After Lecture 6

Q1. **Choose the correct answer**

(i) A hydraulic turbine rotates at N rpm operating under a net head H and having a discharge Q while developing an output power P. The specific speed is expressed as

(a) $\dfrac{N\sqrt{P}}{\rho^{1/2}\left(gH\right)^{5/4}}$

(b) $\dfrac{N\sqrt{Q}}{\left(gH\right)^{3/4}}$

(c) $\dfrac{N\sqrt{P}}{\rho^{1/2}\left(gH\right)^{3/4}}$

(d) $\dfrac{N\sqrt{Q}}{\rho^{1/2}\left(gH\right)^{5/4}}$

[*Ans.(*a)]

(ii) Two hydraulic turbines are similar and homologous when there are geometrically similar and have

(a) the same specific speed
(b) the same rotational speed
(c) the same Froude number
(d) the same Thoma's number

[*Ans.(*a)]

Q2.

A radial flow hydraulic turbine is required to be designed to produce 20 MW under a head of 16 m at a speed of 90 rpm. A geometrically similar model with an output of 30 kW and a head of 4 m is to be tested under dynamically similar conditions. At what speed must the model be run? What is the required impeller diameter ratio between the model and the prototype and what is the volume flow rate through the model if its efficiency can be assumed to be 90%.?

Solution

Equating the power coefficients (π term containing the power P) for the model and prototype, we can write

$$\frac{P_1}{\rho_1 N_1^3 D_1^5} = \frac{P_2}{\rho_2 N_2^3 D_2^5}$$

(where subscript 1 refers to the prototype and subscript 2 to the model).

Considering the fluids to be incompressible, and same for both the prototype and model, we have

$$D_2/D_1 = \left(P_2/P_1\right)^{1/5}\left(N_1/N_2\right)^{3/5}$$
$$= \left(0.03/20\right)^{1/5}\left(N_1/N_2\right)^{3/5}$$

or $$D_2/D_1 = 0.272\left(N_1/N_2\right)^{3/5} \qquad (1)$$

Equating the head coefficients (π term containing the head *H*)

$$\frac{gH_1}{\left(N_1D_1\right)^2} = \frac{gH_2}{\left(N_2D_2\right)^2}$$

Then

$$D_2/D_1 = \left(H_2/H_1\right)^{1/2}\left(N_1/N_2\right)$$

or $$D_2/D_1 = \left(4/16\right)^{1/2}\left(N_1/N_2\right) \qquad (2)$$

Therefore, equating the diameter ratios from Eqs (1) and (2), we have

$$0.272\left(N_1/N_2\right)^{3/5} = \left(4/16\right)^{1/2}\left(N_1/N_2\right)$$

or $$\left(N_2/N_1\right)^{2/5} = 1.84$$

Hence $$N_2 = N_1\left(1.84\right)^{5/2} = 90\times\left(1.84\right)^{5/2} = 413.32 \text{ rpm}$$

From Eq.(1)

$$D_2/D_1 = 0.272\left(90/413.32\right)^{3/5} = 0.11$$

Model efficiency $$= \frac{\text{Power output}}{\text{Water power input}}$$

Hence, $$0.9 = \frac{30\times10^3}{\rho QgH}$$

or $$Q = \frac{30\times10^3}{0.9\times10^3\times9.81\times4} = 0.85 \text{ m}^3\text{/s}$$

Therefore, model volume flow rate = 0.85 m^3/s

After Lecture 8

Q1. **Choose the correct answer**

(i) Governing of turbines means

(a) the discharge is kept constant under all conditions

(b) the speed is kept constant under all conditions(loads)

(c) allow the turbine to run at ‘ runaway’ speed

(d) the power developed is kept constant under all conditions

[*Ans.(*b)]

Q2.

A Pelton wheel works at the foot of a dam because of which the head available at the nozzle is 400 m. the nozzle diameter is 160 mm and the coefficient of velocity is 0.98. the diameter of the wheel bucket circle is 1.75 m and the buckets deflect the jet by 150°. The wheel-to-jet speed ratio is 0.46. Neglecting friction, calculate (i) the power developed by the turbine, (ii) its speed and (iii) hydraulic efficiency.

Solution

Inlet jet velocity is

$$V_1 = C_v\sqrt{2gH} = 0.98\times\sqrt{2\times 9.81\times 400} = 86.82 \text{ m/s}$$

Flow rate $Q = \frac{\pi}{4}d^2V_1$

$$= \frac{\pi}{4}\times(0.16)^2\times 86.82 = 1.74 \text{ m}^3\text{/s}$$

Wheel speed is $U = 0.46\times 86.62 = 39.94$ m/s

Therefore, the rotational speed

$$N = \frac{60U}{\pi D} = \frac{60\times 39.94}{\pi\times 1.75} = 435.9 \text{ rpm}$$

Velocity of jet relative to wheel at inlet

$$V_{r1} = V_1 - U = 86.82 - 39.94 = 46.88 \text{ m/s}$$

In absence of friction

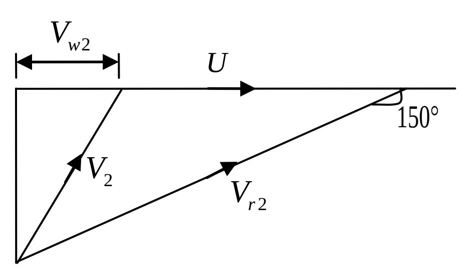
$$V_{r2} = V_{r1} = 46.88 = 46.88 \text{ m/s}$$

(V_{r2} is the velocity of jet relative to wheel at outlet).

Tangential component of inlet jet velocity

$$V_{w1} = V_1 = 86.82 \text{ m/s}$$

From outlet velocity triangle as shown,



$$V_{w2} = V_{r2}\cos\beta_2 - U_2$$
$$= 46.88\cos 30° - 39.94$$
$$= 0.66 \text{ m/s}$$

Power developed $= 10^3\times 1.74\times(86.82 + 0.66)\times 39.94$ W

$$= 6.08 \text{ MW}$$

Hydraulic efficiency $= \dfrac{6.08\times10^6}{10^3\times1.74\times9.81\times400} = 0.8905$ or 89.05%

Q3.

A powerhouse is equipped with Pelton type impulse turbines. Each turbine delivers a power of 14 MW when working under a head of 900 m and running at 600 rpm. Find the diameter of the jet and mean diameter of the wheel. Assume that the overall effiiciency is 89%, velocity coefficient of the jet 0.98, and speed ratio 0.46.

