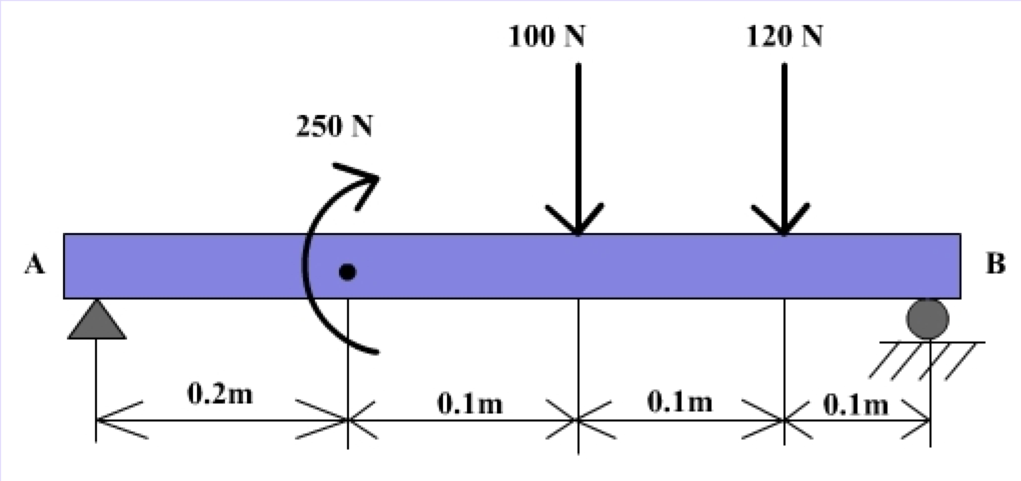
Problem 1: Computation of Reactions

Find the reactions at the supports for a simple beam as shown in the diagram. Weight of the beam is negligible.

Figure:



Concepts involved

- Static Equilibrium equations

Procedure

Step 1:

Draw the free body diagram for the beam.

Step 2:

Apply equilibrium equations

In X direction

$\sum F_X = 0$

$\Rightarrow R_{AX} = 0$

In Y Direction

$\sum FY = 0$

$\Rightarrow R_{AY}+R_{BY} - 100 - 160 = 0$

$\Rightarrow R_{AY}+R_{BY} = 260$

Moment about Z axis (Taking moment about axis pasing through A)

$\sum M_Z = 0$

We get,

$\sum M_A = 0$

$\Rightarrow 0 + 250$ N.m + 100*0.3 N.m + 120*0.4 N.m - R_{BY} *0.5 N.m = 0

$\Rightarrow R_{BY} = 656$ N (Upward)

Substituting in Eq 5.1 we get

$\sum MB = 0$

$\Rightarrow R_{AY} * 0.5 + 250 - 100 * 0.2 - 120 * 0.1 = 0$

$\Rightarrow R_{AY} = -436$ (downwards)

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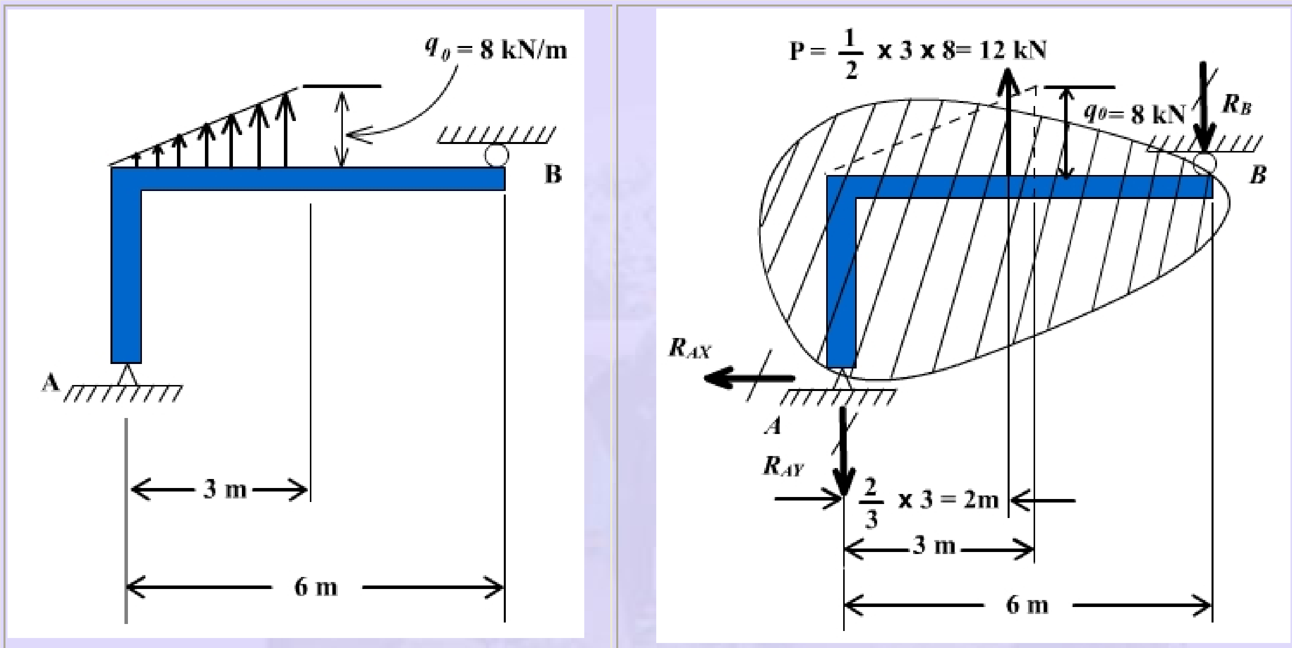
Problem 2: Computation of Reactions

Find the reactions for the partially loaded beam with a uniformly varying load shown in Figure. Neglect the weight of the beam

<u>Figure</u>



(a) (b)

Procedure:

The reactions and applied loads are shown in figure (b). A crude outline of the beam is also shown to indicate that the configuration of the member is not important for finding out the reactions. The resultant force P acting though the centroid of the distributed forces is found out. Once a free body diagram is prepared, the solution is found out by applying the equations of static equilibrium.

