# Pipe Flows (Lectures 45 to 47)

Q1. Choose the correct answer

(i) While deriving an expression for loss of head due to a sudden expansion in a pipe, in addition to the continuity and impulse-momentum equations, one of the following assumptions is made:

(a) head loss due to friction is equal to the head loss is eddying motion
(b) the mean pressure in eddying fluid is equal to the downstream pressure
(c) the mean pressure in eddying fluids is equal to the upstream pressure
(d) head loss in eddies is neglected

[*Ans.(*c)]

(ii) Which of the following statements is correct?

(a) Energy grade line lies above the hydraulic grade line and is always parallel to it
(b)Energy grade line lies above the hydraulic grade line and they are separated from each other by a vertical distance equal to the velocity head
(c)The hydraulic grade line slopes upwards meeting the energy grade line only at the exit of flow
(d) Hydraulic grade line and energy grade line are the same in fluid flow problems.

[*Ans.(*b)]

(iii) For pipes arranged in series

(a) the flow may be different in different pipes
(b) the head loss per unit length must be more in a smaller pipe
(c) the head loss must be the same in all pipes
(d) the flow rate must be the same in all pipes

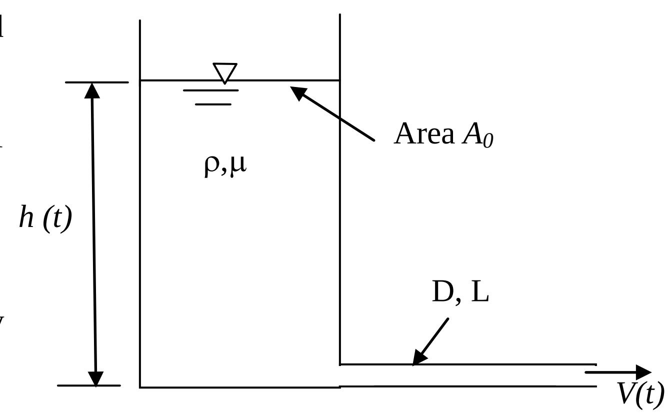
[*Ans.(*d)]

Q2.

A tank of area $A_0$ is draining in laminar flow through a pipe of diameter $D$ and length $L$, as shown in the figure.

(i)Neglecting the exit-jet kinetic energy and assuming the pipe flow is driven by the hydrostatic pressure at its entrance, derive a formula for the tank level $h(t)$ if its initial level is $h_0$, assuming the pipe flow to be laminar.

(ii)Repeat the above derivation without neglecting the exit-jet kinetic energy and assuming the pipe flow to be highly turbulent.

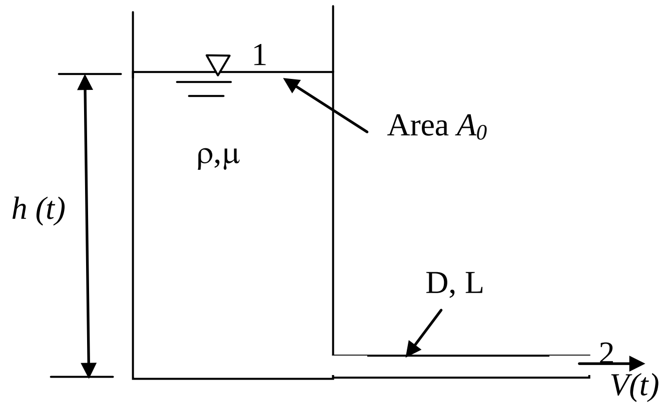



**Solution**

Applying energy equation between sections 1 and 2, we obtain

$$\frac{p_{atm}}{\rho g} + \frac{V_1^2}{2g} + h(t) = \frac{p_{atm}}{\rho g} + \frac{\overline{V}(t)^2}{2g} + 0 + h_l$$