Solution

Power supplied by the water to the turbine

$$= \frac{14\times10^6}{0.89} = 15.73\times10^6 \text{ W}$$

Flow rate $Q = \dfrac{15.73\times10^6}{10^3\times9.81\times900} = 1.78 \text{ m}^3\text{/s}$

Inlet jet velocity is

$$V_1 = C_v\sqrt{2gH} = 0.98\times\sqrt{2\times9.81\times900} = 130.22 \text{ m/s}$$

If d is the diameter of water jet, then

$$1.78 = \frac{\pi}{4}d^2\times130.22$$

or $d = \left(\dfrac{4\times1.78}{\pi\times130.22}\right)^{1/2} = 0.132 \text{ m} = 132 \text{ mm}$

Blade speed is $U = 0.46\times130.22 = 59.90 \text{ m/s}$

Hence mean diameter of the wheel

$$D = \frac{60U}{\pi N} = \frac{60\times59.90}{\pi\times600} = 1.91 \text{ m}$$

After Lecture 12

Q1. **Choose the correct answer**

(i) The use of draft tube in a reaction turbine helps to
(a) provide safety to turbine
(b) increase the flow rate
(c) transport water to downstream without eddies
(d) reconvert residual kinetic energy to pressure energy

[*Ans.(*d)]

(ii) A turbomachine becomes more susceptible to cavitation if
(a) velocity attains a high value
(b) temperature rises above a critical value
(c) pressure falls velow the vapour pressure
(d) Thomas cavitation parameter exceeds a certain limit

[*Ans.(*c)]

(iii) Which place in hydraulic turbine is most susceptible for cavitation
(a) inlet of draft tube
(b) draft tube exit
(c) blade inlet
(d) guide blade

[*Ans.(*a)]

Q2.
Show that when runner blade angle at inlet of a Francis turbine is 90° and the velocity of flow is constant, the hydraulic efficiency is given by $\frac{2}{2+\tan^2\alpha}$, where α is the vane angle.

Solution

Work equivalent head

$$W = \frac{V_1^2 - V_2^2}{2g} + \frac{V_{r2}^2 - V_{r1}^2}{2g} + \frac{U_1^2 - U_2^2}{2g}$$

From the inlet and outlet velocity triangles,

$$V_1 = U_1 \sec\alpha$$

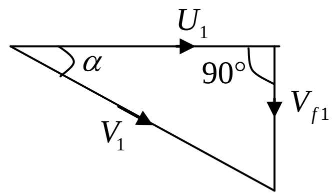
$$V_2 = V_{f2} = U_1 \tan\alpha = U_2 \tan\beta \qquad \left(\text{since, } V_{f2} = V_{f2}\right)$$

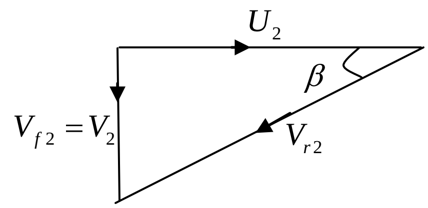
Using these relations into the expression of W, we have

$$W = \frac{1}{2g}\left[U_1^2 \sec^2\alpha - U_1^2 \tan^2\alpha + U_2^2 \sec^2\beta - U_1^2 \tan^2\alpha + U_1^2 - U_2^2\right]$$

$$= \frac{1}{2g}\left[U_1^2 + U_2^2 + U_2^2 \tan^2\beta - U_1^2 \tan^2\alpha + U_1^2 - U_2^2\right]$$

$$= \frac{1}{2g} \times 2U_1^2 = \frac{U_1^2}{g}$$





Inlet velocity triangle

Outlet velocity triangle

Available head $\quad H = \text{Work head} + \text{Energy rejected from turbine}$

$$= \frac{U_1^2}{g} + \frac{V_2^2}{2g}$$

$$= \frac{1}{2g}\left(2U_1^2 + V_2^2\right)$$

$$= \frac{1}{2g}\left(2U_1^2 + U_1^2 \tan^2 \alpha\right)$$

$$= \frac{U_1^2}{2g}\left(2 + \tan^2 \alpha\right)$$

Therefore, hydraulic efficiency

$$\eta_h = \frac{\text{Work equivalent head}}{\text{Available head}}$$

$$= \frac{U_1^2 / g}{U_1^2\left(2 + \tan^2 \alpha\right) / 2g} = \frac{2}{2 + \tan^2 \alpha}$$

Q3.

A Francis turbine has a wheel diameter of 1.2 m at the entrance and 0.6 m at the exit.the blade angle at the entrance is 90° and the guide vane angle is 15°. The water at the exit leaves the blades without any tangential velocity. The available head is 30 m and the radial component of flow velocity is constant. What would be the speed of the wheel in rpm and blade angle at the exit? Ignore friction.

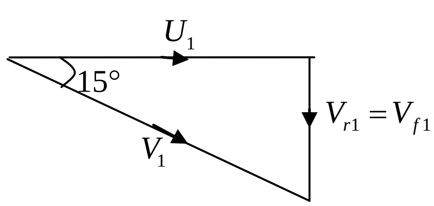
Solution

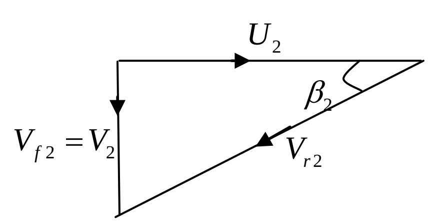
$$\frac{U_1}{U_2} = \frac{D_1}{D_2} = \frac{1.2}{0.6} = 2$$

From the inlet and outlet velocity triangles,

$$V_{f1} = U_1 \tan 15° \text{ and}$$

$$V_{f2} = U_2 \tan \beta_2$$





Inlet velocity triangle

Outlet velocity triangle

Since the flow velocity is constant

$$U_1 \tan 15° = U_2 \tan \beta_2$$

or $$\tan \beta_2 = \frac{U_1}{U_2} \tan 15° = 2 \tan 15° = 0.536$$

Hence $$\beta_2 = \tan^{-1}(0.536) = 28.2°$$

Available head to the turbine

$$H = \frac{V_1^2}{2g} + \frac{p_1}{\rho g}$$

Since the discharge pressure is atmospheric, the inlet pressure head $p_1/\rho g$ (above atmospheric pressure head) can be written as

$$\frac{p_1}{\rho g} = \frac{V_{r2}^2 - V_{r1}^2}{2g} + \frac{U_1^2 + U_2^2}{2g}$$

(friction in the runner is neglected).

Therefore,

$$H = \frac{V_1^2}{2g} + \frac{V_{r2}^2 - V_{r1}^2}{2g} + \frac{U_1^2 + U_2^2}{2g}$$

From the inlet and outlet velocity triangles,

$$V_1 = \frac{U_1}{\cos 15°} = \frac{2U_2}{\cos 15°} = 2.07U_2$$

$$V_{r1} = U_1 \tan 15° = 2U_2 \tan 15° = 0.536U_2$$

$$V_{r2} = \frac{U_2}{\cos 28.2°} = 1.132U_2$$

inserting the values of V_1, V_{r1}, V_{r2} and U_1 in terms of U_2 in the expression of H, we have

$$H = 30 = \frac{U_2^2}{2g}\left[(2.07)^2 + (1.13)^2 - (0.536)^2 + 3\right]$$

$$= \frac{8.27U_2^2}{2g}$$

or $$U_2 = \sqrt{\frac{2 \times 9.81 \times 30}{8.27}} = 8.44 \text{ m/s}$$

Hence, $$D = \frac{60 \times 8.44}{\pi \times 0.6} = 268.65 \text{ rpm}$$

Q4.