$\sum F_x = 0$

$R_{Ax} = 0$

$\sum M_A = 0$ Anticlockwise

$+12 \text{ X } 2 - R_{By} \text{ X } 6 = 0$

R_{By} = 4 kN downwards

$\sum M_B = 12 * (6 - 2) - R_A y * 6$

=> $R_A y = 8$ kN

Check $\sum F_y = 0$

$8 + 12 - 4 = 0$ ok!

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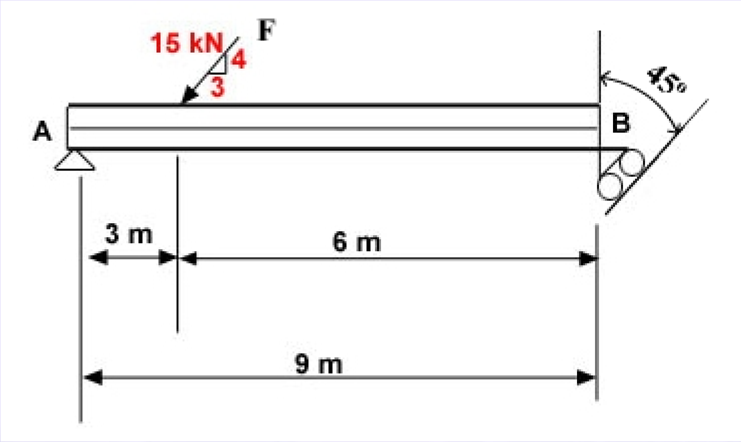
Problem 3: Computation of Reactions

Determine the reactions at A and B for the beam shown due to the applied force.

Figure



Solution

At A, the reaction components is x and y directions are R_{Ax} and R_{Ay}. The reaction R_B acting at B and inclined force F can be resolved into two components along x and y directions. This will simplify the problem.

Calculation:

$F_y = 12$, $F_x = 9$; (By resolving the applied force)

$\sum M_A = 0$

$12 \text{ X } 3 – R_{By} \text{ X } 9 = 0$

$R_{By} = 4 \text{ kN} = R_Bx$

$\sum M_B = 0$

$12 \text{ X } 6 - R_{Ay} \text{ X } 9 = 0$

$R_{Ay} = 8\text{kN}$

$\sum F_x = 0$

$R_{Ax} – 9–4 = 0$

$R_{Ax} = 13\text{kN}$

So,

$$R_A = \sqrt{\left(8^2 + 13^2\right)} = \sqrt{233}\,\text{kN}$$

$$R_B = \sqrt{\left(4^2 + 4^2\right)} = 4\sqrt{2}\ \text{kN}$$

Check:

$\sum F_y = 0$

+ 8 – 12 + 4= 0 ok!

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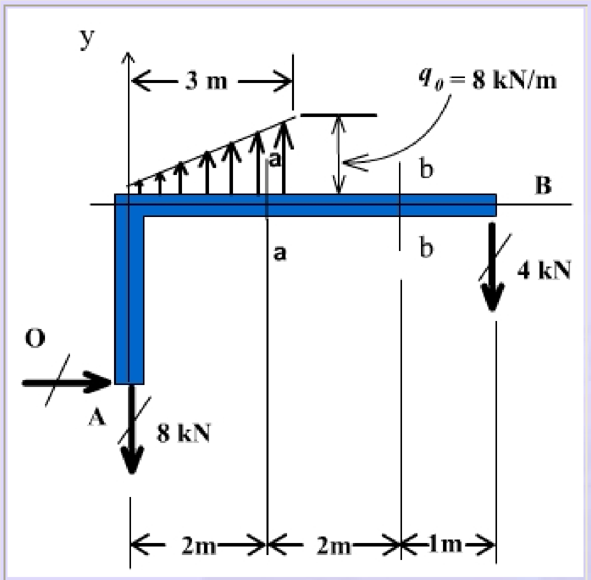
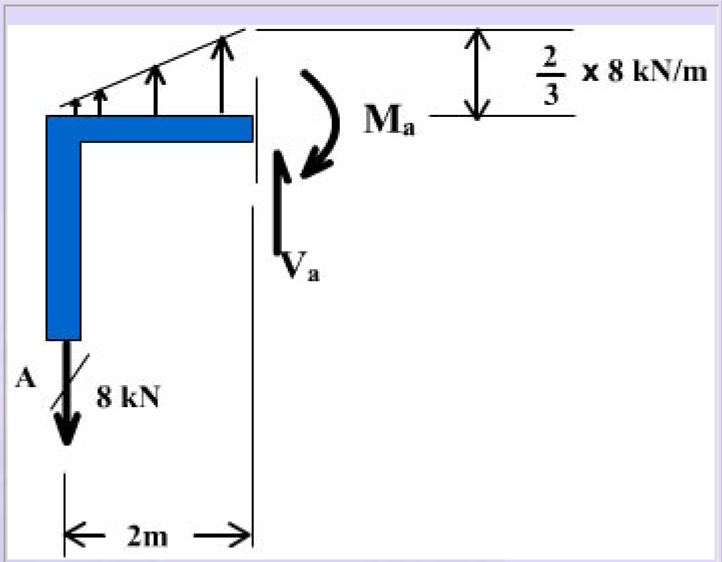
Problem 4: Computation of forces and moments

