Random Vibrations
&
Failure Analysis

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Lecture 1: Introduction

Course Objectives:

The focus of this course is on gaining understanding on how to make an assessment of the risk associated with structures which are excited by random loads.

In keeping with the recent focus on the development of green sources of energy, let us consider as an example an offshore wind turbine.

Offshore wind turbines are constructed in open seas and are usually of enormous dimensions. The main components of the structure include a mast which is either fixed to the ocean surface or is floating, a nacelle, and the turbine blades (see the figure in the next slide). The turbine blades are of enormous dimensions and depending on the rated capacity, could have radial dimensions as 150 m.

For policy and maintenance reasons, the owner would be interested in estimating the risk associated with the structure. The aim of this course is on providing directions which would enable estimating the risk associated with the structure.

Steps involved in failure analysis & risk assessment

Estimating the risk associated with the structure involves the following primary steps:

- Estimate the loads on the structure:

The primary loads acting on the wind turbine are

1. wave loads that act on the submerged part of the wind turbine mast.
2. wind loads that act on the wind turbine blades and the mast that is above the sea level.

Both the wind and wave loads cannot be predicted with certainty and hence can be treated as random.

Steps involved in failure analysis & risk assessment (Cont'd)...

- Estimate the structure behavior and predict the structure response:

The steps involved in analyzing the structure and predicting its behavior include developing an appropriate mathematical model for the structure geometry and the material behavior.

Since any mathematical model is an approximation of the true structure behavior, an analysis based on a model invariably introduces errors that are dependent on the adopted model. Quantifying these errors are essential for an accurate failure analysis of the structure. This requires developing models for these errors.

Discussions on the classification of the errors introduced in modeling the random loads and the uncertain errors due to the mathematical modeling of the structure will be presented in the next lecture.

Steps involved in failure analysis & risk assessment (Cont'd)...

- Quantifying the probability of failure

A structure failure is occurs when the structure response exceeds levels for which it had been designed.

Mathematically, if the loads are defined as L and the structure response Y is obtained as a function of L through the relation

$$Y = f(L)$$

a failure is defined to occur if $Y \geq \alpha$.

Here, the function $f(\,\cdot\,)$ in its most general form could represent a set of simultaneous partial differential equations, and α is a safe threshold limit.

Since both L and $f(\,\cdot\,)$ cannot be defined with accuracy and hence need to be treated as random, one needs to use probability theories to model these quantities. Consequently, the response Y is also defined probabilistically and the occurrence of $Y \geq \alpha$ needs to be estimated using probability theories.

Course contents:

The focus of this course will be on introducing the necessary techniques that will enable the student to make an assessment of the failure probability of structures that are subjected to uncertain loads.

This course is divided into the following *6* modules, with each module having several lectures, the details of which are given below:

1. Module 1: Introduction, Probability Theory, Random variables
2. Module 2: Stochastic processes
3. Module 3: Crossing statistics and failure probability
4. Module 4: Review of Linear Vibration Analysis
5. Module 5: Random vibrations
6. Module 6: Advanced topics

Module 1: Probability theory, random variables

1. **Lecture 1**: Introduction, course objectives, course contents, references
2. **Lecture 2** :Introduction to the concept of uncertainty, classification of uncertainties
3. **Lecture 3**: Introduction to Probability -I: probability space, definitions of probability,
4. **Lecture 4**: Introduction to Probability -II: sample space, measure theory, axioms of probability, Borel field, examples
5. **Lecture 5**: Introduction to Probability -III: Permutations and combinations, product spaces, Bernoulli's theorem, De Moivre Laplace theorem, examples.
6. **Lecture 6**: Introduction to Probability -IV: Law of large numbers, Poisson's theorem, conditional probability, Baye's theorem, independence, examples
7. **Lecture 7:** Problem set -1

Module 1: Probability theory, random variables

8. **Lecture 8**: Random variables -I: Definition of random variables, discrete and continuous random variables, probability distribution function, probability density function, some common distributions
9. **Lecture 9**: Random variables -II: statistics of random variables, moments, moment generating function, characteristic function, cumulants and cumulant generating function, the importance of probability space
10. **Lecture 10**: Functions of random variables-I: functions of one random variable, bivariate probability distribution and density functions,
11. **Lecture 11**: Functions of random variables-II: functions of two random variables, extension to higher dimensions, covariance, conditional distributions for correlated random variables
12. **Lecture 12**: Monte Carlo simulations of random variables: history, pseudo random number generators, simulating from uniformly distributed random variables, MATLAB scripts, validation
13. **Lecture 13**: Problem set -2