A Kaplan turbine develops 10 MW under a head of 4.3 m. taking a speed ratio of 1.8, flow ratio of 0.5, boss diameter 0.35 times the outer diameter and overall efficiency of 90%, find the diameter and speed of the runner.

Solution

$$\text{Power available} = \frac{\text{Power delivered}}{\text{overall efficiency}}$$

Therefore,

$$10^3 \times \dot{Q} \times 9.81 \times 4.3 = \frac{10 \times 10^6}{0.9}$$

or $$\dot{Q} = 263.4 \text{ m}^3\text{/s}$$

The axial velocity $$V_a = 0.5 \times \sqrt{2 \times 9.81 \times 4.3} = 4.59 \text{ m/s}$$

Let the outer diameter of the runner be d_o.

Then,

$$\dot{Q} = \frac{\pi}{4} d \left\{ {}^{2}_{o} - \left(0.35 d_o\right)^2 \right\} \times V_a$$

or $$d_o = \sqrt{\frac{4 \times 263.4}{\pi \left\{1 - \left(0.35\right)^2\right\} \times 4.59}} = 9.12 \text{ m}$$

Blade speed at outer diameter

$$U = 1.8 \times \sqrt{2 \times 9.81 \times 4.3} = 16.53 \text{ m/s}$$

Hence $$N == \frac{60 \times 16.53}{\pi \times 9.12} = 34.6 \text{ rpm}$$

Q5.

The following data refer to an elbow type draft tube:

Area of circular inlet = 25 m^2

Area of rectangular outlet = 116 m^2

Velocity of water at inlet to draft tube = 10 m/s

The frictional head loss in the draft tube equals to 10% of the inlet velocity head.

Elevation of inlet plane above tail race level = 0.6 m

Determine (i) vacuum or negative head at the inlet, and (ii) power thrown away in tail race.

Solution

Velocity of water at outlet from draft tube

$$= \frac{10 \times 25}{116} = 2.15 \text{ m/s}$$

Let p_1 be the pressure at inlet to the draft tube. Applying energy equation between the inlet and outlet of the draft tube, we have

$$\frac{p_1}{\rho g} + \frac{10^2}{2 \times 9.81} + 0.6 = 0 + \frac{\left(2.15\right)^2}{2 \times 9.81} + 0 + 0.1 \times \frac{10^2}{2 \times 9.81}$$

or $$\frac{p_1}{\rho g} = \frac{\left(2.15\right)^2}{2 \times 9.81} - 0.9 \times \frac{10^2}{2 \times 9.81} - 0.6$$

or $$\frac{p_1}{\rho g} = -4.95 \text{ m}$$

(ii) Power thrown away in tail race

$$= 10^3 \times 10 \times 25 \times \frac{2.15^2}{2} \text{ W}$$

$$= 578 \text{ kW}$$

After Lecture 18

Q1. **Choose the correct answer**

(i) A hydraulic pump rotates at N rpm operating under a net head H and having a discharge Q . The specific speed is expressed as

(a) $\dfrac{N\sqrt{P}}{\rho^{1/2}\left(gH\right)^{5/4}}$

(b) $\dfrac{N\sqrt{Q}}{\left(gH\right)^{3/4}}$

(c) $\dfrac{N\sqrt{P}}{\rho^{1/2}\left(gH\right)^{3/4}}$

(d) $\dfrac{N\sqrt{Q}}{\rho^{1/2}\left(gH\right)^{5/4}}$

[*Ans.(b*)]

(ii) The relation between mechanical (η_m), manometric (η_{mano}) and overall efficiency (η_o) is

(a $\eta_o = \dfrac{\eta_{mano}}{\eta_m}$

(b) $\eta_o = \eta_m \times \eta_{mano}$

(c) $\eta_o = \eta_{mano} \times \eta_m$

(d) $\eta_o = \dfrac{\eta_m}{\eta_{mano}}$

[*Ans.(b*)]

(iii) The comparison between pumps operating in series and in parallel is

(a) pumps operating in series boost the head, whereas pumps operating in parallel boost the discharge

(b) pumps operating in series boost the discharge, whereas pumps operating in parallel boost the head

(c) in both cases there would be a boost in head only

(d) in both cases there would be a boost in discharge only

[*Ans.(*a)]

(iv) For a centrifugal pump the net positive suction head (NPSH) is defined as,

(a) NPSH= (velocity head + pressure head) at suction.

(b) NPSH= (velocity head + pressure head) at discharge

(c) NPSH= (velocity head + pressure head - vapour pressure of the liquid) at suction.

(d) NPSH= (velocity head + pressure head - vapour pressure of the liquid) at discharge.

[*Ans.(*c)]

(v) Which of the following pumps is preferred for flood control and irrigation applications?
(a) Centrifugal pump
(b) Axial flow pump
(c) Mixed flow pump
(d) Reciprocating pump

[*Ans.(*b)]

Q2.
A centrifugal pump handles liquid whose kinematic viscosity is three times that of water. The dimensionless specific speed of the pump is 0.183 rev and it has to discharge 2 m^3/s of liquid against a total head of 15 m. Determine the speed, test head and flow rate for a one-quarter scale model investigation of the full size pump if the model uses water.

Solution

Since the viscosity of the liquid in the model and prototype vary significantly, equality of Reynolds number must apply for dynamic similarity. Let subscripts 1 and 2 refer to prototype and model respectively.

Equating Reynolds number,

$$N_1 D_1^2 / \nu_1 = N_2 D_2^2 / \nu_2$$

or $$N_2 / N_1 = (4)^2 / 3 = 5.333$$

Equating the flow coefficients

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$$

or $$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}\left(\frac{D_2}{D_1}\right)^3$$

$$= 5.333/(4)^3 = 0.0833$$

Equating head coefficients

$$\frac{H_1}{(N_1 D_1)^2} = \frac{H_2}{(N_2 D_2)^2}$$

or $$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

$$\frac{H_2}{H_1} = (5.333/4)^2 = 1.776$$

Dimensionless specific speed of the pump can be written as

$$K_{sp} = \frac{N_1 \sqrt{Q_1}}{(gH\ )^{3/4}}$$

or $$N_1 = \frac{K_{sp}(gH_1)^{3/4}}{Q_1^{1/2}}$$

$$= \frac{0.183(9.81 \times 15)^{3/4}}{2^{1/2}} = 5.47 \text{ rev/s}$$

Therefore, model speed $N_2 = 5.47 \times 5.33 = 29.15$ rev/s

Model flow rate $Q_2 = 0.0833 \times 2 = 0.166$ m^3/s

Model head $H_2 = 15 \times 1.776 = 26.64$ m

Q3.