In the earlier Example, determine the internal system of forces at sections a-a and b-b; see Figure

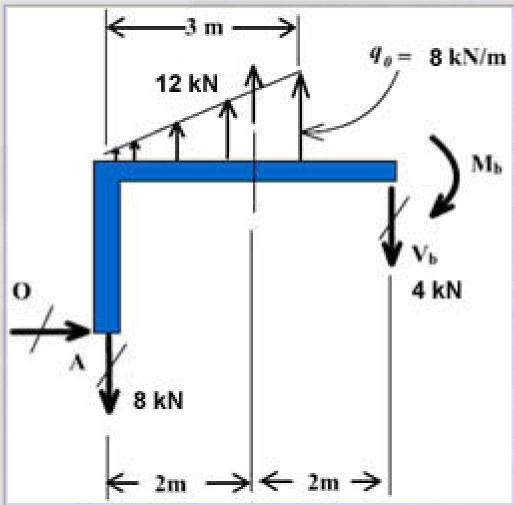
Figure



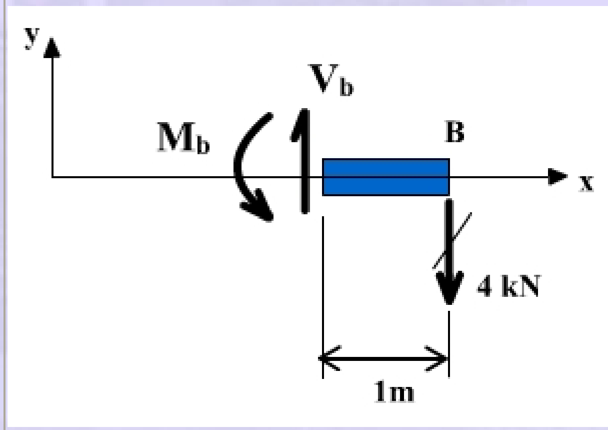
(a)



(b)



(c)



(d)

Solution:

A free-body diagram for the member, including reactions, is shown in Fig. (a). A free-body to the left of section a-a in Fig. 5-20(b) shows the maximum ordinate for the isolated part of the applied load. Using this information,

V_a = -8 + 1/2 * 2 * 2/3 * 8 = -2.67 kN

and

M_a = -8 * 2 + {1/2 * 2 * (2/3 * 8)} * {1/3 * 2} = -14.45 kN

These forces are shown in the figure.

A free-body diagram to the left of section b-b is shown in Figure. This gives

V_b = - 4 kN

And

M_b = -4 x 2 = - 8 kN-m

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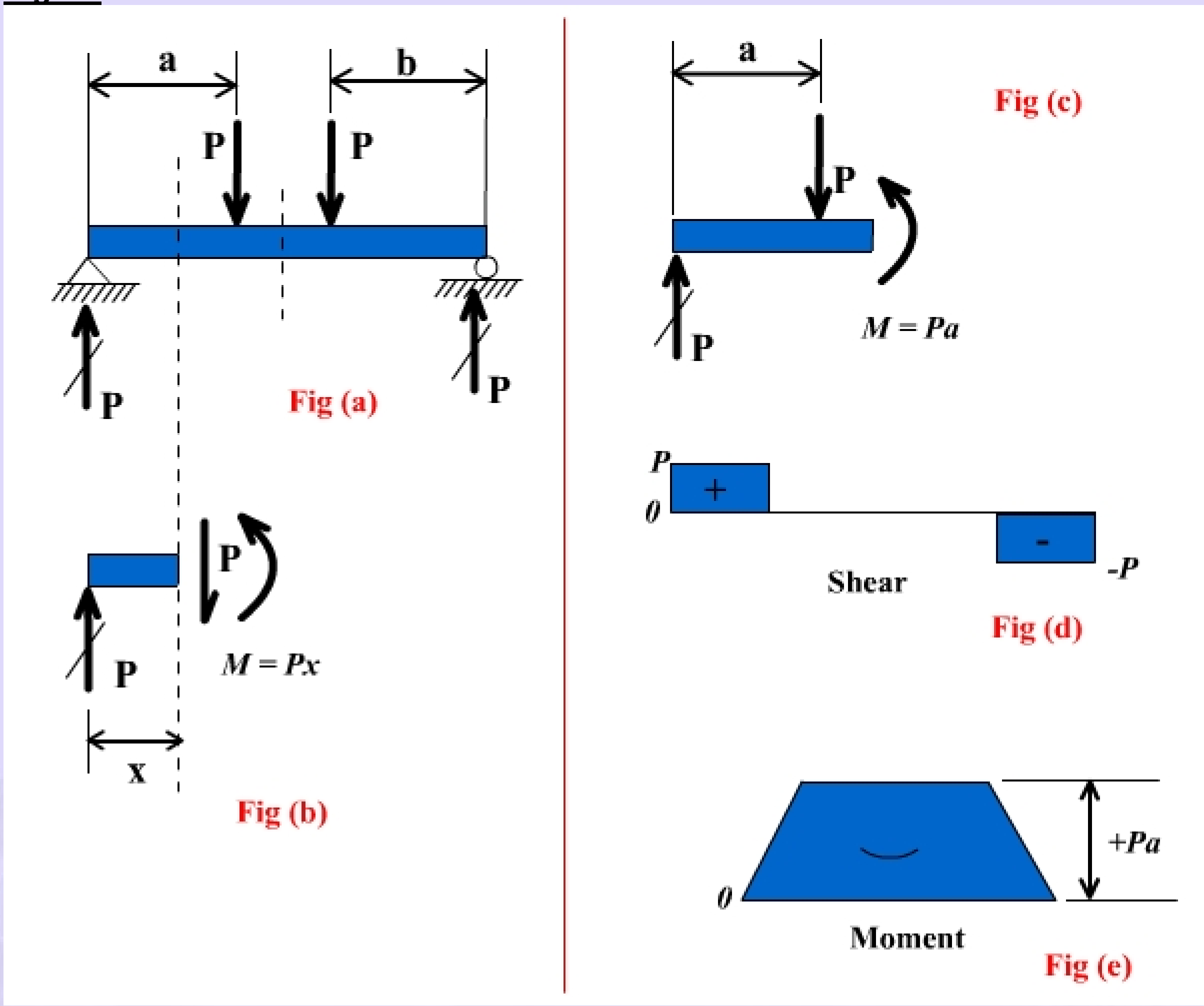
Problem 5: Bending Moment and Shear force

