of the air is 300°C. At steady state, the plate temperature is 40°C. If the convective heat transfer coefficient is 250 W/m².K, determine the heat transfer rate from the air to one side of the plate.

Known: air flow over a plate with prescribed air and surface temperature and convection heat transfer coefficient.

Find: heat transfer rate from the air to the plate

Schematic:

Assumptions: (1) temperature is uniform over plate area, (2) heat transfer coefficient is uniform over plate area

Analysis: the heat transfer coefficient rate by convection from the airstreams to the plate can be determined from Newton's law of cooling written in the form,

$$q = q'' \cdot A = hA(T_{\infty} - T_s)$$

where A is the area of the plate. Substituting numerical values,

$$q = 250\text{W/m}^2\text{K} * (0.25 * 0.50)\text{m}^2 (300 - 40)^{\circ}\text{C}$$

$$q = 8125\text{W}$$

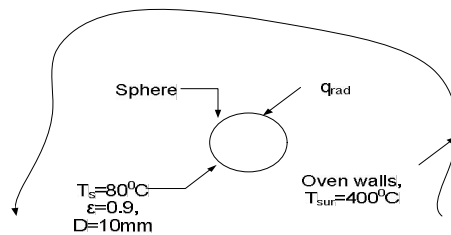
Comments: recognize that Newton's law of cooling implies a direction for the convection heat transfer rate. Written in the form above, the heat rate is from the air to plate.

Problem 4 :

A sphere of diameter 10 mm and emissivity 0.9 is maintained at 80°C inside an oven with a wall temperature of 400°C. What is the net transfer rate from the oven walls to the object?

Known: spherical object maintained at a prescribed temperature within a oven.

Find: heat transfer rate from the oven walls to the object

Schematic:

Assumptions: (1) oven walls completely surround spherical object, (2) steady-state condition, (3) uniform temperature for areas of sphere and oven walls, (4) oven enclosure is evacuated and large compared to sphere.

Analysis: heat transfer rate will be only due to the radiation mode. The rate equation is

$$q_{\text{rad}} = \varepsilon A_s \sigma (T_{\text{sur}}^4 - T_s^4)$$

Where $A_s = \pi D^2$, the area of the sphere

$$q_{\text{rad}} = 0.9 * \pi (10 * 10^{-3})^2 \text{ m}^2 * 5.67 * 10^{-8} \text{ W / m}^2 \cdot \text{K} [(400 + 273)^4 - (80 + 273)^4] \text{ K}^4$$

$$q_{\text{rad}} = 3.04 \text{ W}$$

Discussions:

(1) this rate equation is applicable when we are calculating the net heat exchange between a small object and larger surface that completely surrounds the smaller one.

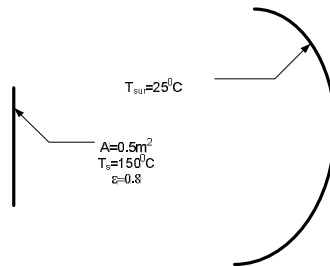
(2) When performing radiant heat transfer calculations, it is always necessary to have temperatures in Kelvin (K) units.

Problem 5:

A surface of area 0.5m^2 , emissivity 0.8 and temperature 150°C is placed in a large, evacuated chamber whose walls are maintained at 25°C . Find the rate at which radiation is emitted by the surface? What is the net rate of radiation exchange between the surface and the chamber walls?

Known: Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

Find: (a) rate of surface radiation emission, (b) net rate of radiation exchange between the surface and chamber walls.

Schematic:

Assumptions: (1) area of the enclosed surface is much less than that of chamber walls.

Analysis (a) the rate at which radiation is emitted by the surface is emitted

$$q_{\text{emit}} = q_{\text{emit}} \cdot A = \varepsilon A \sigma T_s^4$$

$$q_{\text{emit}} = 0.8(0.5\text{m}^2)5.67 \cdot 10^{-8} \text{ W / m}^2 \cdot \text{K}^4 [(150 + 273)\text{K}]^4$$

$$q_{\text{emit}} = 726\text{W}$$

(b) The net rate at which radiation is transferred from the surface to the chamber walls is

$$q = \varepsilon A \sigma (T_s^4 - T_{\text{surr}}^4)$$

$$q = 0.8(0.5\text{m}^2)5.67 \cdot 10^{-8} \text{ W / m}^2 \cdot \text{K}^4 [(423\text{K})^4 - (298\text{K})^4]$$