The basic design of a centrifugal pump has a dimensionless specific speed of 0.075 rev. the blades are forward facing on the impeller and the outlet angle is 120° to the tangent, with an impeller passage width at the outlet being equal to one-tenth of the diameter. The pump is to be used to raise water through a vertical distance of 35 m at a flow rate of 0.04 m^3/s. The suction and delivery pipes are each of 150 mm diameter and have a combined length of 40 m with a friction factor of 0.005. Other losses at the pipe entry, exit, bends, etc. are three times the velocity head in the pipes. If the blades occupy 6% of the circumferential area and the hydraulic efficiency (neglecting slip) uis 76%, what will be the diameter of the pump impeller?

Solution

Velocity in the pipes $v = \dfrac{0.04 \times 4}{\pi \times (0.15)^2} = 2.26$ m/s

Total losses in the pipe

$$h_1 = \frac{4fl}{2gd}v^2 + \frac{3v^2}{2g} = \left[\frac{4 \times 0.005 \times 40}{0.15} + 3\right]\frac{(2.26)^2}{2 \times 9.81} = 2.17 \text{ m}$$

Therefore, total head required to be developed = 35+2.17 = 37.17 m

The speed of the pump is determined from the consideration of specific speed as

$$0.075 = \frac{N(0.04)^{1/2}}{(9.81 \times 37.17)^{3/4}}$$

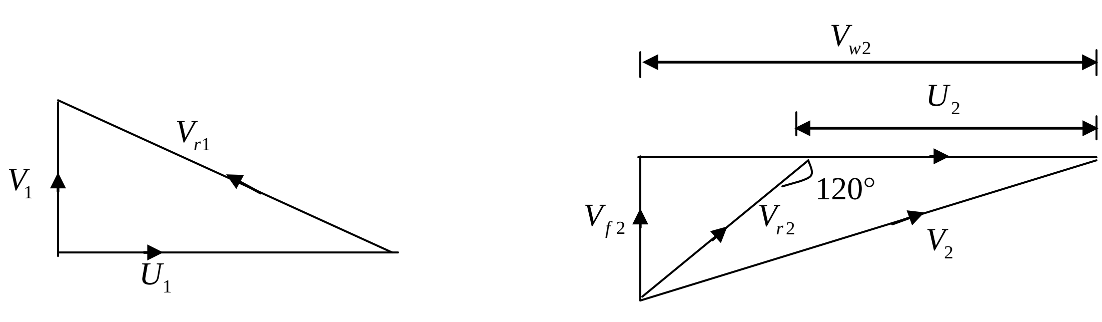
or $$N = \frac{0.075(9.81 \times 37.17)^{3/4}}{(0.04)^{1/2}} = 31.29 \text{ rev/s}$$

Let the impeller diameter be D.

Flow area perpendicular to the impeller outlet periphery

$$0.075 = \pi D \times D/10 \times 0.94 = 0.295D^2$$

The inlet and outlet velocity triangles are drawn below:



Inlet Outlet

$$V_{f2} = \frac{Q}{0.295D^2} = \frac{0.04}{0.295D^2} = \frac{0.135}{0D^2} \text{ m/s}$$

$$U_2 = \pi DN = \pi \times D \times 31.29 = 98.3D \text{ m/s}$$

Hydraulic efficiency $\eta_h = \frac{gH}{V_{w2}U_2}$

or $0.76 = \frac{9.81 \times 37.17}{98.3D \times V_{w2}}$

which gives $V_{w2} = \frac{4.88}{D}$ m/s

From the outlet velocity triangle,

$$\tan 60^\circ = \frac{V_{f2}}{V_{w2} - U_2} = \frac{0.135}{D^2 \left[4.88/D - 98.3D \right]}$$

or $D^3 = 0.0496D - 0.0008$

which gives $D = 0.214$ m

Q4.

Show that the pressure rise in the impeller of a centrifugal pump with backward curved vane can be expressed as

$$\frac{p_2}{\rho g} - \frac{p_1}{\rho g} = \frac{1}{2g}\left[V_{f1}^2 + U_2^2 - V_{f2}^2 \text{cosec}^2 \beta_2 \right]$$

where the subscripts 1 and 2 represents the inlet and outlet conditions of the impeller respectively, β is the blade angle, U is the tangential velocity of the impeller and V_f is the flow velocity. Neglect the frictional and other losses in the impeller.

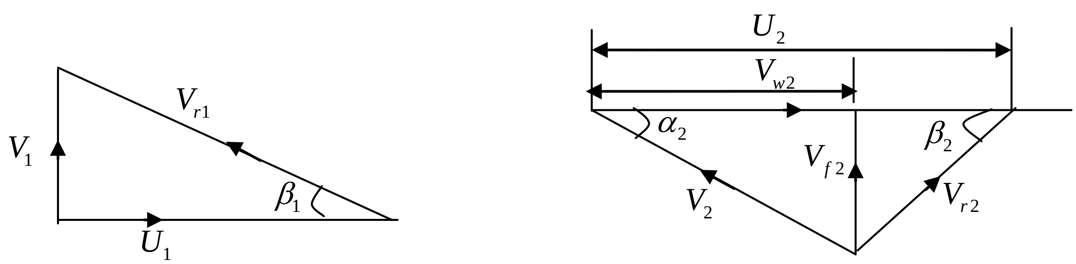
Solution

Applying energy equation between inlet and outlet of the impeller, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 - \frac{V_{w2}U_2}{g}$$

or, $$\frac{p_2}{\rho g} - \frac{p_1}{\rho g} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{V_{w2}U_2}{g} \qquad (z_1 = z_2) \qquad (1)$$

The inlet and outlet velocity triangles are shown in the figure below.



Inlet Outlet

From the inlet velocity triangle, we have

$$V_1 = V_{f1}$$

From the outlet velocity triangle, we have

$$V_{w2} = U_2 - V_{f2} \cot \beta_2$$

Now,

$$V_2^2 = V_{w2}^2 + V_{f2}^2$$
$$= \left(U_2 - V_{f2} \cot \beta_2\right)^2 + V_{f2}^2$$
$$= U_2^2 - 2U_2 V_{f2} \cot \beta_2 + V_{f2}^2 \cot^2 \beta_2 + V_{f2}^2$$
$$= U_2^2 - 2U_2 V_{f2} \cot \beta_2 + V_{f2}^2 \text{cosec}^2 \beta_2$$

Substituting the values of V_1, V_2 and V_{w2} in Eq. (1), we have

$$\frac{p_2}{\rho g} - \frac{p_1}{\rho g} = \frac{V_{f1}^2}{2g} - \frac{U_2^2 - 2U_2 V_{f2} \cot \beta_2 + V_{f2}^2 \text{cosec}^2 \beta_2}{2g} + \frac{\left(U_2 - V_{f2} \cot \beta_2\right) U_2}{g}$$
$$= \frac{1}{2g}\left[V_{f1}^2 - U_2^2 + 2U_2 V_{f2} \cot \beta_2 - V_{f2}^2 \text{cosec}^2 \beta_2 + 2U_2^2 - 2U_2 V_{f2} \cot \beta_2\right]$$

or

$$\frac{p_2}{\rho g} - \frac{p_1}{\rho g} = \frac{1}{2g}\left[V_{f1}^2 + U_2^2 - V_{f2}^2 \text{cosec}^2 \beta_2\right]$$

Q5.