Construct shear and bending-moment diagrams for the beam loaded with the forces shown in the figure.

Figure



Solution.

Taking an arbitrary section at a distance x from the left support isolates the beam segment shown in Fig.(b). This section is applicable for any value of x just to the left of the applied force P. The shear, remains constant and is +P. The bending moment varies linearly from the support, reaching a maximum of +Pa.

An arbitrary section applicable anywhere between the two applied forces is shown in Fig.(c). Shear force is not necessary to maintain equilibrium of a segment in this part of the beam. Only a constant bending moment of +Pa must be resisted by the beam in this zone. Such a state of bending or flexure is called pure bending.

Shear and bending-moment diagrams for this loading condition are shown in Figs (d) and (e). No axial-force diagram is necessary, as there is no axial force at any section of the beam

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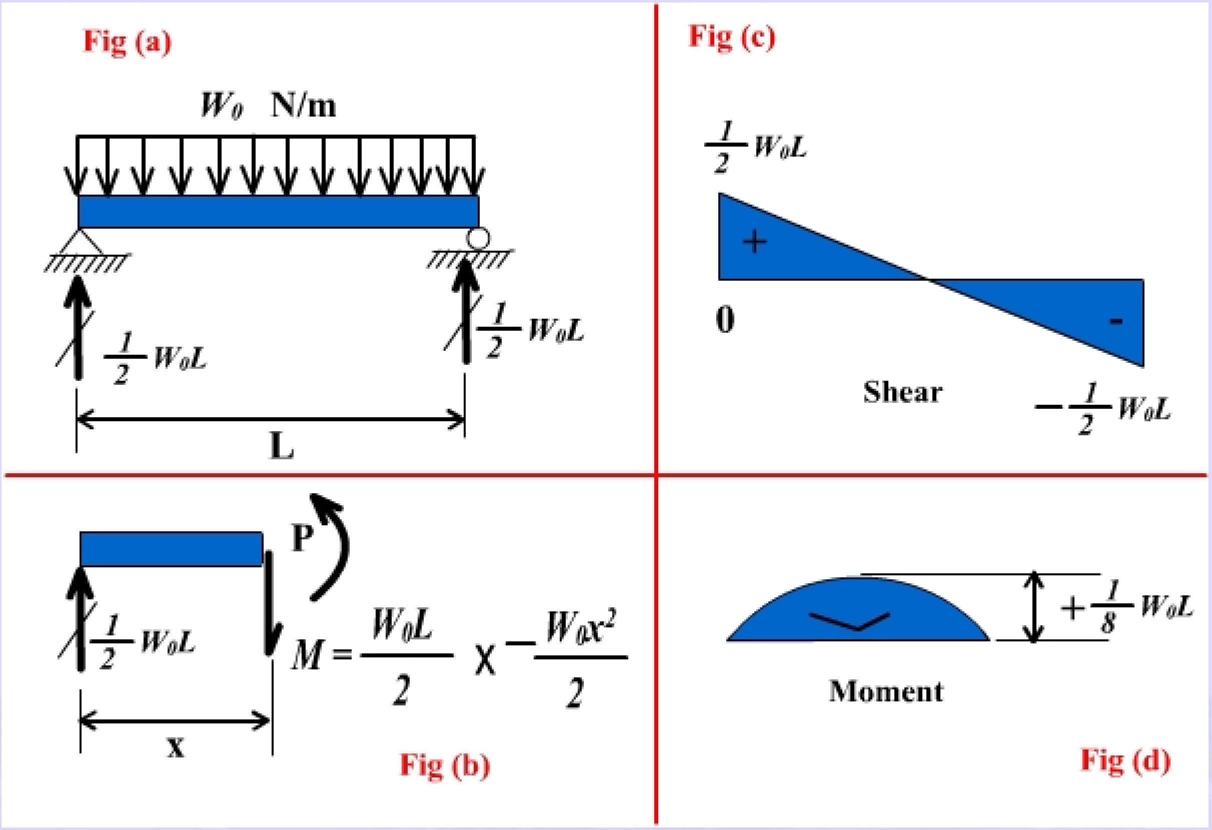
Problem 6: Bending Moment Diagram

Plot shear and bending-moment diagrams for a simply supported beam with a uniformly distributed load; see Figure.

