

Exercises for Module 4

of

Micro and Smart Systems NPTEL Course

(Some of the following problems are taken from instructors' book entitled "Micro and Smart Systems", John Wiley, 2012.)

4.1 State the process involved in the MEMS device fabrication and the role of finite element method in the MEMS design Process.

4.2 What are the differences between the mathematical model and numerical model? Can there be different mathematical and numerical model to a given problem?

4.3 Numerically integrate $\int_{-1}^1 \frac{\sin x}{x} dx$. How many segments are needed to get an accurate solution.

4.4 Why is the design rule check is critical for MEMS design process?

4.5 What do you understand by multi-physics problem? Outline the philosophy of coupling different domains in FEM.

4.6 State the advantages of discretizing the domain into smaller domain for problem solving?

4.7 What do you understand by essential (Dirichlet) boundary conditions and Natural (Neumann) boundary conditions in the context of finite element analysis?

4.8 The Strong form of a differential equation of a certain beam of Young's Modulus E , moment of inertia I and undergoing transverse motion $w(x)$ is given by

$$EI \frac{d^4 w}{dx^4} + \lambda w + q = 0$$

In the above equation λ is some known constant. Derive the weak form of the above equation.

4.9 Solve the following differential equation using the method of moments.

$$x^2 \frac{d^2 y}{dx^2} = 1$$

4.10 The force displacement relation for a given 1-D solid is given by

$$F = ku^{3/5}$$

where F is the force, u is the displacement and k is some known constant. Determine the work and complimentary work of the solid.

4.11 Does the following differential equation have a functional? If so determine the functional form of this differential equation

$$\frac{d^2}{dx^2} \left(x^2 \frac{d^2 y}{dx^2} \right) + 6x + 1 = 0$$

4.12 Derive the strain energy expression for a tapered rod of length L undergoing axial deformation $u(x)$ and having depth varying from h_0 at the left end. The beam depth varies as a function of axial coordinate $h(x)=h_0(1+x/L)$ and is made of a material having Young's modulus Y .

4.13 What is the difference between C_0 and C_1 continuous elements? If a four noded element can support three degrees of freedom per node, namely, $w, dw/dx, dw/dy$, respectively. Is the element C_0 or C_1 continuous element?

4.14 It is necessary to formulate a three-noded bar with its nodes at the ends and the midpoint. Assume, each node can carry one degree of freedom, namely, the axial deformation. Assume a suitable polynomial that can be used to formulate this element and also compute its shape functions.

4.15 The voltage field in a two-noded piezoelectric element is given by

$$v(x) = Ae^{-ikx} + Be^{-ik(L-x)}$$

where k is some known constant, L the length of the piezoelectric element, and i a complex number = $\sqrt{-1}$. Determine the shape functions for the voltage field.

4.16 What are area coordinates? Why are area coordinates preferred for triangular elements? Write the shape functions of a 6-noded triangle in terms of area coordinates.

4.17 What is iso-parametric finite element formulation? How is it different from sub-parametric and super-parametric formulations?

4.18 Can the Jacobian be negative? If it is negative, what are its implications?

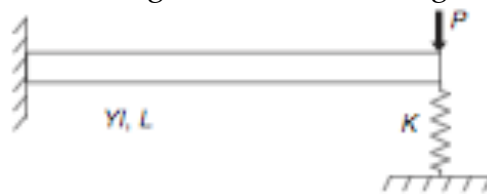
4.19 A quadrilateral element has the following coordinates for the four nodes: (0,0), (10,2), (15,6) and (-5,4). Using four-noded isoparametric shape functions, determine the value of the Jacobian.

4.20 Numerically integrate the integral given below using Gaussian integration procedure.

$$\int_{-1}^1 \frac{1 + \xi + 2\xi^2}{4 + 7\xi^2}$$

How many integration points are required to integrate and why?

4.21 A spring-supported beam of length L and flexural rigidity YI is shown below.



(a) State the essential and natural boundary conditions for this problem.

(b) Solve this problem using a single-beam element and determine the deflection at the midpoint and at the spring location of the beam (beam tip).

4.22 The sensing and actuation constitutive laws are assumed uncoupled? Under what circumstances, these laws get coupled?

4.23 State the utility of sensing and actuation constitutive laws in smart applications.

4.24 Derive the expression for strain energy stored in an piezoelectric Bimorph cantilevered beam of length L and area of rectangular cross section A . The thickness of the piezoelectric patch is t . The Young's modulus of beam is Y and the Young's modulus and the electro-mechanical coupling coefficient of the piezo patch are Y_m and d_{31} , respectively. The bimorph beam is poled such that the beam produces only axial strain, that is the strain along the axis of the beam.