MODULE 5

FINITE ELEMENT METHOD: ASSIGNMENT

V.1 Outline the steps involved in numerical simulation of a problem using finite element method. Use Galerkin formulation to obtain the discretized equations for steady state heat conduction problem in a two-dimensional domain based on linear triangular elements.

V.2 Viscous incompressible flow between two infinitely broad parallel plates is governed by the differential equation

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} - \frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x} = 0$$

where u is the velocity component parallel to the plates (i.e. in x-direction), y is the direction perpendicular to the plates, μ is viscosity and ($\mathrm{d}p/\mathrm{d}x$) is the constant pressure gradient. Assume that the bottom plate is at y = 0, and top plate is at y = 2. Flow is symmetric with respect to the centerline of the channel formed by the plates. Hence, we need to obtain the solution only for the bottom half. Write the appropriate boundary conditions for the problem (i.e. at y = 0 and y = 1). Using polynomial trial functions and Galerkin finite method, obtain the solution of the flow problem. Consider only **two** trial functions.

V.3 Write a finite element code for the potential flow (or steady state heat conduction) problem. Use bilinear finite elements and Gaussian quadrature for evaluation of finite element integrals. Study the effect of order of Gaussian integration (i.e. choice of number of Gaussian points used in quadrature) on accuracy of numerical solution. Your code should be able to deal with different types of boundary conditions for a rectangular domain. Choose pre-conditioned conjugate gradient method for solution of discrete linear system in your code.

V.4 Consider fully developed laminar viscous flow between two large parallel walls. Starting from Navier-Stokes equations, derive simplified governing equations for this problem. Derive discretized equations and write a computer code to solve this problem using finite element method. Compare your results with analytical solution.

V.5 Consider one dimensional transient heat conduction governed by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 \phi}{\partial x^2}.$$

Derive discretized equations and write a computer code to solve this problem using finite element method. Your code should provide options for (a) linear as well as quadratic finite elements, and (b) different time integration methods. Choose a test case for which analytical solution is easily available in any heat transfer text book, and compare your results with analytical solution. Study the effect of element size, order of elements and time step on accuracy of the solution and stability of the solution process.

V.6 In a heat-treatment process, a very long slab of width 8 m has been heated to 400 °C. It is then cooled by bringing both of its sides in contact with a fluid bath at 0 °C. Assuming constant material properties, write down the governing equation, and initial and boundary conditions for the problem. Use linear finite elements for spatial discretization and two-level methods for time integration. Assume the diffusivity $\kappa = 1$ m^2/s. Choose $\Delta x = 1$ m and $\Delta t = 0.25$ s. Write a computer program to obtain the temperature distribution in the slab at time $t = 0.25$ s and $t = 0.5$ s using (a) forward Euler method, and (b) Crank-Nicolson method. Use TDMA for solution of linear system if required.

V.7 Consider the steady state heat conduction in a slab of width $l = 0.5$ m with heat generation. The left end of the slab ($x = 0$) is maintained at T = 373 K. The right end of the slab ($x = 0.5$ m) is being heated by a heater for which the heat flux is 1 kW/m^2. The heat generation in the slab is temperature dependent and is given by $Q = (1273 - T)$ W/m^3. Thermal conductivity is constant at $k = 1$ W/(m-K).Write down the governing equation and boundary conditions for the problem. Use the finite element method to obtain an approximate numerical solution of the problem. Choose $\Delta x = 0.1$, and **use the TDMA.**

V.8 Consider steady state heat conduction with constant heat generation in a slab of constant conductivity k = 1 bounded by planes x = 0 and x = 4. Both ends of the slab are kept at zero temperature. Write down the governing equation and boundary conditions for the problem. Obtain an approximate numerical solution of the problem using the finite element method Choose $\Delta x = 1$, and use the TDMA. For numerical solution, first do hand calculations and then write a computer program which can take different values of Δx. Compare results with analytical solution.

NOTE: Compare FEM solution of problems in this module to FDM solution obtained in Module III and FVM solution in Module IV.