MODULE VI

SOLUTION OF ALGEBRAIC SYSTEMS: ASSIGNMENT

VI.1. TDMA can be used for solution of the algebraic system of equations arising from the finite difference discretization of
(a) one dimensional Laplace or Poisson's equation
(b) two dimensional Laplace or Poisson's equation
(c) three dimensional Laplace or Poisson's equation
(d) multi-dimensional heat conduction equation if ADI method is used.

VI.2. Consider the sparse linear algebraic system $\mathbf{Ax} = \mathbf{b}$ obtained from central difference discretization of three dimensional Poisson equation. Which of the following would be the most suitable solution method(s) when the number of grid points is ***very large***?
(a) one dimensional Laplace or Poisson's equation
(b) two dimensional Laplace or Poisson's equation
(c) three dimensional Laplace or Poisson's equation
(d) multi-dimensional heat conduction equation if ADI method is used.

VI.3. For a sparse linear algebraic system $\mathbf{Ax} = \mathbf{b}$, which iterative solver amongst the following would exhibit the fastest convergence?
(a) Jacobi's method.
(b) Gauss-Seidel method.
(c) SOR method.
(d) GMRES method.

VI.4. Which of the following iterative methods can be used for the solution of asymmetric linear systems?
(a) Bi-conjugate gradient method.
(b) Conjugate gradient method.
(c) SOR method.
(d) GMRES method.

VI.5. The most effective method for the solution of the sparse linear algebraic system $\mathbf{Ax} = \mathbf{b}$ obtained from central difference discretization of **one dimensional** Poisson equation is
(a) LU decomposition.
(b) Conjugate gradient method.
(c) Multigrid method.
(d) TDMA.

VI.6. The convergence of the preconditioned conjugate gradient iterations depends on the preconditioner. Which of the following statements is true with respect to convergence rate of PCG iterations while comparing Jacobi's (diagonal) preconditioner and incomplete Cholesky preconditioner?
(a) Diagonal preconditioner is better than incomplete Cholesky preconditioner.
(b) Incomplete Cholesky preconditioner is better than the diagonal preconditioner.
(c) Both provide the same rate of convergence.
(d) None of the above.

VI.7. Use of finite difference or finite volume method for the solution of generic transport equation for transport of a scalar φ yield a sparse linear system of equation given by

$$\mathbf{A\Phi} = \mathbf{Q}$$

where **A** is the system matrix, $\mathbf{\Phi}$ is the vector of nodal unknowns, and **Q** is the known load vector. Iterative solvers are usually preferred for solution this sparse system. Provide the basic equations (algorithms) for the following basic solvers

- (i) Jacobi Method
- (ii) Gauss-Seidel Method
- (iii) SOR method

Which of these three would exhibit the best convergence behavior? Would you use any of these for solution of very large (say, order of a million) system?

VI.8. Provide the definition of projection method for the solution of a linear system. What is a Krylov subspace method? What is general form of the approximate solution obtained from a Krylov subspace method? For which choice of the constraint subspace does this method reduce to conjugate gradient method, and GMRES. Write the algorithm for the preconditioned conjugate gradient (PCG) method. What are the conditions on the system matrix **A** for application of conjugate gradient method? What are the situations in which GMRES (or Bi-CG) must be used?

VI.9. Consider the finite difference (or finite volume) solution of 2-D steady state heat conduction problem governed by the PDE

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

using central difference scheme on a uniform Cartesian grid. Outline an iterative solution procedure for solving the resulting penta-diagonal system of equations using TDMA (tri-diagonal matrix algorithm). Note that such an algorithm is popularly referred as ADI or alternating sweep-direction method.

VI.10. Can you extend the ADI type method of the previous problem for finite difference solution of 3-D steady state heat conduction problem

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$