MODULE IX

NAVIER STOKES EQUATIONS: ASSIGNMENT

IX.1. In numerical solution of Navier-Stokes equations for an incompressible flow
- a) there is no separate equation for pressure.
- b) pressure is computed from equation of state.
- c) equation for pressure is derived by combining continuity and momentum equations.
- d) solution of pressure is not required.

IX.2. Numerical approximation of Navier-Stokes equations on staggered grid
- a) provides strong coupling between velocity and pressure.
- b) is preferred over collocated grid.
- c) leads to oscillations in pressure.
- d) should be avoided.

IX.3 Consider the numerical solution of incompressible Navier-Stokes equations. Semi-discretized (i.e. discrete in space) momentum equation can be written as

$$\frac{\partial(\rho u_i)}{\partial t} = H_i - \frac{\delta p}{\delta x_i}$$

where $\delta() / \delta x_i$ represents a discretized spatial derivative, and H_i is a short-hand notation for the advective and diffusive terms. Explicit methods are very popular for time integration, especially for high Re flows. Usually, the calculation at the first step (t = 0) is started using explicit Euler method whereas from second time step onwards, second order Adams-Bashforth method is used for time advancement. Note that for the initial value problem

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = f(\phi, t); \quad \phi(t_0) = \phi_0$$

second order Adams-Bashforth method is given by

$$\phi^{n+1} = \phi^n + \frac{\Delta t}{2}\,[3f(\phi^n, t_n) - f(\phi^{n-1}, t_{n-1})].$$

(a) Derive the equations for time advancement of the velocity field at the first (explicit Euler) as well as subsequent time steps (Adams-Bashforth).
(b) Derive the discrete Poisson equation(s) for pressure for this composite algorithm.
(c) Briefly outline the steps involved in this solution algorithm.
(d) Can a code based on the above algorithm be used for numerical simulation of turbulent flows using (i) DNS and (ii) LES? If no, what modifications/additions would be required?

IX.4 Why is a staggered grid preferred in finite difference or finite volume solution of Navier-Stokes equations?

IX.5 Why we need to derive a Poisson equation for pressure in numerical simulation of incompressible flows? Derive this equation in general context.

IX.6 Provide an outline of (a) an explicit and (b) an implicit Navier-Stokes solvers for incompressible flows.

IX.7 Can you devise a solution strategy for solution of compressible flows using an explicit time integration scheme? Provide the algorithmic outline of your strategy.

IX.8 Write the general outline of SIMPLE type algorithms (SIMPLE, SIMPLER, SIMPLEC). Which of these three iterative algorithms has worst convergence behaviour and which is likely to exhibit the best convergence? Rank these algorithms in order of their computational expense (from the least expensive to the most expensive).