Answer key to Mid sem Exam "Microfluidics", Feb 2015

Q1 (a) (approx) $\mu L/min$

(b) gases are less ~~than~~ dense than liquids → weaker intermolecular attraction makes it easier to dislodge gas molecule from solid boundary

(c) Phosphate group of DNA gives it −ve charge ⇒ can be manipulated by electric field.

(d) Energy supplied → in tune with energy gap of TiO_2 ⇒ electron - hole reactions begin immediately → net alteration in surface charge and hence surface energy.

(e) Surface has affinity for the droplet, whereas shear in the flow tries to break it off

(f) vesicle is essentially enveloped by a membrane, whereas droplet is seperated by a fluid-fluid interface

(g) cellular length scale ~ 10 μm, cells in microconfinement mimic physiological condition in microcapillaries

(h) wick free and large surface area to volume ratio → for micro heat pipes

(i) $Q = CV$, $\frac{dC}{dt}$ due to $\frac{d\epsilon}{dt}$ (evaporation creates change in ϵ) creates $\frac{dQ}{dt}$ and hence current

(j) Bubble enters ~~junction~~ channel with less resistance, after which resistance builds up and bubble enters a different (narrower) channel.

Q2 $N = m$, $n = 1$

$$\underbrace{0}_{\left.\frac{dm}{dt}\right|_{system}} = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho(\vec{V}_r \cdot \hat{n})\, dA$$

Non deformable ⇒ $\frac{\partial}{\partial t}$ goes inside the integral

$$\Rightarrow 0 = \int_{CV} \frac{\partial}{\partial t}(\rho)\, d\forall + \underbrace{\int_{CV} \nabla\cdot(\rho \vec{V}_r)\, d\forall}_{\text{using divergence theorem}} = 0$$

$$\Rightarrow 0 = \int_{CV} \left[\frac{\partial \rho}{\partial t} + \nabla\cdot\{\rho(\vec{V} - \vec{V}_{cv})\}\right] d\forall = 0$$

since cv is arbitrary

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho(\vec{V} - \vec{V}_{cv})) = 0.$$

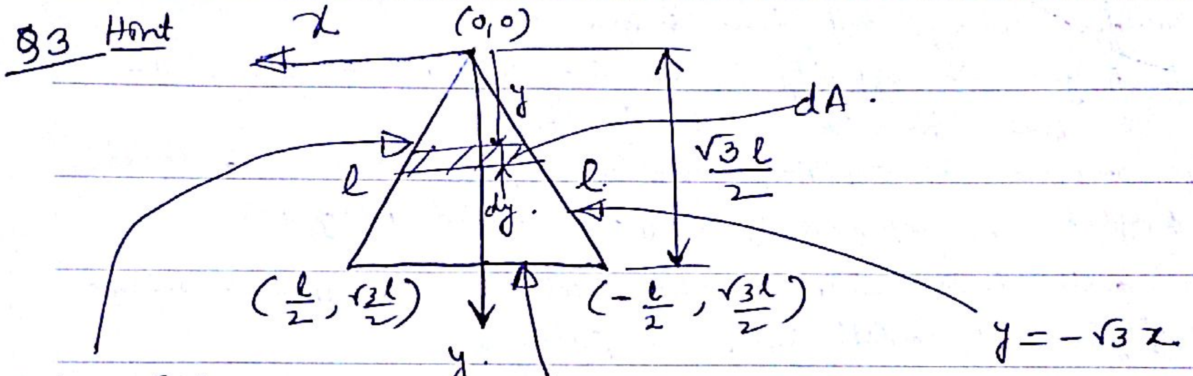
Q1 (b)(i) → Non accelerating flow
→ no body couple

(ii) Newtonian fluid

(iii) Simple, compressible, pure substance with no phase change, no viscous dissipation

(iv) no generation or destruction of species, species average velocity is same as fluid velocity, Ficks law valid

Q3 Hint



$$y = \frac{\frac{\sqrt{3}l}{2}}{\frac{l}{2}} x = \sqrt{3}x \qquad y = \frac{\sqrt{3}l}{2}$$

let $\quad \omega = a(y-\sqrt{3}x)(y+\sqrt{3}x)\left(y-\frac{\sqrt{3}l}{2}\right) = a(y^2-3x^2)\left(y-\frac{\sqrt{3}l}{2}\right)$

$$0 = -\frac{dp}{dz} + \mu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$\frac{\partial \omega}{\partial x} = a(-6x)\left(y-\frac{\sqrt{3}l}{2}\right) \Rightarrow \frac{\partial^2 \omega}{\partial x^2} = -6a\left(y-\frac{\sqrt{3}l}{2}\right)$$

$$\frac{\partial \omega}{\partial y} = a(2y-3x^2)\left(y-\frac{\sqrt{3}l}{2}\right) + a(y^2-3x^2)$$

$$\frac{\partial^2 \omega}{\partial y^2} = a(2-3x^2)\left(y-\frac{\sqrt{3}l}{2}\right) + a(2y-3x^2) + 2ay$$

$$\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} = -6a\left(y-\frac{\sqrt{3}l}{2}\right) + \cancel{a(2y} - a(2-3x^2)\left(y-\frac{\sqrt{3}l}{2}\right) + a(2y-3x^2) + 2ay$$

