Fluid Statics and Fluid Under Rigid Body Motion (Lectures 7 to 10)

Q1. **Choose the correct answer**

(i) A static fluid can have
(a) non-zero normal and shear stress
(b) zero egative normal stress and zero shear stress
(c) non-zero normal stress and zero shear stress
(d) zero normal stress and non-zero shear stress

[*Ans.(*c)]

(ii) The piezometric head in a static liquid
(a) remains constant at all points in the liquid
(b) increases linearly with depth below a free surface
(c) decreases linearly with depth below a free surface
(d) remains constant only in a horizontal plane

[*Ans.(*b)]

(iii) The centre of pressure of a liquid on a plane surface immersed vertically in a static body of liquid, always lies below the centroid of the surface area, because
(a) there is no shear stress in liquids at rest
(b) the liquid pressure is constant over depth
(c) in liquids the pressure acting is same in all directions
(d) the liquid pressure increases linearly with depth

[*Ans.(*d)]

(iii) The line of action of buoyancy force acts through the
(a) centre of gravity of any submerged body
(b) centroid of the volume of any floating body
(c) centroid of the displaced volume of fluid
(d) centroid of the volume of fluid vertically above the body

[*Ans.(*c)]

(iv) How is the metacentric height, *GM* expressed?

(a) $GM = BG + \frac{I}{\forall}$ (b) $GM = \frac{\forall}{I} - BG$

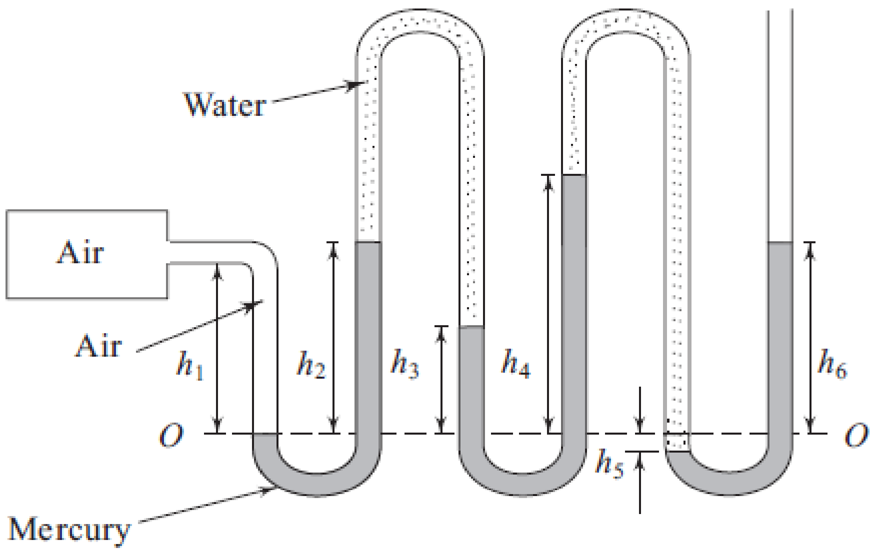
(c) $GM = \frac{I}{\forall} - BG$ (d) $GM = BG - \frac{\forall}{I}$

where I = Moment of inertia of the plan of the floating body at the water surface
$\forall$ = Volume of the body submerged in water
BG = Distance between the centre of gravity (G) and the centre of buoyancy (B).

[*Ans.(*c)]

Q2.

A multitube manometer using water and mercury is used to measure the pressure of air in a vessel, as shown in the figure below. For the given values of heights, calculate the gauge pressure in the vessel. $h_1 = 0.4$ m, $h_1 = 0.5$ m, $h_1 = 0.3$ m, $h_1 = 0.7$ m, $h_1 = 0.1$ m and $h_1 = 0.5$ m.



Solution

Let the gauge pressure of air in the vessel is p_A

Starting from the air vessel, the pressure is continuously tracked through the liquid path in the manometer to reach at the open end where the gauge pressure is zero.

