Compressible Flows (Lectures 54 to 58)

Q1. Choose the correct answer

(i) Select the expression that gives the speed of a sound wave relative to the medium of propagation which is an ideal gas($\gamma = c_p / c_v$)

(a) $\sqrt{\gamma RT}$

(b) $\sqrt{\gamma \rho / p}$

(c) $\sqrt{\partial p / \partial \rho}$

(d) $\sqrt{c_p RT}$

[*Ans.(*a)]

(ii) The flow upstream of a shock is always

(a) subsonic

(b) supersonic

(c) sonic

(d) incompressible

[*Ans.(*b)]

Q2.

Air flows steadily and isentropically into an aircraft inlet at a rate of 100 kg/s. At section 1 where the cross-sectional area is 0.464 m^2, the Mach number, temperature and absolute pressure are found to be 3, -60°C and 15.0 kPa respectively. Determine the velocity and cross-sectional area downstream where $T = 138°\text{C}$.

Solution

We know that

$$T_{01} = T_1\left[1 + \frac{\gamma - 1}{2} Ma_1^2\right]$$

$$= 213\left[1 + \frac{1.4-1}{2}(3.0)^2\right] = 596 \text{ K}$$

Let the downstream where $T = 138°\text{C}$ be designated by 2. Then, one can write

$$\frac{T_{02}}{T_2} = \left[1 + \frac{\gamma - 1}{2} Ma_2^2\right]$$

For isentropic flow, we get

$$T_{02} = T_{01}$$

Hence,
$$Ma_2 = \left[\frac{2}{\gamma - 1}\left\{\frac{T_{01}}{T_2} - 1\right\}\right]^{1/2} = \left[\frac{2}{1.4-1}\left\{\frac{596}{411} - 1\right\}\right]^{1/2} = 1.5$$

Velocity of air at downstream is found to be

$$V_2 = Ma_2 C_2 = Ma_2 \left(\gamma R T_2\right)^{1/2} = 1.5 \times \left(1.4 \times 287 \times 411\right)^{1/2} = 610 \text{ m/s}$$

Now,
$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma - 1}} = \left(\frac{411}{213}\right)^{\frac{1}{1.4-1}} = 5.17$$

The density of air at section 1 is given by

$$\rho_1 = \frac{p_1}{RT_1} = \frac{15\times10^3}{287\times213} = 0.245 \text{ kg/m}^3$$

Mass flow rate can be expressed as

$$\dot{m} = \rho_2 V_2 A_2$$

Cross-sectional area at downstream is

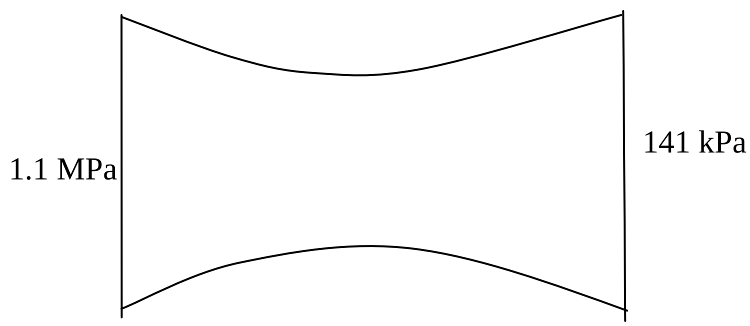
$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = \frac{\dot{m}}{5.17\rho_1 V_2} = \frac{100}{5.17\times0.245\times610} = 0.129 \text{ m}^2$$

Q3.

Air is to be expanded through a converging-diverging nozzle by a frictionless adiabatic process from a pressure of 1.10 MPa (abs) and a temperature of 115°C to a pressure of 141 kPa(abs). Determine the throat and exit areas for a well designed shockless nozzle if the mass flow rate is 2 kg/s.

Solution

The flow situation being considered is shown in the figure below.



We know that

$$\frac{p_0}{p} = \left[1+\frac{\gamma-1}{2}Ma^2\right]^{\frac{\gamma}{\gamma-1}}$$

Mach number at the exit is

$$Ma_1 = \left[\frac{2}{\gamma-1}\left\{\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}\right]^{1/2}$$

$$= \left[\frac{2}{1.4-1}\left\{\left(\frac{1.1}{0.141}\right)^{\frac{1.4-1}{1.4}} - 1\right\}\right]^{1/2} = 2.0$$

We know that

$$\frac{T_0}{T_1} = 1+\frac{\gamma-1}{2}Ma_1^2$$

Temperature of air at the exit is

$$T_1 = \frac{T_0}{1+\frac{\gamma-1}{2}Ma_1^2} = \frac{388}{1+\frac{1.4-1}{2}(2)^2} = 216\ \text{K}$$

Velocity of air at exit is found to be

$$V_1 = Ma_1 C_1 = Ma_1(\gamma R T_1)^{1/2} = 2.0\times(1.4\times 287\times 216)^{1/2} = 589\ \text{m/s}$$

The density of air at exit is given by

$$\rho_1 = \frac{p_1}{RT_1} = \frac{141\times 10^3}{287\times 216} = 2.27\ \text{kg/m}^3$$

Since $Ma_1 = 2.0$, nozzle must be chocked and $Ma_t = 1.0$.