Figure



Solution

A section at a distance x from the left support is taken as shown in figure (b). The shear is found out by subtracting the load to the left of the section from the left upward reaction. The internal bending moment M resists the moment caused by the re-action on the left less the moment caused by the forces to the left of the same section. The summation of moments is performed around an axis at the section.

Similar expressions may be obtained by considering the right-hand segment of the beam, while taking care of the sign conventions. The plot of the V and M functions is shown in Figs(c) and (d)

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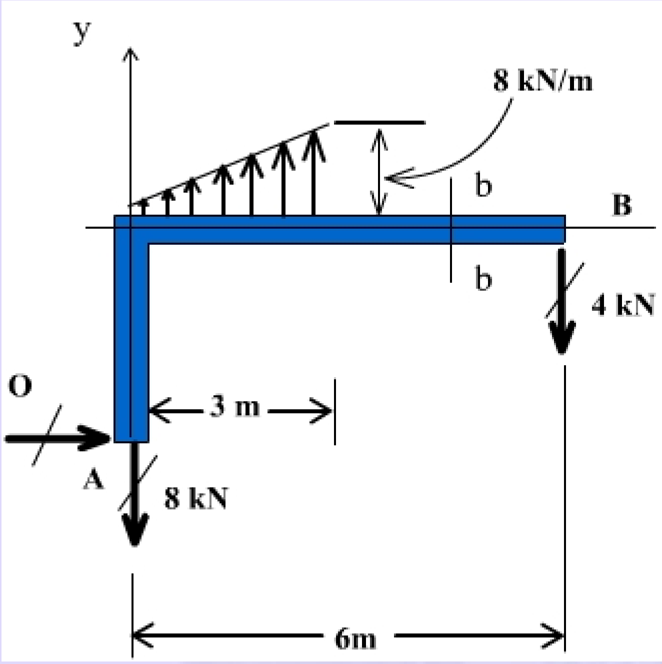
Problem 7: Bending Moment and Shear force

For the beam as shown in Fig 5, express the shear V and the bending moment M as a function of x along the horizontal member.

Figure:



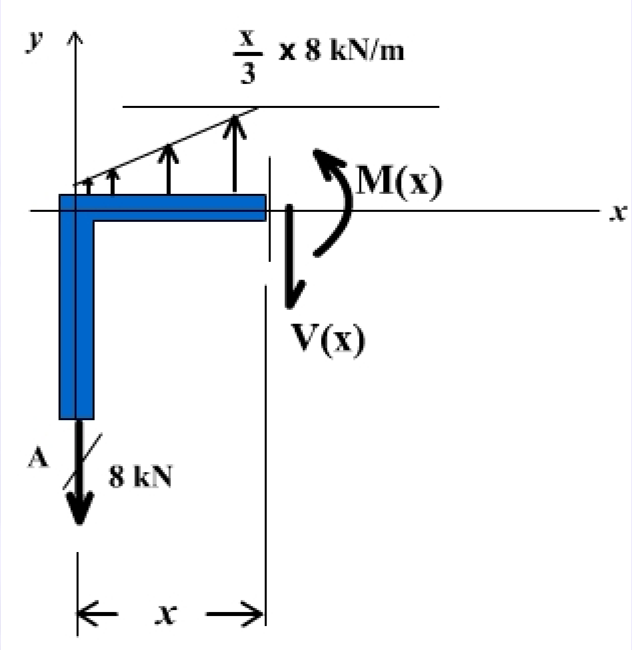


Fig (a)

Solution

A load discontinuity occurs at x == 3 m, in this problem. Hence, the solution is determined in two parts for each of which the functions V and M are continuous.

$V(x) = -8 + 1/2 \times [(x/3) * 8)] = -8 + (4/3)\, x^2$ kN for $0 < x < 3$

$M(x) = -8x + 1/2x[(x/3) * 8 *(x/3)] = -8x + 4/9x^3$ kN.m for $0 < x < 3$

$V(x) = -8 + 12 = +4$KN For $3<x<6$,

$M(x) = -8x + 12(x-2) = 4x - 24$kN.m For $3<x<6$,

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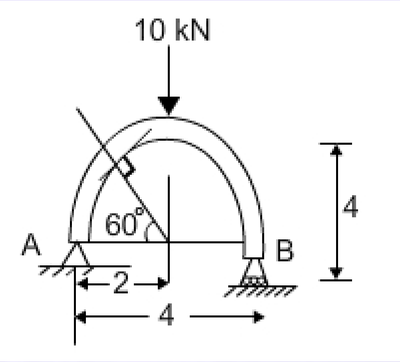
Problem 8: Bending Moment and shear force

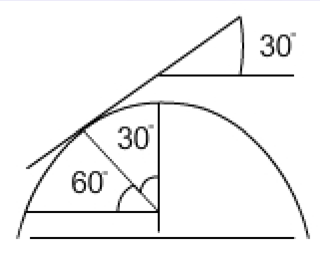


A parabolic beam is subjected to a 10kN force as shown above. Find axial force, shear force and bending moment at a section, as shown above.