The impeller of a centrifugal pump is 0.5 m in diameter and rotates at 1200 rpm. Blades are curved back to an angle of 30° to the tangent at outlet tip. If the measured velocity of flow at the outlet is 5 m/s, find the work input per kg of water per second. Find the theoretical maximum lift to which the water can be raised if the pump is provided with whirlpool chamber which reduces the velocity of water by 50%.

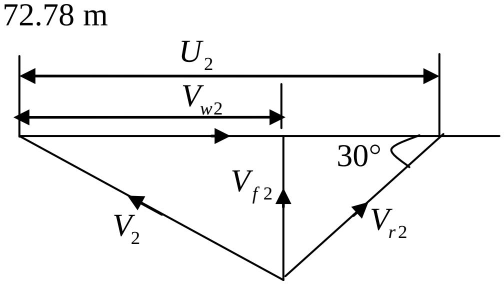
Solution

The peripheral speed at impeller at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1200}{60} = 31.4 \text{ m/s}$$

Work input per unit weight of water

$$\frac{V_{w2} U_2}{g} = \frac{\left(31.4 - 5 \cot 30°\right) \times 31.4}{9.81} = 72.78 \text{ m}$$



Under ideal condition (without loss), the total head developed by the pump = 72.78 m
From the outlet velocity triangle as shown in the figure, velocity of water at impeller outlet

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{\left(31.4 - 5 \cot 30°\right)^2 + 5^2} = 23.28 \text{ m/s}$$

After the whirlpool chamber, the velocity of water at delivery $= 0.5 \times 23.28$ m/s
Therefore, the pressure head at impeller outlet

$$= 72.78 - \frac{(0.5 \times 23.28)^2}{2 \times 9.81} = 65.87 \text{ m/s}$$

Hence, the theoretical maximum lift = 65.87 m

Q6.

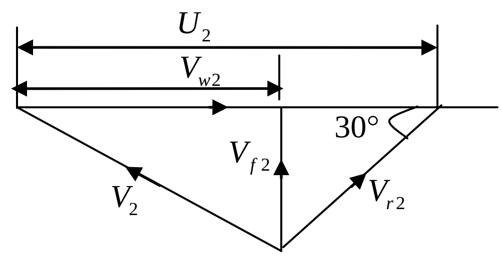
The impeller of a centrifugal pump is 0.3 m in diameter and runs at 1450 rpm. The pressure gauges on suction and delivery sides show the difference of 25 m. the blades are curved back to an angle of 30°. The velocity of flow through impeller, being constant, equals to 2.5 m/s, find the manometric efficiency of the pump. If the frictional losses in impeller amounts to 2 m, find the fraction of total energy which is converted into pressure energy by impeller. Also find the pressure rise in pump casing.

Solution

The peripheral speed at impeller at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1450}{60} = 22.78 \text{ m/s}$$

From the outlet velocity triangle as shown in the figure,



$$V_{w2} = U_2 - V_{f2} \cot \beta_2$$

$$= 22.78 - 2.5 \cot 30° = 18.45 \text{ m/s}$$

Input power per unit weight of water

$$= \frac{V_{w2} U_2}{g} = \frac{18.45 \times 22.78}{9.81} = 42.84 \text{ m}$$

Head developed by the pump = 25 m

(difference in the kinetic heads in suction and delivery pipe is neglected)

Hence, the manometric efficiency

$$\eta_m = \frac{H_m}{\frac{V_{w2} U_2}{g}} = \frac{25}{42.84} = 58.35\%$$

From the outlet velocity triangle

$$V_2^2 = V_{f2}^2 + V_{w2}^2 = (2.5)^2 + (18.45)^2 = 346.65 \text{ m}^2/\text{s}^2$$

The velocity head at impeller outlet $\frac{V_2^2}{2g} = \frac{346.65}{2 \times 9.81}$ =17.65 m

Therefore, the increase in pressure head in the impeller

$$= 42.84 - 2 - 17.67$$

$= 23.17$ m which is $23.17 \times 100 / 42.84 = 54.1\%$ of the total energy.

Pressure rise in the pump casing $= 25 - 23.17 = 1.83$ m

Q7.

During a laboratory test on a pump, appreciable cavitation began when the pressure plus the velocity head at inlet was reduced to 3.62 m while the change in total head across the pump was 36.5 m and the discharge was 48 litres/s. Barometric pressure was 750 mm of Hg and the vapour pressure of water 1.8 kPa. What is the value of critical cavitation parameter σ_c? If the pump is to give the same total head and discharge in a location where the normal atmospheric pressure is 622 mm of Hg and the vapour pressure of water is 830 Pa, by how much must the height of the pump above the supply level be reduced?

Solution

For cavitation to begin

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = 3.62 \text{ (given)}$$

where subscript 1 refers to inlet condition to the impeller.

In this situation, $p_1 = 1.8$ kPa

Hence, $\frac{p_1}{\rho g} = \frac{1.8 \times 10^3}{10^3 \times 9.81} = 0.183$ m

Therefore, $\frac{V_1^2}{2g} = 3.62 - 0.183 = 3.08$ m

Thus, $\frac{V_1^2}{2gH} = \frac{3.03}{36.5} = 0.084$

Let z_1 be the initial height of the pump above the supply level. Then, applying Bernoulli's equation between the supply level and impeller inlet, we have

$$0.75 \times 13.6 = 3.26 + z_1 + h_f$$

where h_f is the frictional head loss.

or $z_1 + h_f = 6.94$ m

In the second case, the total head H and the dimensionless cavitation parameter σ_c remains the same. Hence the velocity head at impeller inlet remains the same as 3.08 m.

Therefore, applying Bernoulli's equation between the supply level and the impeller inlet, we have

$$0.622 \times 13.6 = \frac{830}{10^3 \times 9.81} + 3.08 + \left(z_2 + h_f\right)$$

(the head loss due to friction is considered to be the same for the same pump with the same discharge).

This gives $z_2 + h_f = 5.29$ m

Hence $z_1 - z_2 = 6.94 - 5.29 = 1.65$ m

Therefore the height of the pump above the supply level has to be reduced by 1.65 m.

After Lecture 20

Q1. **Choose the correct answer**

(i) In a single acting reciprocating pump without air vessel, the average velocity in the suction pipe is given by (A and a_s are the cross-sectional area of the piston and the suction pipe respectively, ω is the angular velocity of the crank and r is the radius of the crank)

(a) $\frac{A}{a_s}\omega r$

(b) $\frac{A}{a_s}\pi\omega r$

(c) $\frac{A}{a_s}\frac{\omega r}{\pi}$

(d) $\frac{A}{a_s}\frac{\pi}{\omega r}$

[*Ans.(*c)]

(ii) In a reciprocating pump without air vessels, the acceleration head in the suction/delivery pipe is maximum when the crank angle is

(a) 0°
(b) 90°
(c) 120°
(d) 180°

[*Ans.(*a)]

(iii) In a reciprocating pump without air vessels, the friction head in the suction/delivery pipe is maximum when the crank angle is

(a) 0°
(b) 90°
(c) 120°
(d) 180°

[*Ans.(*b)]

(iv) Indicator diagram shows for one complete revolution of crank the

(a) variation of kinetic head in the cylinder
(b) variation of pressure head in the cylinder
(c) variation of kinetic and pressure head in the cylinder
(d) none of the above

[*Ans.(*b)]

(v) Air vessel in a reciprocating pump is used

(a) to obtain a continuous supply of water at uniform rate
(b) to reduce suction head
(c) to increase the delivery head
(d) to increase pump efficiency

[*Ans.(*a)]

Q2.

