MODULE IV

FINITE VOLUME METHOD: ASSIGNMENT

IV.1. The finite volume method normally uses the following form of conservation equation as the starting point:
a) differential equation in strong conservation form.
b) integral form of conservation law.
c) differential equation in non-conservation form.
d) any of the above (i.e. either of (a), (b) or (c)).

IV.2. In finite volume method, approximation of the surface integral: $F_e = \int_{S_e} f \, \mathrm{d}S \approx f_e S_e$
a) is first order accurate.
b) is second order accurate.
c) is fourth order accurate.
d) is third order accurate.

IV.3. The upwind difference scheme which approximates ϕ_e by the value at upstream node (i.e. $\phi_e \approx \phi_U$)
a) is numerically diffusive.
b) may yield oscillatory solution.
c) is first order accurate.
d) never yields oscillatory solution.

IV.4. In finite volume method, approximation of the volume integral, $Q_P = \int_\Omega q \, \mathrm{d}\Omega \approx q_P \Delta\Omega$ where q_P is the value of q at CV centre and $\Delta\Omega$ is the volume of the CV, is
a) in general, second order accurate.
b) in general, fourth order accurate.
c) exact if q is constant or varies linearly in CV.
d) in general, first order accurate.

IV.5. Outline the steps involved in numerical simulation of a problem using finite volume method.

IV.6. Define the upwind interpolation (UDS) for approximating the value of a variable at the east face of a control volume. Show that this scheme is first order accurate, and is numerically diffusive with a coefficient of numerical diffusion $\Gamma_e^{num} = (\rho u)_e \Delta x / 2$.

IV.7. Consider the generic transport equation for a scalar quantity ϕ, and its finite volume formulation using a standard cell-centered grid in which ϕ is defined at the cell centroid and velocity field values are taken at the faces of the control volume. For convection-dominated problems, upwind interpolations are commonly used to approximate the value of ϕ at face centers. A quadratic upwind interpolation (QUICK) scheme can be derived using polynomial fitting. Show that the QUICK interpolation formulation on a uniform Cartesian grid is given by

$$\phi_e = \frac{6}{8}\phi_U + \frac{3}{8}\phi_D - \frac{1}{8}\phi_{UU}$$

where D, U and UU denote the downstream, the first upstream and the second upstream node respectively (E,P, and W OR P, E and EE depending on the flow direction).

IV.8. Use finite volume method to formulate the discrete equations for potential flow problem, and write an appropriate code. The same code can also be used to solve steady state heat conduction problems. Choose a few problems for which exact solution is available in a heat transfer book, and compare your numerical results with the analytical solution. Study the effect of grid size on accuracy of the numerical solution.

IV.9. Consider fully developed laminar viscous flow between two large parallel walls. Starting from Navier-Stokes equations, derive simplified governing equations for this problem. Derive discretized equations and write a computer code to solve this problem using finite volume method. Compare your results with analytical solution.

IV.10. Consider one dimensional transient heat conduction governed by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 \phi}{\partial x^2}.$$

Derive discretized equations and write a computer code to solve this problem using finite volume method. Your code should provide options for different time integration methods. Choose a test case for which analytical solution is easily available in any heat transfer text book, and compare your results with analytical solution. Study the effect of grid size and time step on accuracy of the solution and stability of the solution process.

IV.11. In a heat-treatment process, a very long slab of width 8 m has been heated to 400 °C. It is then cooled by bringing both of its sides in contact with a fluid bath at 0 °C. Assuming constant material properties, write down the governing equation, and initial and boundary conditions for the problem. Use the finite volume method for spatial discretization and two-level methods for time integration. Assume the diffusivity $\kappa = 1$ m^2/s. Choose $\Delta x = 1$ m and $\Delta t = 0.25$ s. Write a computer program to obtain the temperature distribution in the slab at time $t = 0.25$ s and $t = 0.5$ s using (a) forward Euler method, and (b) Crank-Nicolson method. Use TDMA for solution of linear system if required.

IV.12. Consider the steady state heat conduction in a slab of width $l = 0.5$ m with heat generation. The left end of the slab ($x = 0$) is maintained at T = 373 K. The right end of the slab ($x = 0.5$ m) is being heated by a heater for which the heat flux is 1 kW/m^2. The heat generation in the slab is temperature dependent and is given by $Q = (1273 - T)$ W/m^3. Thermal conductivity is constant at $k = 1$ W/(m-K).Write down the governing equation and boundary conditions for the problem. Use the finite volume method to obtain an approximate numerical solution of the problem. Choose $\Delta x = 0.1$, and **use the TDMA.**

IV.13. Consider steady state heat conduction with constant heat generation in a slab of constant conductivity k = 1 bounded by planes x = 0 and x = 4. Both ends of the slab are kept at zero temperature. Write down the governing equation and boundary conditions for the problem. Obtain an approximate numerical solution of the problem using the finite volume method. Choose Δx = 1, and use the TDMA. For numerical solution, first do hand calculations and then write a computer program which can take different values of Δx. Compare results with analytical solution.

NOTE: Compare FVM solution of problems in this module to FDM solution obtained in Module III and FEM solution in Module V.