Module 2: Stochastic processes

1. **Lecture 14**: Stochastic processes-I: definition, classification of random processes, description of random processes, stationarity and ergodicity, example problems
2. **Lecture 15**: Stochastic processes - II: calculus of stochastic processes, definitions of convergence, differentiation and integration of stochastic processes, example problems
3. **Lecture 16**: Stochastic processes - III: review of Fourier series, Fourier transform of a stochastic process, power spectral density and correlation functions,
4. **Lecture 17:** Stochastic processes - IV: properties of power spectral density function, narrow band and broad band random processes, measures of bandwidth of a process
5. **Lecture 18**: Problem set - 3
6. **Lecture 19**: Monte Carlo simulations of stochastic processes: theory and background for simulating random processes using spectral decomposition method, MATLAB scripts

Module 3: Crossings of stochastic processes

1. **Lecture 20**: Crossings of stochastic process: Failures in vibrating systems, extreme value distributions, crossing statistics, Rice's integral, crossings of Gaussian processes.

2. **Lecture 21**: Crossings of non-Gaussian processes: crossing statistics of transformed Gaussian processes, translation processes, monotonic and non-monotonic transformations.

3. **Lecture 22**: Peak distributions of stochastic processes: peak distributions of stochastic processes, narrow band approximations, fractional occupation time, example

4. **Lecture 23**: Random fatigue: Introduction to random fatigue, Palmgern-Miner damage accumulation rule, S-N diagrams, peak counting method, range counting method, level crossing method, rain-flow cycle counting method, spectral method for rain flow counting, bounds on expected random fatigue damage

Module 4: Review of linear vibrations

1. **Lecture 24**: Review of linear vibrations - I: single degree of freedom systems, time domain analysis of free and forced vibrations, damping, damped oscillations, impulse response function, Duhamel's integral

2. **Lecture 25**: Review of linear vibrations - II: Duhamel's integral, frequency domain analysis, generalisation to multi degree of freedom discrete systems, concept of natural coordinates and modal analysis, eigenvalues and eigenvectors

4. **Lecture 26**: Review of linear vibrations - III: Continuous systems, Lagrangian formulation, governing equations of vibrating beams, solution methods, transcendental equations, eigenvalues and eigenfunctions, mode shapes, orthogonality of mode shapes, analysis, Approximate methods: Rayleigh's quotient

6. **Lecture 27**: Review of linear vibrations - IV: Rayleigh-Ritz method, Approximate methods, Method of weighted residuals, Galerkin's method, finite element method for vibrating beams, Steps in FEM.

Module 5: Random vibrations

1. **Lecture 28**: Random vibrations of a sdof oscillator: Time domain analysis for the random vibrations of a sdof oscillator, auto and cross-correlation functions, frequency domain approach, auto and cross psd of response, approximations

2. **Lecture 29**: Random vibrations of higher order systems: generalisation of time and frequency domain random vibration analysis to higher order vibrating systems, PSD matrices, modal approach in random vibrations

3. **Lecture 30**: Nonlinear systems: Random vibration analysis in nonlinear systems, equivalent linearization, stochastic averaging

4. **Lecture 31**: Problem set -4

Module 6: Special topics

1. **Lecture 32**: Karhunen Loeve expansions: Hilbert space, orthogonal basis, Karhunen-Loeve expansions

2. **Lecture 33**: Polynomial chaos expansions: Generalization of K-L expansion, Polynomial chaos expansion, Stochastic Galerkin approach, example problem

3. **Lecture 34**: Markov processes -I: Definition, Chapman-Kolmogorov equation, continuity of Markov process, Differential form of Chapman-Kolmogorov equation

4. **Lecture 35**: Markov processes - II: Jump processes, Diffusion process, Liouville's equation, Stationary and homogenous Markov process, Wiener process, Random walk, Ornstein-Uhlenbeck process

5. **Lecture 36**: Fokker Planck-Kolmogorov equation

6. **Lecture 37**: Stochastic calculus: stochastic differential equation, Steltjes integral, Ito integral, Stratonovitch integral, numerical methods

References:

Following is a list of text books/references that are relevant to this course:

1. A. Papoulis. (1997). Probability, random variables and stochastic processes. McGraw-Hill, New York.

2. Y.K. Lin (1967). Probabilistic theory of structural dynamics. McGraw-Hill, New York.

3. N.C. Nigam (1983). Introduction to random vibrations. MIT Press, Massachusetts.

4. L.D. Lutes, S. Sarkani (2004). Random vibrations: analysis of structural and mechanical systems, Elsevier, Amsterdam.

5. J. Solnes (1997). Stochastic processes and random vibrations: Theory and practice. John Wiley & Sons, Chichester.