(i)Neglecting $\frac{\bar{V}(t)^2}{2g}$ and entry loss ( given) and as $V_1$ is negligible as compared to $\bar{V}(t)$, the above equation becomes




$$h(t) = h_l$$

Considering laminar flow, we have

$$h_l = \frac{32\mu \bar{V} L}{\rho g D^2}$$

Thus, one can write

$$h(t) = \frac{32\mu \bar{V} L}{\rho g D^2}$$

or
$$\bar{V} = \frac{\rho g D^2}{32\mu \bar{V} L} h(t)$$

Again, from continuity equation, we get

$$A_{pipe}\bar{V} = -A_0 \frac{dh(t)}{dt}$$

or
$$\frac{\pi}{4} D^2 \frac{\rho g D^2}{32\mu L} h(t) = -A_0 \frac{dh(t)}{dt}$$

Integrating the above equation, we obtain

$$h(t) = h_0 \exp\left(\frac{-\pi D^4 \rho g t}{128\mu L A_0}\right)$$

(ii) Given that the flow is highly turbulent, therefore friction factor $f \approx$ constant .
From energy equation with the consideration of exit kinetic energy, we have

$$h(t) = \frac{\bar{V}(t)^2}{2g} + f\frac{L}{D}\frac{\bar{V}(t)^2}{2g}$$

or
$$\bar{V}(t) = \sqrt{\frac{2gh(t)}{1 + f\frac{L}{D}}}$$

Again, from continuity equation, we get

$$A_{pipe}\bar{V} = -A_0 \frac{dh(t)}{dt}$$

or
$$\frac{\pi}{4} D^2 \sqrt{\frac{2gh(t)}{1 + f\frac{L}{D}}} = -A_0 \frac{dh(t)}{dt}$$

Integrating the above equation, we obtain

$$h(t) = \sqrt{h_0} - \frac{\pi}{8}\frac{D^2 t}{A_0}\sqrt{\frac{2g}{1 + f\frac{L}{D}}}$$

Q3.

A single uniform pipe joins two reservoirs. Calculate the percentage increase of flow rate obtainable if, from the midpoint of this pipe, another of the same diameter is added in parallel to it. Neglect all losses except pipe friction and assume a constant and equal $f$ for both pipes.

**Solution**

Let the diameter of the pipe be *D*.

Case 1:

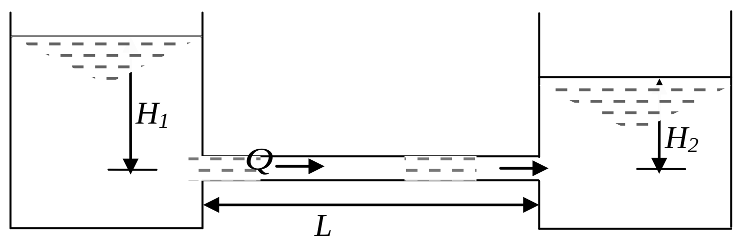
When the single pipe joins two reservoirs, as shown in the figure below, the loss of head is

$$h_{f1} = f\frac{L}{D}\frac{V^2}{2g}$$

where is $V$ the average velocity of fluid in the pipe.

For this case, the discharge is given by

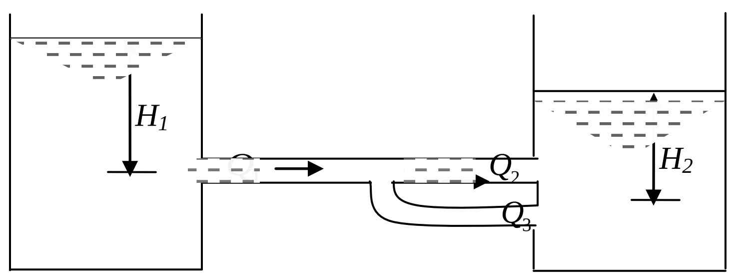
$$Q = AV$$




Case 2:

When another pipe is added in parallel to the main pipe from the midpoint as shown in the figure below, the loss of head is

$$h_{f2} = f\frac{\frac{L}{2}}{D}\frac{V_1^2}{2g} + f\frac{\frac{L}{2}}{D}\frac{V_2^2}{2g} \qquad (1)$$