A reciprocating pump has a suction head of 6 m and delivery head of 15 m. It has a bore of 150 mm and stroke of 250 mm and piston makes 60 double strokes in a minute. Calculate the force required to move the piston during (i) suction stroke, and (ii) during the delivery stroke. Find also the power to drive the pump.

Solution

(i)

Suction pressure $= 6\times10^3 \times 9.81 \text{ N/m}^3$

The force required to move the piston during suction stroke

$$= 6\times10^3 \times 9.81\times\pi\times\frac{(0.15)^2}{4} \text{ N} = 1040 \text{ N} = 1.04 \text{ kN}$$

(ii) Delivery pressure $= 6\times10^3 \times 9.81 \text{ N/m}^3$

The force required to move the piston during suction stroke

$$= 15\times10^3 \times 9.81\times\pi\times\frac{(0.15)^2}{4} \text{ N} = 2600 \text{ N} = 2.6 \text{ kN}$$

The rate of discharge $= \pi\times\frac{(0.15)^2}{4}\times0.25\times\frac{120}{60} = 0.0088 \text{ m}^3\text{/s}$

The power required to drive the pump

$$= 0.0088\times10^3\times(15+6)\times9.81 \text{ W} = 1.81\text{kW}$$

Q3.

An air vessel fitted on the delivery side to a single-acting reciprocating pump, with plunger diameter of 30 cm and crank radius of 25 cm. The length and diameter of the delivery pipe are 40 m and 10 cm respectively. If the pump runs at 50 rpm, find the power saved in overcoming the friction by fitting the air vessel. Assume atmospheric pressure head as 10.3 m of water and Darcy's friction factor as 0.03.

Solution

Angular speed is given by

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times50}{60} = 5.236 \text{ rad/s}$$

Without air vessel

The maximum loss of head due to friction in delivery pipe is computed as

$$h_{fd,\max} = \frac{fl_d}{d_d\times2g}\left(\frac{A}{a_d}\omega r\right)^2$$

where d_d is the diameter of delivery pipe, a_d is the area of delivery pipe, f is the Darcy's friction factor, l_d is the length of delivery pipe, A is the area of piston, ω is the angular velocity of the crank and r is the radius of the crank.

Substituting the respective values, we have

$$h_{fd,\max} = \frac{0.03\times40}{0.1\times2\times9.81}\left(\frac{\frac{\pi}{4}(0.3)^2}{\frac{\pi}{4}(0.1)^2}\times5.236\times0.25\right)^2 = 85 \text{ m}$$

Power required in overcoming the friction is found to be

$$P_{without} = \rho g Q \times \frac{2}{3} h_{fd,\max} = \frac{\rho g A L N}{60} \times \frac{2}{3} h_{fd,\max} \qquad \left[\because Q = \frac{ALN}{60}\right]$$

$$= \frac{1000 \times 9.81 \times 0.0707 \times 0.5 \times 50}{60} \times \frac{2}{3} \times 85 = 16375 \text{ W}$$

With air vessel

The loss of head due to friction in delivery pipe by fitting an air vessel is computed as

$$h_{fd} = \frac{f l_d v_{av}^2}{d_d \times 2g} = \frac{f l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

$$= \frac{0.03 \times 40}{0.1 \times 2 \times 9.81} \times \left(\frac{\frac{\pi}{4}(0.3)^2}{\frac{\pi}{4}(0.1)^2} \times \frac{5.236 \times 0.25}{\pi}\right)^2 = 8.613 \text{ m}$$

Power required in overcoming the friction is found to be

$$P_{with} = \rho g Q \times h_{fd} = \frac{\rho g A L N}{60} \times h_{fd}$$

$$= \frac{1000 \times 9.81 \times 0.0707 \times 0.5 \times 50}{60} \times 8.613 = 2489 \text{ W}$$

The power saved in overcoming the friction by fitting the air vessel is

$$P_{without} - P_{with} = 16375 - 2489 = 13886\text{W} = 13.886\text{kW}$$

After Lecture 25

Q1.

Determine the pressure ratio developed and the specific work input to drive a centrifugal air compressor of an impeller diameter of 0.5 m and running at 7000 rpm. Assume zero whirl at the entry and $T_{1t} = 290$ K . The slip factor and power input factor to be unity, the process of compression is isentropic and for air c_p 1005 J/kgK, $\gamma = 1.4$.

Solution

The impeller tip speed

$$U_2 = \frac{\pi \times 0.5 \times 7000}{60} = 183.26 \text{ m/s}$$

The pressure ratio can be written as

$$\frac{p_2}{p_1} = \left[1 + \frac{U_2^2}{c_p T_{1t}}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \left[1 + \frac{(183.26)^2}{1005 \times 290}\right]^{\frac{1.4}{1.4-1}} = 1.46$$

The specific work input is

$$= U_2^2 = (183.26)^2 = 33.58 \times 10^3 \text{ J/kg} = 33.58 \text{ kJ/kg}$$

Q2.

Air at a temperature of 27°C flows into a centrifugal compressor at 20000 rpm. The following data are given:

Slip factor	0.80
Power input factor	1
Isentropic efficiency	80%
Outer diameter of blade tip	0.5 m

Assuming the absolute velocities of air entering and leaving the compressor are same, find (i) static temperature rise of air passing through the compressor, and (ii) the static pressure ratio. c_p of air is 1005 J/kgK.

Solution

Velocity of the blade tip

$$U_2 = \frac{\pi \times 0.5 \times 20000}{60} = 523.6 \text{ m/s}$$

Stagnation temperature rise

$$T_{2t} - T_{1t} = \frac{\psi \sigma U_2^2}{c_p}$$

$$= \frac{0.80 \times 1 \times (523.6)^2}{1005} = 218.23°\text{C}$$

Since the absolute velocities at the inlet and the outlet of the stage are the same, the rise in stagnation temperature equals to that in static temperature.

The static pressure ratio can be written as

$$\frac{p_2}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(1 + \frac{\eta_c\left(T_2 - T_1\right)}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(1 + \frac{0.8 \times 218.23}{300}\right)^{\frac{1.4}{1.4-1}} = 4.98$$

Q3.

The conditions of air at the entry of an axial flow compressor stage are $p_1 = 100\ \mathbf{kN/m^2}$ and $T_1 = 300\ \mathbf{K}$. The air angles are $\beta_1 = 51°$, $\beta_2 = 10°$, $\alpha_1 = \alpha_3 = 8°$.