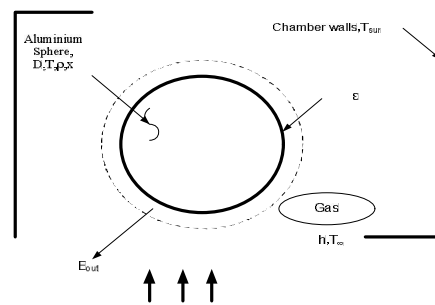
$$q = 547\text{W}$$

Problem 6:

A solid aluminium sphere of emissivity ϵ , initially at a high temperature, is cooled by convection and radiation in a chamber having walls at a lower temperature. Convective cooling is achieved with a gas passing through the chamber. Write a differential equation to predict the variation of sphere temperature with time during the cooling process.

Known: Initial temperature, diameter and surface emissivity of a solid aluminium sphere placed in a chamber whose walls are maintained at lower temperature. Temperature and convection coefficient associated with gas flow over the sphere.

Find: equation which could be used to determine the aluminium temperature as a function of time during the cooling process.

Schematic:

Assumptions: (1) at any time t , the temperature T of the sphere is uniform, (2) constant properties; (3) chamber walls are large relative to sphere.

Analysis: applying an energy balance at an instant of time to a control volume about the sphere, it follows that

$$\dot{E}_{st} = -\dot{E}_{out}$$

Identifying the heat rates out of the CV due to convection and radiation, the energy balance has the form

$$\frac{d}{dt}(\rho V c T) = -(q_{conv} + q_{rad})$$

$$\frac{dT}{dt} = -\frac{A}{\rho V c} [h(T - T_{\infty}) + \epsilon \sigma (T^4 - T_{surr}^4)]$$

$$\frac{dT}{dt} = -\frac{6}{\rho c D} [h(T - T_{\infty}) + \epsilon \sigma (T^4 - T_{surr}^4)]$$

Where $A = \pi D^2$, $V = \pi D^3/6$ and $A/V = 6/D$ for the sphere.

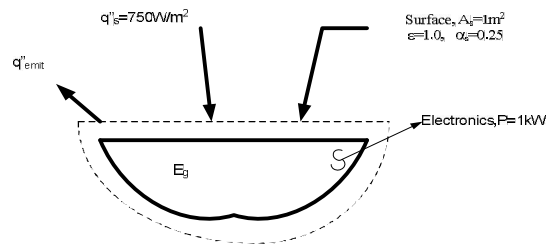
Problem 7: An electronic package dissipating 1 kW has a surface area 1m^2 . The package is mounted on a spacecraft, such that the heat generated is transferred from the exposed surface by radiation into space. The surface emissivity of the package is 1.0. Calculate the steady state temperature of the package surface for the following two conditions:

- the surface is not exposed to the sun
- The surface is exposed to a solar flux of $750\text{W}/\text{m}^2$ and its absorptivity to solar radiation is 0.25?

Known: surface area of electronic package and power dissipation by the electronics. Surface emissivity and absorptivity to solar radiation. Solar flux.

Find: surface temperature without and with incident solar radiation.

Schematic:



Assumptions: steady state condition

Analysis: applying conservation of energy to a control surface about the compartment, at any instant

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = 0$$

It follows that, with the solar input,

$$\alpha_s A_s q''_s - A_s q''_{\text{emit}} + P = 0$$

$$\alpha_s A_s q''_s - A_s \epsilon \sigma T_s^4 + P = 0$$

$$T_s = \left(\frac{\alpha_s A_s q''_s + P}{A_s \epsilon \sigma} \right)^{\frac{1}{4}}$$

In the shade ($q''_s = 0$)

$$T_s = \left(\frac{1000\text{W}}{1\text{m}^2 * 1 * 5.67 * 10^{-8} \text{W} / \text{m}^2 \cdot \text{K}^4} \right)^{\frac{1}{4}} = 364\text{K}$$

In the sun,

$$T_s = \left(\frac{0.25 * 1\text{m}^2 * 750\text{W} / \text{m}^2 + 1000\text{W}}{1\text{m}^2 * 1 * 5.67 * 10^{-8} \text{W} / \text{m}^2 \cdot \text{K}^4} \right)^{\frac{1}{4}} = 380\text{K}$$