From continuity equation, we have

$$Q_1 = Q_2 + Q_3$$

Since the diameter and length of both the parallel pipes are same, we have

$$Q_2 = Q_3$$

$\therefore \qquad Q_2 = Q_3 = \dfrac{Q_1}{2}$

or $\qquad V_2 = V_3 = \dfrac{V_1}{2}$

Substituting the value of $V_2$ in Eq.(1), we get

$$h_{f2} = f\,\frac{\frac{L}{2}}{D}\frac{V_1^2}{2g} + f\,\frac{\frac{L}{2}}{D}\frac{\frac{V_1^2}{4}}{2g} = \frac{5}{8}f\,\frac{L}{D}\frac{V_1^2}{2g}$$

Equating the head losses, we have

$$h_{f_1} = h_{f2}$$

$$f\,\frac{L}{D}\frac{V^2}{2g} = \frac{5}{8}f\,\frac{L}{D}\frac{V_1^2}{2g}$$

or $\qquad V_1 = 1.26V$

For this case, the discharge is given by

$$Q_1 = AV_1 = 1.26AV$$

Therefore, the percentage increase in the flow rate is given by

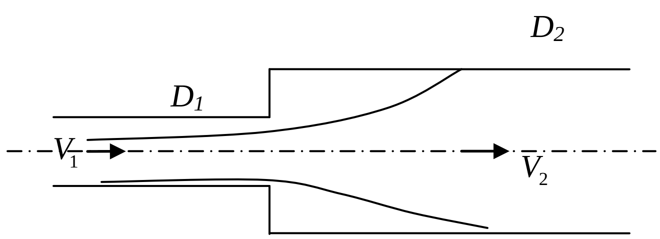
$$\frac{Q_1 - Q}{Q} = \frac{1.26AV - AV}{AV} = 0.26 \text{ or } 26\%$$

Q4.

There is a sudden increase in the diameter of a pipe from $D_1$ to $D_2$. If the minor loss is independent of the direction of flow, what would be the value of $\dfrac{D_1}{D_2}$? Assume coefficient of contraction $C_c = 0.62$.

***Solution***

The schematic diagram of the pipe is shown in the figure below.




Let the velocities corresponding to diameters $D_1$ and $D_2$ be $V_1$ and $V_2$ respectively.

From continuity equation, we have

$$Q = A_1V_1 = A_2V_2$$

or $$V_2 = \frac{A_1}{A_2}V_1 = \frac{\frac{\pi}{4}D_1^2}{\frac{\pi}{4}D_2^2}\times V_1 = \frac{D_1^2}{D_2^2}V_1$$

Loss of head due to sudden enlargement is given by

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{\left(V_1 - \frac{D_1^2}{D_2^2} V_1\right)^2}{2g} = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 \frac{V_1^2}{2g}$$

If the direction of flow is reversed, there will be a sudden contraction from $D_2$ to $D_1$. Then the loss of head due to sudden contraction is given by

$$h_c = \frac{V_1^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{V_1^2}{2g}\left[\frac{1}{0.62} - 1\right]^2 = 0.6126\frac{V_1^2}{2g}$$

When the minor loss is independent of the direction of flow, the loss of head due to sudden enlargement should be equal to the loss of head due to sudden contraction. Therefore, we have

$$\left(1 - \frac{D_1^2}{D_2^2}\right)^2 \frac{V_1^2}{2g} = 0.6126\frac{V_1^2}{2g}$$

or $$\left(1 - \frac{D_1^2}{D_2^2}\right)^2 = 0.6126$$

or $$1 - \frac{D_1^2}{D_2^2} = \sqrt{0.6126} = 0.7827$$

or $$\frac{D_1^2}{D_2^2} = 1 - 0.7827 = 0.2173$$

or $$\frac{D_1}{D_2} = \sqrt{0.2173} = 0.466$$