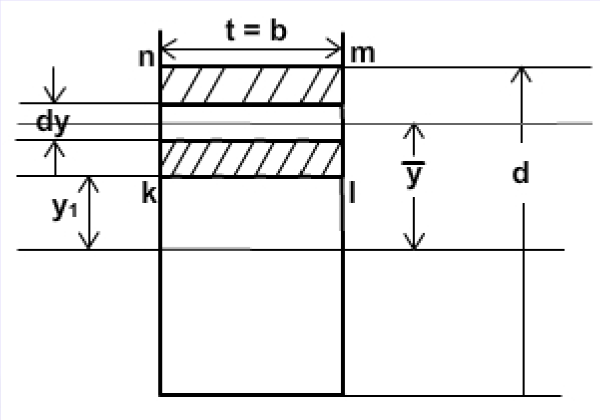
Problem 1: Derivation of Shear stress in rectangular crosssection

Derive an expression for the shear stress distribution in a beam of solid rectangular cross-section transmitting a vertical shear V.



The cross sectional area of the beam is shown in the figure. A longitudinal cut through the beam at a distance y_1, from the neutral axis, isolates area klmn. (A_1).

Shear stress,

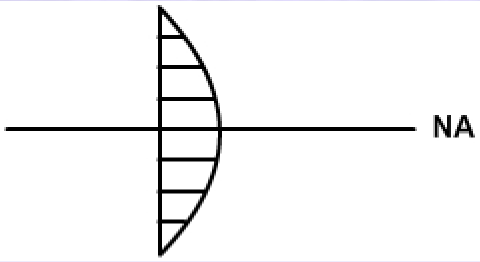
$$\tau = \frac{VQ}{It}$$

$$= \frac{V}{It}\int_{A_1} y.dA$$

$$= \frac{V}{Ib}\int_{y_1}^{d/2} by\,dy$$

$$= \frac{V}{2I}\left[(d/2)^2 - (y_1)^2\right] \text{ ---------------------- (1)}$$

The Shear Stress distribution is as shown below



Max Shear Stress occurs at the neutral axis and this can be found by putting y = 0 in the equation 1.

$$\tau_{max} = \frac{Vd^2}{8I}$$

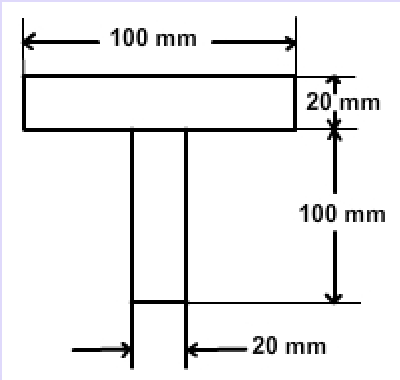
$$= \frac{3}{2}\frac{V}{bh}$$

$$= \frac{3}{2}\frac{V}{A}$$

Top

Problem 2: Computation of Shear stresses

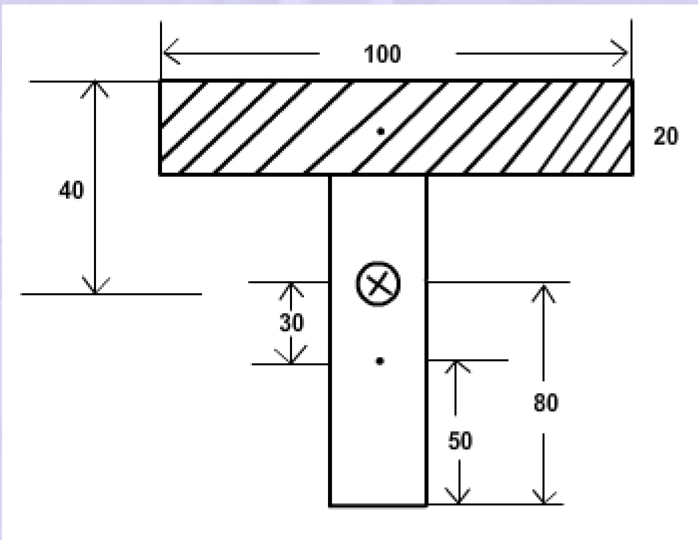
A vertical shear force of 1KN acts on the cross section shown below. Find the shear at the interface (per unit length)



Solution:

Formula used: q = VQ/I

We first find the distance of the neutral axis from the top fiber.



All dimensions in mm

$$y_{NA} = \frac{20\times100\times10 + 20\times100\times70}{20\times100 + 20\times100} = 40\text{mm}$$

$Q = \int ydA$ of shaded area about neutral axis.