Therefore,

$$p_A - h_2\rho_m g + (h_2 - h_3)\rho_w g - (h_4 - h_3)\rho_m g + (h_4 + h_5)\rho_w g - (h_5 + h_6)\rho_m g = 0$$

or

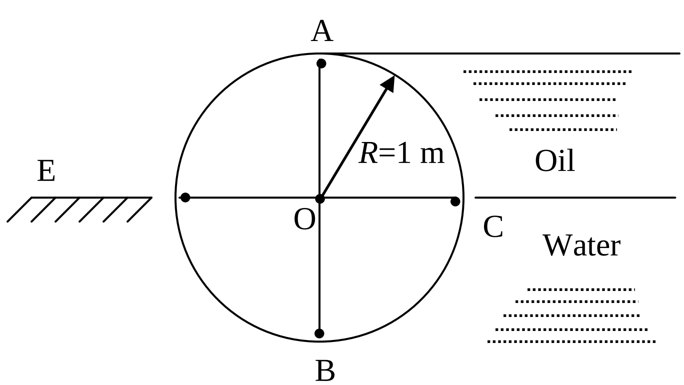
$$p_A = g\left[(\rho_m - \rho_w)(h_2 - h_3 + h_4 + h_5) + \rho_m h_6\right]$$

$$= 9.81\left[(13.6-1)\times 10^3(0.5-0.3+0.7+0.1)+13.6\times 10^3\times 0.5\right]$$

$$= 190314 \text{ N/m}^2 = 190.31 \text{ kN/m}^2$$

Q3.

Find the weight of the cylinder (dia=2 m) per meter length if it supports water and oil (Sp. gr. 0.82) as shown in Fig. below. Assume contact with wall as frictionless.



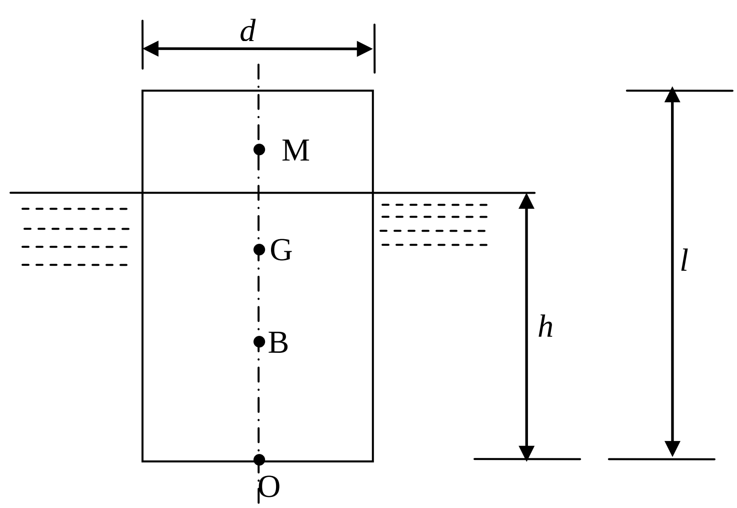
Solution
In absence of friction at contact with wall, the net upward vertical component of the hydrostatic force will be balanced by the weight of the cylinder.
The net upward vertical component of hydrostsatic force F_V equals to the weight of water corresponding to the volume OCBO - the weight of oil corresponds to the volume of ACOA
Hence,

$$F_V = \frac{\pi \times (1)^2}{4} \times 10^3 \times 9.81 - \frac{\pi \times (1)^2}{4} \times 0.82 \times 10^3 \times 9.81$$
$$= \frac{\pi}{4} \times 9.81 \times 0.18 \times 10^3 \text{ N}$$
$$= 1.39 \times 10^3 \text{ N} = 1.39 \text{ kN}$$