Solution :

We first find the equation of the beam and the point where section is cut. With origin at A, equation of the parabola can be written as

$$y = kx(4-x). \text{It remains to find } k.$$
$$\text{wrt } y = 4 \text{ when } x = 2 \Rightarrow k = 1$$
$$\text{so } y = x(4-x).$$
$$\text{slope, } y' = \frac{dy}{dx} = 4 - 2x = \tan 30 = \frac{1}{\sqrt{3}}$$
$$\Rightarrow x = \frac{\left(4\sqrt{3}+1\right)}{2\sqrt{3}}$$

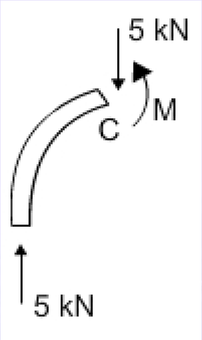


Now, we only have to resolve the force at the section along the normal and tangential directions. But before that, we first have to find the support reactions.

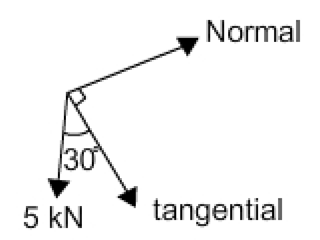
$$\sum F_x = 0 \Rightarrow R_{Ax} = 0$$
$$\sum M_A = 0 \Rightarrow 10 \times 2 = R_{By} \times 4 \; R_{By} = 5kN.$$

By symmetry (since R_{AX} = 0), $R_{Ay} = 5kN$ $[\text{or } R_{Ay} = 10 - R_{By}]$

We now draw the free body diagram of the cut body



We now resolve the 5kN force



$$\text{Shear force} = 5\cos 30 = 5\frac{\sqrt{3}}{2} = 4.33 \text{ kN.}$$
$$\text{Axial force} = -5\cos 30 = -2.50 \text{ kN(compression)}$$
$$\sum M_c = 0 \Rightarrow 5 \times \frac{\left(4\sqrt{3}+1\right)}{2\sqrt{3}} = M = 11.44 \text{ kNm}$$

The Reader can repeat the exercise for a general angle θ and check where each of quantities reaches their zeros and maxima.

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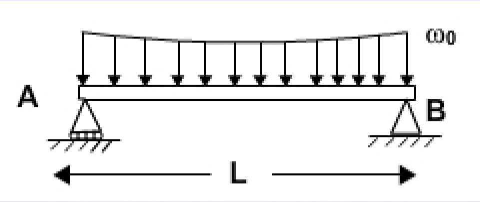
Problem 9: Bending Moment and Shear force

A constant load of ω_0 per unit length is applied on a simply supported beam as shown below. Draw the shear force and bending moment diagram by

a. Method of sections

b. Integration method

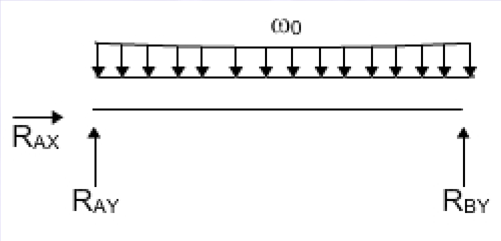
Solution:



Formulas used:

$$\frac{dv}{dx} = q \qquad \frac{dm}{dx} = V.$$

We first find the support reactions which are necessary for both the methods.

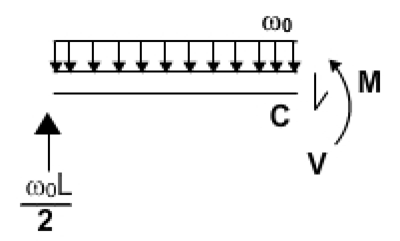


$$\sum F_x = 0 \Rightarrow R_{Ax} = 0$$

$$\text{By symmetry, } R_{Ay} = R_{By} = \frac{\omega_0 L}{2}$$

a. Method of section

We cut a section at a distance 'x' from left and of the beam.



$$\sum F_y = 0 \Rightarrow V = \frac{\omega_0 L}{2} - \omega_0 x$$

$$\sum M_c = 0 \Rightarrow M = \frac{\omega_0 L}{2} \times x - \omega_0 \times x \times \frac{x}{2} = \omega_0 \frac{x}{2}(L - x)$$

b. Method of Integration

$$\frac{dv}{dx} = q.$$

$$\text{Here, } q = -\omega_0$$

$$\Rightarrow \frac{dv}{dx} = -\omega_0$$

$$\text{Integrating } V = -\omega_0 x + C_1$$

$$\text{wkt V at } x = 0 \text{ is } \frac{\omega_0 L}{2}. \text{ Putting } x = 0 \text{ in above equation,}$$

$$\text{We get } C_1 = \frac{\omega_0 L}{2}$$

$$V = \frac{\omega_0}{2}(L - 2x)$$

$$\frac{dm}{dx} = V = \frac{\omega_0}{2}(L - 2x)$$

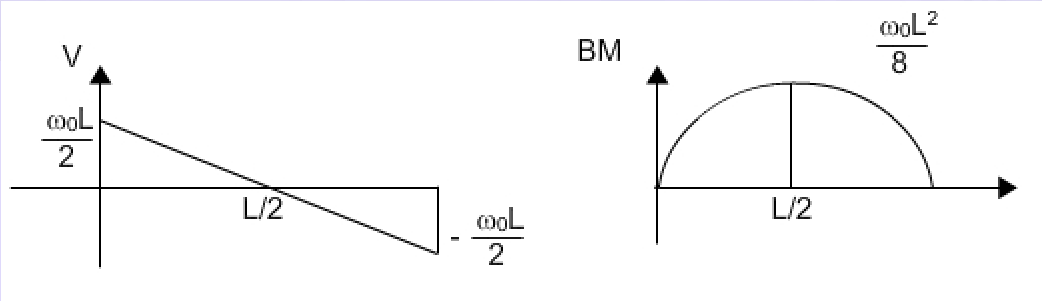
$$M = \frac{\omega_0}{2}\left(Lx - x^2\right) + C_2$$

wkt, for a simply supported beam, bending moment is zero at the two ends.