Q = 20 x 100 x 30 = 6 x 10^4

V = 1KN

$$I = \frac{20 \times 100^3}{12} + 20 \times 100 \times 30^2 + \frac{100 \times 20^3}{12} + 100 \times 20 \times 30^2$$

$$= 5.33 \times 10^6$$

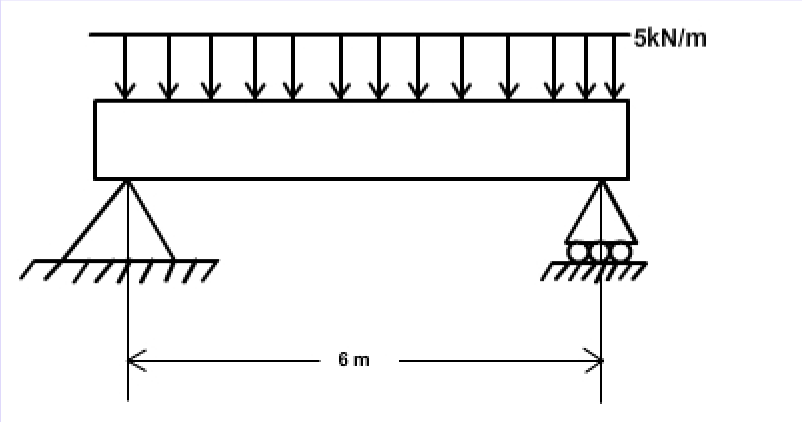
$$q = \frac{VQ}{I} = \frac{10^3 \times 6 \times 10^4 \times \phi(10^{-3})^3}{5.33 \times 10^6 \times \left(10^{-3}\right)^4} = 1.125 \times 10^4 \, \frac{N}{m}$$

$$= 11.25 \, \frac{KN}{m}$$

Top

Problem 3: Computation of Shear stresses

A 6m long beam with a 50 mm × 50 mm cross section is subjected to uniform loading of 5KN/m. Find the max shear stress in the beam

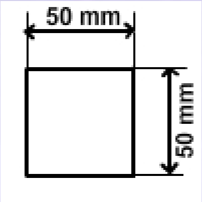


Solution:

$$\tau_{max} = \frac{3V}{2A}$$

We first find the section of maximum shear force. We know this is at the supports and is equal to

$$\frac{5 \times 6}{2} = 15KN$$



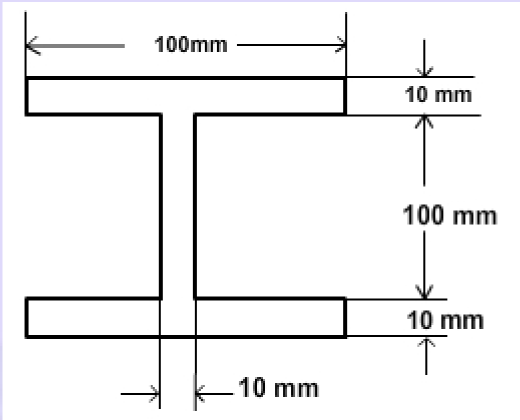
We also know that max.shear stress occurs at the centre (for a rectangular cross section) and is 1.5 times the average stress.

So, $\tau_{max} = \dfrac{3 \times 15 \times 10^3}{2 \times 50 \times 50 \times 10^{-6}} = 9\,Mpa$

Top

Problem 4: Computation of Shear stresses

The cross section of an I beam is shown below. Find the max.shear stress in the flange if it transmits a vertical shear of 2KN.



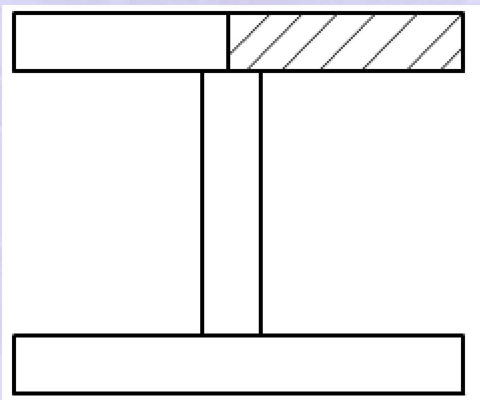
Solution:

Formula used: $\tau = \dfrac{VQ}{It}$

$$V = 2KN$$

$$I = \frac{10\times100^3}{12} + \left(\frac{100\times10^3}{12} + 100\times10\times55^2\right)\times 2 = 6.9\times10^6 \text{ mm}^4$$

Q is maximum at the midpoint as shown below



Q = 50 × 10 × 55

$$\tau_{max} = \frac{2\times10^3\times50\times10\times55\times\left(10^{-3}\right)^3}{\left(6.9\times10^6\right)\left(10^{-3}\right)^4\times10\times10^{-3}} = 0.79 \text{ MPa}$$

Top