Pressure at throat is

$$p_t = \frac{p_0}{\left[1+\frac{\gamma-1}{2}Ma_t^2\right]^{\frac{\gamma}{\gamma-1}}} = \frac{1.1\times 10^6}{\left[1+\frac{1.4-1}{2}(1)^2\right]^{3.5}} = 581\ \text{kPa}$$

Temperature at throat is

$$T_t = \frac{T_0}{1+\frac{\gamma-1}{2}Ma_t^2} = \frac{388}{1+\frac{1.4-1}{2}(1)^2} = 323\ \text{K}$$

The density of air at throat is given by

$$\rho_t = \frac{p_t}{RT_t} = \frac{581\times 10^3}{287\times 323} = 6.27\ \text{kg/m}^3$$

Velocity of air at throat is found to be

$$V_t = Ma_t C_t = Ma_t(\gamma R T_t)^{1/2} = 1.0(1.4\times 287\times 323)^{1/2} = 360\ \text{m/s}$$

Mass flow rate of air can be expressed as

$$\dot{m} = \rho_1 V_1 A_1 = \rho_t V_t A_t$$

Cross-sectional area at throat is

$$A_t = \frac{\dot{m}}{\rho_t V_t} = \frac{2}{6.27\times 360} = 8.86\times 10^{-4}\ \text{m}^2$$

Cross-sectional area at exit is

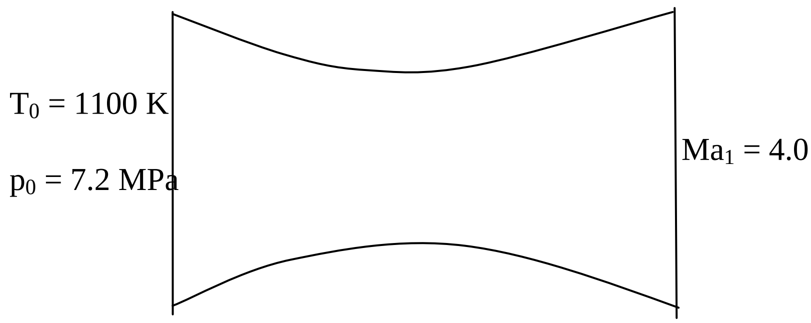
$$A_1 = \frac{\dot{m}}{\rho_1 V_1} = \frac{2}{2.27\times 589} = 1.5\times 10^{-3}\ \text{m}^2$$

Q4.

Air, at a stagnation pressure of 7.2 MPa (abs) and a stagnation temperature of 1100 K, flows isentropically through a converging-diverging nozzle having a throat area of 0.01 m^2. Determine the velocity at the downstream section where the Mach number is 4.0. Also find the mass flow rate.

Solution

The flow situation being considered is shown in the figure below.



We know that

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} Ma^2$$

Temperature of air at the exit is

$$T_1 = \frac{T_0}{1 + \frac{\gamma-1}{2} Ma_1^2} = \frac{1100}{1 + \frac{1.4-1}{2}(4)^2} = 262 \text{ K}$$

Velocity of air at exit is found to be

$$V_1 = Ma_1 C_1 = Ma_1 \left(\gamma R T_1\right)^{1/2} = 4.0\left(1.4 \times 287 \times 262\right)^{1/2} = 1300 \text{ m/s}$$

Since $Ma_1 = 4.0$, nozzle must be chocked and $Ma_t = 1.0$

Pressure and temperature at throat are found to be

$$p_t = \frac{p_0}{\left[1 + \frac{\gamma-1}{2} Ma_t^2\right]^{\frac{\gamma}{\gamma-1}}} = \frac{7.2 \times 10^6}{\left[1 + \frac{1.4-1}{2}(1)^2\right]^{3.5}} = 3.8 \text{ MPa}$$

$$T_t = \frac{T_0}{1 + \frac{\gamma-1}{2} Ma_t^2} = \frac{1100}{1 + \frac{1.4-1}{2}(1)^2} = 917 \text{ K}$$

The density of air at downstream is given by

$$\rho_1 = \frac{p_1}{RT_1} = \frac{3.8 \times 10^6}{287 \times 917} = 14.4 \text{ kg/m}^3$$

Velocity of air at throat is found to be

$$V_t = Ma_t C_t = Ma_t \left(\gamma R T_t\right)^{1/2} = 1.0\left(1.4 \times 287 \times 917\right)^{1/2} = 607 \text{ m/s}$$

Mass flow rate of air is

$$\dot{m} = \rho_t V_t A_t = 14.4 \times 607 \times 0.01 = 87.4 \text{m/s}$$