$$M = 0 \text{ at } x = 0 \Rightarrow C_2 = 0$$

$$\Rightarrow M = \frac{\omega_0}{2}\left(Lx - x^2\right) = \frac{\omega_0 x}{2}(L - x)$$

We see that the expression for shear force and bending moment is the same using the two methods. It only remains to plot them.



Points to ponder:

The relation $\frac{dm}{dx} = V$ can be construed from two diagrams as below.

As shear force decreases (with increasing x), the slope of the bending moment diagram also decreases.

Further the bending moment is maximum when its derivative, the shear force is zero.

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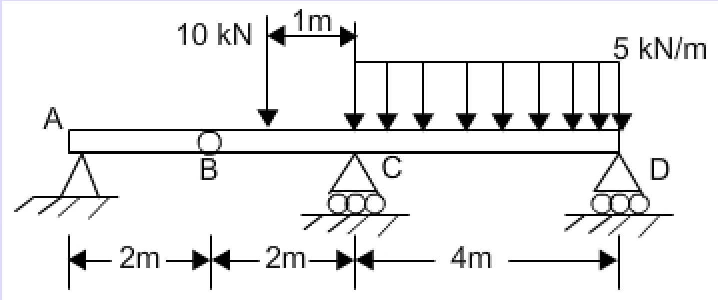
Problem 10: Bending Moment and Shear force



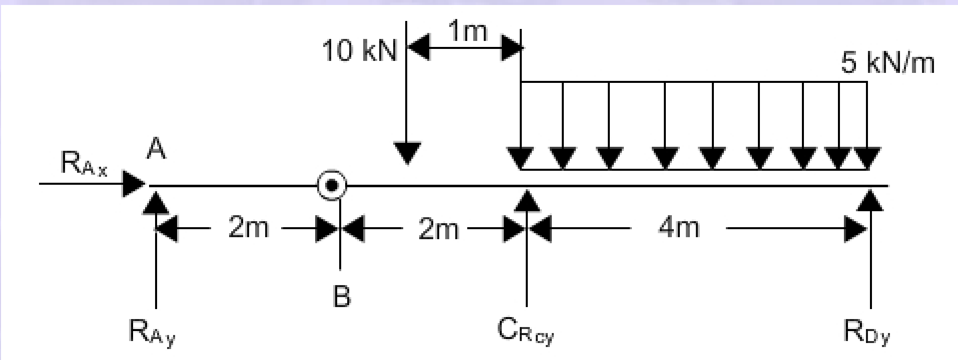
A beam with a hinge is loaded as above. Draw the shear force and bending moment diagram.

Solution:

Concept: A hinge can transfer axial force and shear force but not bending moment. So, bending moment at the hinge location is zero.

Also, without the hinge, the system is statically indeterminate (to a degree 1). The hinge imposes an extra condition thus rendering the system determinate.

We first find the support reactions.



$$\sum F_x = 0 \Rightarrow R_{Ax} = 0.$$

$$M_B = 0 \Rightarrow R_{Ay} \times 2 = 0 \Rightarrow R_{Ay} = 0 \left[\text{Bending moment at hinge } = 0\right]$$

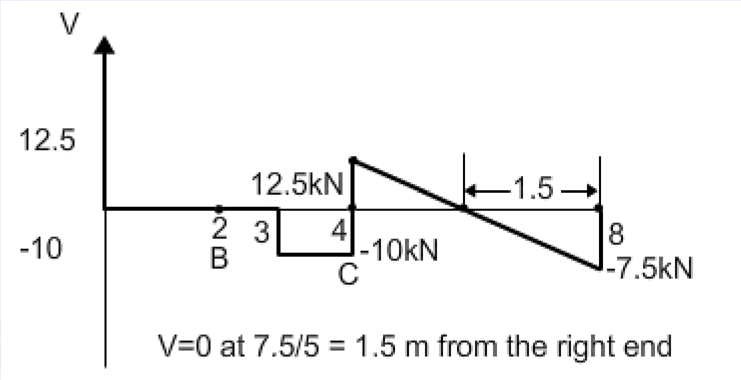
$$\sum M_D = 0 \Rightarrow 10 \times 5 + 5 \times 4 \times 2 = R_{cy} \times 4$$

$$R_{cy} = 22.5\text{kN}$$

$$R_{Dy} = 10 + 5 \times 4 - 22.5 = 7.5\text{kN}$$

Shear force Diagram

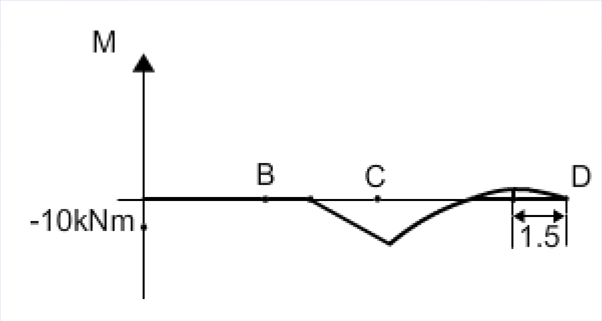
The shear force remains zero till the 10kN load is reached. It is then constant and equal to -10 kN till it reaches the point C where it jumps up by the value of Rcy. From C to D it decreases linearly at 5kN/m. From above considerations, the shear force diagram is as below.