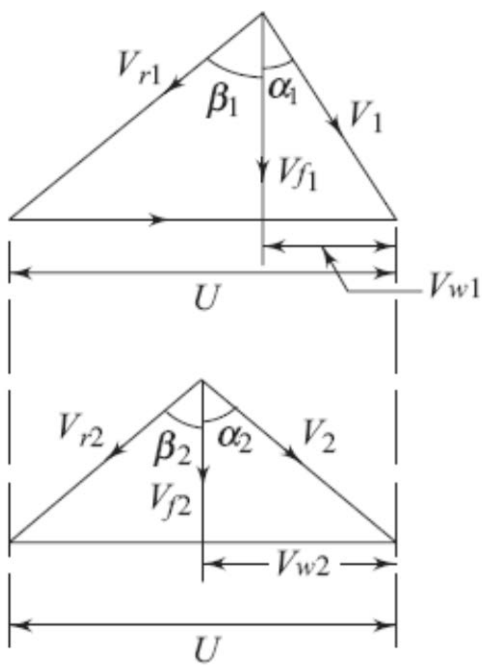
The mean diameter and peripheral speed are 0.5 m and 150 m/s respectively. Mass flow rate through the stage is 30 kg/s; the work done factor is 0.95 and mechanical efficiency is 90%. Assuming an isentropic stage efficiency of 85%, determine (i) blade height at entry (ii) stage pressure ratio, and (iii) the power required to drive the stage. For air, $\gamma = 1.4$, $R = 287$ J/kg K

Solution

The density of air at the entry is found to be

$$\rho_1 = \frac{p_1}{RT_1} = \frac{100 \times 10^3}{287 \times 300} = 1.16\ \mathbf{kg/m^3}$$

The velocity triangles of a stage of an axial flow compressor are shown in the figure below.



From the inlet velocity triangle, we get

$$\frac{U}{V_f} = \tan\alpha_1 + \tan\beta_1$$

Hence, $$V_f = \frac{150}{\tan 8° + \tan 51°} = 109.06 \text{ m/s}$$

Mass flow rate of air is

$$\dot{m} = V_f \rho_1 \left(\pi d h_1\right)$$

or $$30 = 109.06 \times 1.16 \times \pi \times 0.5 h_1$$

or $$h_1 = 0.15 \text{ m}$$

(ii)Static temperature rise of the stage can be written as

$$\Delta T_{st} = \frac{\lambda U V_f}{c_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$= \frac{0.95 \times 150 \times 190.06}{1005}\left(\tan 51° - \tan 10°\right) = 16.37°\text{C}$$

The pressure ratio can be written as

$$R_s = \left(1 + \frac{\eta_c\left(T_2 - T_1\right)}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(1 + \frac{0.85 \times 16.37}{300}\right)^{\frac{1.4}{1.4-1}} = 1.17$$

(iii) Power required to drive the stage is

$$P = \frac{\dot{m} w}{\eta_m} = \frac{\dot{m} c_p \Delta T_{st}}{\eta_m}$$

$$= \frac{30 \times 1005 \times 16.37}{0.9} = 548.39 \times 10^3 \text{ W} = 548.39 \text{ kW}$$

After Lecture 37

Q1. **Choose the correct answer**

(i) Select the expressions that do not give the speed of a sound wave relative to the medium of propagation which is an ideal gas($\gamma = c_p / c_v$)

(a) $\sqrt{\gamma RT}$

(b) $\sqrt{\gamma\rho/p}$

(c) $\sqrt{\partial p/\partial\rho}$

(d) $\sqrt{\gamma p/\rho}$

[*Ans.(*b) and (c)]

(ii) Shock waves are highly localized irreversibilities in the flow. While passing through a normal shock wave, the flow changes from

(a) a supersonic to a subsonic state
(b) a subsonic to a supersonic state
(c) a subsonic state to a sonic state
(d) a supersonic to a hypersonic state

[*Ans.(*a)]

(iii) The flow upstream of a shock is always

(a) supersonic
(b) subsonic
(c) sonic
(d) none of these

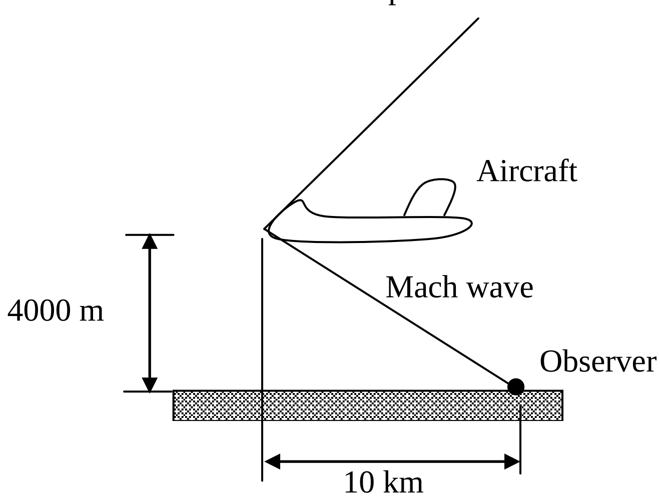
[*Ans.(*a)]

Q2.

A man on the ground observes that an airplane flying horizontally at an altitude of 4000 m has traveled 10 km from the overhead position before the sound of the airplane is first heard. Estimate the speed of the airplane. The temperature in the atmosphere is given by $T = 288.16 - 0.0065H$ (for $0 \le H \le 11019$ m, the altitude H is measured from the seal level).

Solution

The speed of sound is determined at the temperature at mean altitude to describe the Mach wave.



At the mean altitude of 2000 m, the temperature is

$$T = 288.16 - 0.0065 \times 2000 = 275.16 \text{ K}$$

Hence, the mean speed of sound is given by

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 275.16} = 332.5 \text{ m/s}$$

If α is the Mach angle based on the mean speed of found, then

$$\tan\alpha = \frac{4000}{10000} = 0.4$$

However, since $\sin\alpha = 1/M$, it follows that $\tan\alpha = 1\big/\sqrt{M^2 - 1}$

Thus $$M = \sqrt{\left(1/0.4\right)^2 + 1} = 2.69$$

Hence, velocity of aircraft is $= 2.69 \times 332.5 = 894.4$ m/s

Q3.

A pitot-static tube is placed in a subsonic airflow. The static pressure and temperature in the flow are 100 kPa and 27°C respectively. The difference between the pitot and static pressures is measured and found to be 30 kPa. Find the air velocity (i) assuming an incompressible flow, (ii) assuming compressible flow.