$$= a\left[-6y + 3\sqrt{3}l + 2y - \sqrt{3}l - 3x^2y + \frac{3\sqrt{3}}{2}lx^2 + 2y - 3x^2 + 2y\right.$$

$$\frac{\partial w}{\partial y} = 2ay\left(y - \frac{\sqrt{3}l}{2}\right) + a(y^2 - 3x^2)$$

$$\frac{\partial^2 w}{\partial y^2} = 2a\cdot\left(y - \sqrt{3}\frac{l}{2}\right) + 2ay + 2ay$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -6a\left(y - \frac{\sqrt{3}l}{2}\right) + 2a\left(y - \frac{\sqrt{3}l}{2}\right) + 4ay$$

$$= 6a\frac{\sqrt{3}l}{2} - 2a\frac{\sqrt{3}l}{2} = 2a\sqrt{3}l$$

$$\mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{dp}{dz}$$

$$\Rightarrow 2a\sqrt{3}l\,\mu = \frac{dp}{dz} \Rightarrow a = \frac{1}{2\sqrt{3}\mu l}\frac{dp}{dz}$$

$$\therefore w = -\frac{1}{2\sqrt{3}\mu l}\frac{dp}{dz}\left(y - \frac{\sqrt{3}l}{2}\right)(3x^2 - y^2)$$

$$Q = 2\int_{y=0}^{y=\frac{\sqrt{3}l}{2}}\int_{x=0}^{x=\frac{y}{\sqrt{3}}} w\,dx\,dy$$

$$\Rightarrow Q = 2a\int_{y=0}^{y=\frac{\sqrt{3}l}{2}}\left[y^3x - \frac{\sqrt{3}y^2xl}{2} - \frac{3x^3}{3}y + \frac{3\sqrt{3}}{2}\frac{x^3}{3}l\right]_0^{\frac{y}{\sqrt{3}}} dy$$

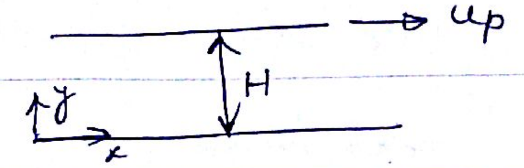
$$= 2a\int_0^{\frac{\sqrt{3}l}{2}}\left(\frac{2y^4}{3\sqrt{3}} - \frac{y^3 l}{3}\right) dy$$

$$= 2a\left(\frac{3}{80} - \frac{3}{64}\right)l^5$$

Substituting the value of a

$$Q = \frac{\sqrt{3}l^4}{320\mu}\left(-\frac{dp}{dz}\right)$$

Q4



(no imposed pr grad along x)

$$\rho\left[\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + v\frac{\partial u}{\partial y}\right] = -\cancel{\frac{\partial p}{\partial x}} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right)$$

(steady flow) 0 (translational invariance along x) 0

$$\Rightarrow \rho v \frac{du}{dy} = \mu \frac{d^2u}{dy^2}$$

continuity: $\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} = 0$ (2D + incompressible)

$\Rightarrow v \neq v(y)$, $v = -v_s$ at $y = 0, H \Rightarrow v = -v_s\ \forall y$

$$\Rightarrow -\rho v_s \frac{du}{dy} = \mu \frac{d^2u}{dy^2}$$

Let $\bar{u} = \frac{u}{u_p}$, $\bar{y} = \frac{y}{H}$ $\Rightarrow -\frac{\rho v_s u_p}{H}\frac{d\bar{u}}{d\bar{y}} = \frac{\mu u_p}{H^2}\frac{d^2\bar{u}}{d\bar{y}^2}$

$$\Rightarrow \frac{d^2\bar{u}}{d\bar{y}^2} + \epsilon\frac{d\bar{u}}{d\bar{y}} = 0 \qquad \text{where } \epsilon = \frac{\rho v_s H}{\mu}$$

Let $\bar{u} = u_0 + \epsilon u_1$

$$\Rightarrow \frac{d^2u_0}{d\bar{y}^2} + \epsilon\frac{d^2u_1}{d\bar{y}^2} + \epsilon\frac{du_0}{d\bar{y}} + \epsilon^2\frac{d^2u_1}{d\bar{y}^2} = 0$$

$O(\epsilon^0) \longrightarrow \frac{d^2u_0}{d\bar{y}^2} = 0 \Rightarrow u_0 = C_1\bar{y} + C_2$

bc (i) at $\bar{y} = 0, \bar{u}_0 = 0 \Rightarrow C_2 = 0$ (ii) at $\bar{y} = 1$, $u_0 = 1 \Rightarrow u_0 = \bar{y}$

$$O(\epsilon) \longrightarrow \frac{d^2u_1}{d\bar{y}^2} + \frac{du_0}{d\bar{y}} = 0$$

$\Rightarrow \frac{d^2u_1}{d\bar{y}^2} + 1 = 0 \Rightarrow \frac{du_1}{d\bar{y}} = -\bar{y} + C_3 \Rightarrow u_1 = -\frac{\bar{y}^2}{2} + C_3\bar{y} + C_4$

bc (i) At $\bar{y} = 0$, $u_1 = 0 \Rightarrow C_4 = 0$.

(ii) At $\bar{y} = 1$, $u_1 = 0 \Rightarrow C_3 = \frac{1}{2}$

$= \frac{\bar{y}}{2}(1 - \bar{y})$