Bending moment diagram:

Let us draw the bending moment diagram from the shear force diagram, keeping in mind the fact that the slope of bending moment diagram at any point must be equal to the shear at that point. Further, we know that the bending moment is zero at end supports.

The bending moment remains zero till the 10kN force, as shear is zero. It then decreases linearly at 10 kNm/m up to the point C. From the point C, it is parabolic till it finally reaches zero at the right support D. Further, it reaches a maximum where shear is zero, keeping these in mind, the BM diagram is as below.



$$M_{max} = 7.5 \times 1.5 - 5 \times 1.5 \times \frac{1.5}{2} = 5.625 \text{ kNm}$$

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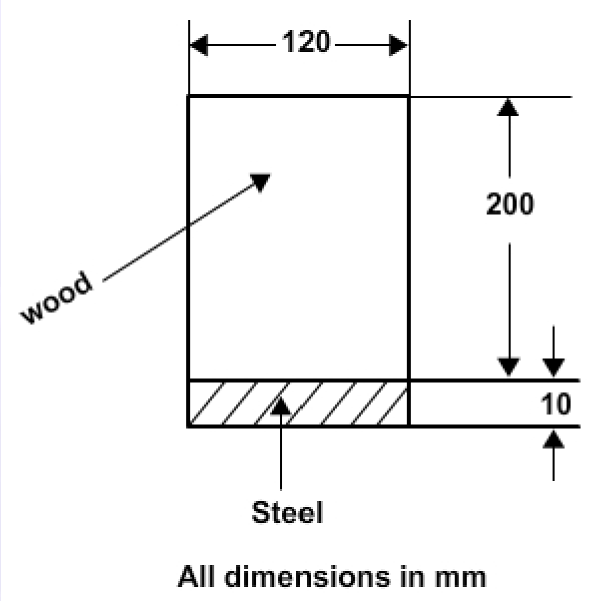
Problem 11: Beams of Composite Cross section

The composite beam shown in figure is made up of two materials. A top wooden portion and a bottom steel portion. The dimensions are as shown in the figure. Take young's modulus of steel as 210 GPa and that wood as 15 GPa. The beam is subjected to a bending moment of 40 kNm about the horizontal axis. Calculate the maximum stress experienced by two sections.



Solution:

The solution procedure involves hiding an equivalent dimension for one of the materials keeping the other as reference.

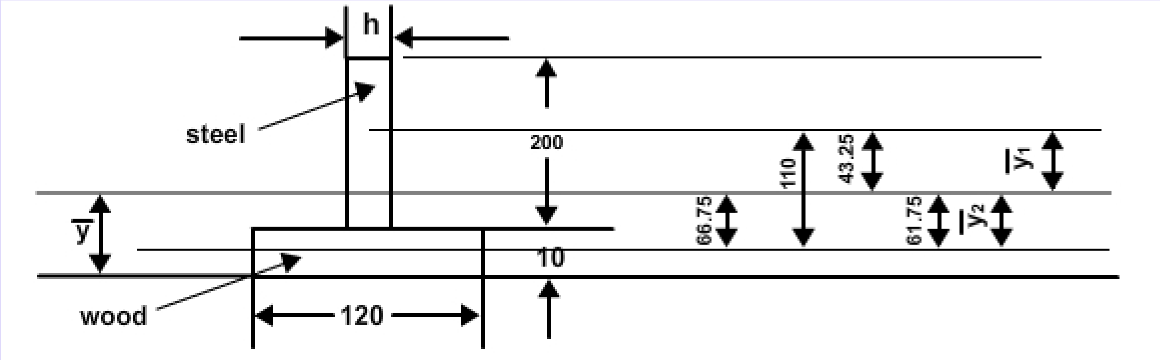
Let us take, Reference Material as steel.

Ratio of modulii,

$$r = \frac{E_{wood}}{E_{steel}} = \frac{15}{210} = .00714$$

$$= 7.14 \times 10^{-2}$$

The equivalent section is as shown in following Figure.



The equivalent thickness of wood sections, h

= 120 × r

= 120 × 7.14 × 10^{-2}

= 8.568 mm

Distance of the neutral axis from the bottom of the beam,

$$\bar{y} = \frac{(120\times10)\times5+(200\times8.568)\times(100+10)}{(120\times10)+(200\times8.568)}$$

$= 66.75$ mm

$$\text{Moment of Inertia } I = \sum\frac{b_i d_i^3}{12} + A_i \bar{y}_i^2$$

$$= \frac{8.568 \times (200)^3}{12} + (8.568 \times 200)\times(43.25)^2 + \frac{(120)\times(10)^3}{12} + (120)\times(10)\times(61.75)^2$$

$= 13.5 \times 10^6 \text{mm}^4$

Stresses in beams

$$(\sigma_{steel})_{max} = \frac{M.y}{I}$$

$$= \frac{40 \times 10^3 \times 66.75 \times 10^{-3}}{13.5 \times 10^6 \times (10^{-3})^4}$$

$= 197.78$ MPa

$$(\sigma_{wood})_{max} = (\sigma_{steel})_{max} \times r$$

$= 14.12$ MPa

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