Q4.
A uniform wooden cylinder has a specific gravity of 0.6. Find the ratio of diameter to length of the cylinder so that it will just float upright in a state of neutral equilibrium in water.
Solution
Let the submerged length of the cylinder be h.
From the condition of floating equilibrium with the axis vertical,

$$\frac{\pi d^2}{4} \times l \times 0.6 \times 10^3 \times 9.81 = \frac{\pi d^2}{4} \times h \times 10^3 \times 9.81$$

or $\quad h = 0.6l$
(l is the length of the cylinder and d is the diameter of the cylinder)
Let B, G and M be the centre of buoyancy, centre of gravity and metacentre of the cylinder (shown in the figure below) respectively.
Distance of centre of gravity from the base $OG = 0.5l$
Distance of centre of buoyancy from the base $OB = \frac{h}{2} = \frac{0.6l}{2} = 0.3l$
$\therefore \quad BG = OG - OB = 0.5l - 0.3l = 0.2l$



Then, metacentric height is given by

$$GM = BM - BG = \frac{I_{YY}}{\forall} - BG$$

where I_{YY} is the moment of inertia of the plane of floatation about the centroidal axis perpendicular to the plane of rotation and is given by

$$. \; I_{YY} = \frac{\pi d^4}{64}$$

where, L be the length of the block in a direction perpendicular to the plane of the figure and $\forall$ is the volume of submerged potion and is given by

$$\forall = \frac{\pi d^4}{4} \times 0.6l$$

Thus, $$BM = \frac{I_{YY}}{\forall} = \frac{\frac{\pi d^4}{64}}{\frac{\pi d^4}{4} \times 0.6l} = \frac{d^2}{9.6l}$$

Therefore, $$GM = \frac{I_{YY}}{\forall} - BG$$

$$= \frac{d^2}{9.6l} - 0.2l$$

For the state of neutral equilibrium,

$$GM = 0$$

Hence, $$\frac{d^2}{9.6l} - 0.2l = 0$$

or $$\frac{d}{l} = 1.386$$

Q5.

Find the minimum apex angle of a solid cone of specific gravity 0.8 so that it can float in stable equilibrium in fresh water with its axis vertical and vertex downward.

Solution

Let 2α and H be the apex angle and the height of the cone respectively. If h is the depth of submergence of the cone in water, then from floating equilibrium with the axis vertical and vertex downwards, we have

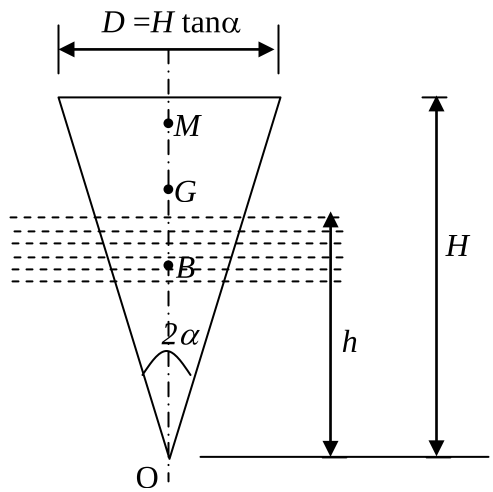
$$\frac{1}{3}\pi\left(H \tan\alpha\right)^2 H \times 0.8 = \frac{1}{3}\pi\left(h \tan\alpha\right)^2 h$$

or $$h = \sqrt[3]{0.8}H = 0.928H$$

The distance between the centre of buoyancy and centre of gravity along the axis can be written as

$$BG = \frac{3}{4}\left(H - h\right)$$

$$= \frac{3}{4}\left(1 - 0.928\right)H = 0.054H$$



The distance between the centre of buoyancy B and metacentre M can be written as

$$BM = \frac{\frac{\pi}{4}\left(h\tan\alpha\right)^4}{\frac{\pi}{3}\left(h\tan\alpha\right)^2 \times h} = \frac{3h}{4}\tan^2\alpha$$

