MODULE II

MATHEMATICAL MODELING: ASSIGNMENT

II.1. Symbol A_i represents

(a) a scalar quantity.
(b) a first order tensor.
(c) a tensor of order i.
(d) a vector.

II.2. Consider Kronecker delta δ_{ij}. Value of $\delta_{ij}\delta_{ij}$ is

(a) 0
(b) 1
(c) 2
(d) 3

II.3. Which of the following statements are true for alternating tensor ε_{ijk}?

(a) It is an isotropic tensor of order 3.
(b) $\varepsilon_{ijk} = \varepsilon_{kij}$
(c) $\varepsilon_{ijk} = -\varepsilon_{kij}$
(d) $\varepsilon_{ijk} = -\varepsilon_{ikj}$.

II.4. Gradient of a vector **v** can be represented as

(a) $v_{i,j}$.
(b) grad **v**.
(c) $v_i v_j$.
(d) $v_{i,i}$.

II.5. Which of the following statement(s) is/are true for the following form of momentum equation:

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{b}$$

a) It's strong conservation form.
b) It is valid only for Newtonian fluids.
c) It's non-conservation form.
d) It is valid for any fluid.

II.6. Three dimensional steady-state heat conduction equation $\nabla^2 T + \dot{q} = 0$ is

a) a parabolic equation.
b) a hyperbolic equation.
c) an elliptic equation.
d) is elliptic in x and y, and parabolic in z.

II.7. Navier-Stokes equations are hyperbolic for

a) unsteady inviscid compressible flow.
b) steady inviscid subsonic compressible flow.
c) steady inviscid supersonic compressible flow.
d) steady viscous subsonic compressible flow.

II.8. Using the Cartesian index notation, prove the following vector identities:

a. $\nabla \cdot (A\nabla B - B\nabla A) = A\nabla^2 B - B\nabla^2 A$
b. $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + (\nabla \cdot \mathbf{B})\mathbf{A} - (\nabla \cdot \mathbf{A})\mathbf{B}$
c. $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A}$

II.9. What are the fundamental physical principles which are essential in mathematical modeling of a fluid flow problem? What is the role of constitutive relations?

II.10. Use Reynolds transport theorem to derive integral form of continuity equation. Also obtain the differential form of mass conservation equation starting from the integral form.

II.11. Take an infinitesimal control volume in cylindrical polar coordinates and derive the differential form continuity equation.

II.12. Consider an arbitrary control volume in a time dependent compressible flow. Let e denote the internal energy, $\mathbf{v}$ be the velocity field, $\mathbf{q}$ denote the rate of heat flux and $\boldsymbol{\tau}$ denote the stress tensor. Use the Reynolds transport theorem to derive the integral form of the energy equation (i.e. the first law of thermodynamics). From this integral form, derive the differential form of the first law of thermodynamics.

II.13. Consider a fixed inertial control volume in fluid medium. Derive the integral and differential forms of momentum equation using Reynolds transport theorem. Use Stokes hypothesis to derive Navier-Stokes equations.