Therefore,

the metacentric height $GM = BM - BG = \frac{3h}{4}\tan^2\alpha - 0.054H$

For stable equilibrium, metacentric height should be positive. That means

$$GM > 0$$

Hence,

$$\frac{3h}{4}\tan^2\alpha - 0.054H > 0$$

or $$\frac{3\times 0.928}{4}H\tan^2\alpha - 0.054H > 0$$

or $$\tan^2\alpha > \frac{4\times 0.054}{3\times 0.928}$$

or $$\tan\alpha > 0.278$$

or $$\alpha > 15.56^\circ$$

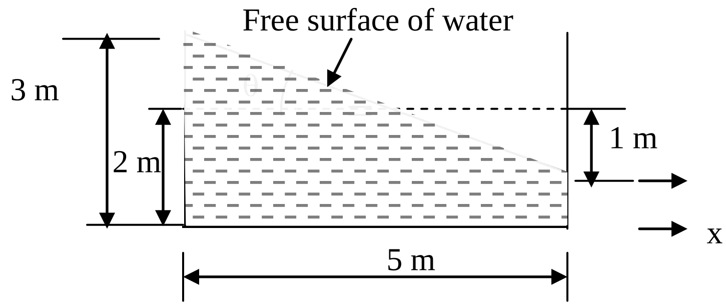
Therefore, the minimum apex angle $2\alpha_{\text{minimum}} = 2\times 15.56^\circ = 31.12^\circ$

Q8.

An open rectangular tank of $5\text{ m}\times 4\text{ m}\times 4\text{ m}$ is 3 m high. It contains water up to a height of 2 m and is accelerated horizontally along the longer side. Determine the maximum acceleration that can be given without spilling the water and also calculate the percentage of water spilt over, if this acceleration is increased by 20%. Derive the formula used for the solution of the problem.

Solution

For the maximum acceleration without spilling the water, the free surface takes the shape as shown in figure below.



From the geometry of the figure above, we have

$$\tan\theta = \frac{a_{x,\max}}{g}$$

or $$\frac{a_{x,\max}}{g} = \frac{1}{2.5} = 0.4$$

or $$a_{x,\max} = 0.4g = 0.4 \times 9.81 = 3.924 \text{ m/s}^2$$

This is the maximum acceleration that can be given without spilling the water.

Now the acceleration is increased by 20%. Therefore, the new acceleration is $a_x = 3.924 \times 1.2 = 4.7088 \text{ m/s}^2$

Now, $$\tan\theta = \frac{h}{L} = \frac{a_x}{g}$$

or $$\frac{h}{5} = \frac{4.7088}{9.81}$$

or $$h = 2.4 \text{ m}$$

The new configuration is shown in the figure below.

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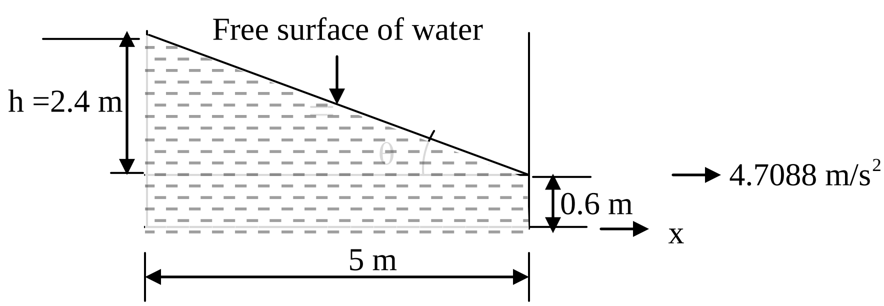

Volume of water left in the tank is

$$= \left[\frac{1}{2} \times 2.4 \times 5 + 0.6 \times 5\right] \times 4$$

$$= 36 \text{ m}^3$$

Initial volume of water in the tank is

$$= 5 \times 4 \times 2 = 40 \text{ m}^3$$

Percentage of water spilt over is

$$\frac{40 - 36}{40} \times 100 = 10\%$$