Solution

The density in the flow is given by

$$\rho = \frac{p}{RT} = \frac{100 \times 10^3}{287 \times 300} = 1.161 \text{ kg/m}^3$$

(i) If incompressible flow is assumed, the velocity is given by

$$V = 2\sqrt{\frac{p_0 - p}{\rho}} = 2\sqrt{\frac{30 \times 10^3}{1.161}} = 321.5 \text{ m/s}$$

(ii) When compressibility effect is considered, the velocity is found by noting that

$$\frac{p_0 - p}{p} = \frac{30}{100}$$

Hence, $$\frac{p_0}{p} = 1.3$$

However, $$\frac{p_0}{p} = \left[1 + \frac{\gamma - 1}{2}M^2\right]^{\frac{\gamma}{\gamma - 1}}$$

Thus, for $p_0/p = 1.3$, the above relation gives

$$M^2 = \frac{2}{0.4} \times \left[1.3^{1/3.5} - 1\right]$$

which gives $M = 0.624$

The velocity is therefore given by

$$V = Ma = 0.624\sqrt{1.4 \times 287 \times 300} = 216.6 \text{ m/s}$$

Q4.

Air is expanded from a large reservoir in which the pressure and temperature are 600 kPa and $40°C$ respectively through a convergent-divergent nozzle. The design back-pressure is 100 kPa. Find

(i) The ratio of the nozzle exit area to the nozzle throat area

(ii) The discharge velocity from the nozzle under design considerations

(iii) At what back-pressure will there be a normal shock at the exit plane of the nozzle?

Solution

Here, $p_0 = 600 \text{ kPa}$ and $T_0 = 40°C$ and the design back-pressure is $p_b = 100 \text{ kPa}$.

(i) When operating at the design conditions $p_e = p_b$, thus

$$\frac{p_e}{p_o} = \frac{100}{600} = 0.1667$$

For $p_e/p_o = 0.1667$, we get from isentropic flow table

$$M_e = 1.83$$

and

$$\frac{A^*}{A_e} = 0.6792$$

Hence, the ratio of the nozzle exit area to the nozzle throat area is

$$\frac{A_e}{A^*_e} = \frac{1}{0.6792} = 1.472$$

(ii) At $M_e = 1.83$, we have from isentropic flow table

$$\frac{a_0}{a_e} = 1.2922$$

Hence,
$$\frac{a_e}{a_0} = 0.7739$$

∴
$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \times 286.8 \times 313} = 345 \text{ m/s}$$

Therefore,

$$V_e = \frac{V_e}{a_e}\frac{a_e}{a_0} a_0 = 1.83 \times 0.7739 \times 354.5 = 502.1 \text{ m/s}$$

The nozzle discharge velocity under design conditions is 502.1 m/s.

(iii) When there is a normal shock wave on the exit plane of the nozzle the design conditions will exist upstream of the shock. Hence, using $M_1 = 1.83$, normal shock wave tables give

$$\frac{p_2}{p_1} = 3.74$$

Hence,

$$p_2 = 3.74 \times 100 = 374 \text{ kPa}$$

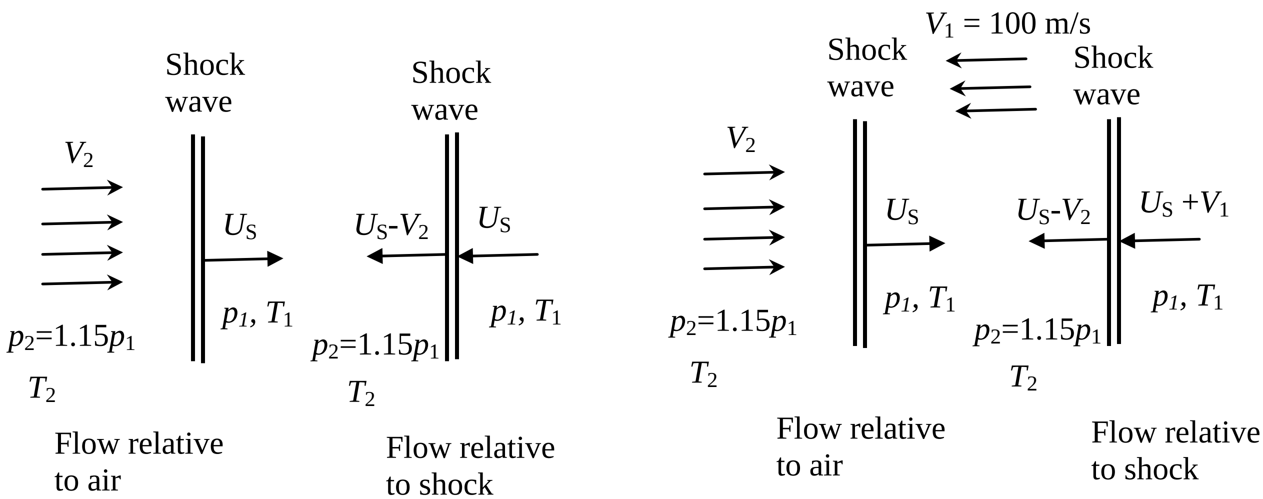
Therefore, there will be a normal shock wave on the exit plane of the nozzle when $p_b = p_2 = 374 \text{ kPa}$

Q5.

A shock wave across which the pressure ratio is 1.15 moves down a duct into still air at a pressure of 50 kPa and a temperature of 30°C. Find the temperature and velocity of the air behind the shock wave. If instead of being at rest, the air ahead of the shock wave is moving toward the wave at a velocity of 100 m/s, what is the velocity of the air behind the shock wave?

Solution

The flow situation being considered is shown in the figure below.



(a) Case with still air

(b) Case with undisturbed air moving towards the wave

For the case of normal shock wave moving into still air (Fig. (a)), we have

$$p_2/p_1 = 1.15,\ p_1 = 50 \text{ kPa},\ T_1 = 30°\text{C}$$

For $p_2/p_1 = 1.15$, we have from normal shock tables

$$M_1 = 1.062,\ M_2 = 0.943,\ T_2/T_1 = 1.041$$

Therefore,

$$T_2 = 1.041 \times (273+30) = 315.4 \text{ K} = 42.4°\text{C} \text{ and } p_2 = 1.15 \times 50 = 57.5 \text{ kPa}$$

Since $\quad M_1 = U_s/a_1$ and $M_2 = (U_s - V)/a_2$

We get

$$V = M_1 a_1 - M_2 a_2$$
$$= 1.062 \times \sqrt{1.4 \times 287 \times 303} - 0.943 \times \sqrt{1.4 \times 287 \times 315.4} = 35.1 \text{ m/s}$$

For the case where the air ahead of the shock is moving towards the wave (Fig.(b)). We get for $p_2/p_1 = 1.15$ from normal shock table

$$M_1 = 1.062,\ M_2 = 0.943,\ T_2/T_1 = 1.041$$

So, $\quad T_2 = 1.041 \times (273+30) = 315.4 \text{ K} = 42.4°\text{C}$

However, since the flow relative to the wave is being considered, it follows that

$$M_1 = \left(U_s + V_1\right) / a_1 \text{ and } M_2 = \left(U_s - V_2\right) / a_2$$

It follows

$$V_2 = U_s - M_2 \times a_2 = \left(M_1 \times a_1 - V_1\right) - M_2 \times a_2$$

$$= \left(1.062 \times \sqrt{1.4 \times 287 \times 303} - 100\right) - 0.943 \times \sqrt{1.4 \times 287 \times 315.4} = -64.9 \text{ m/s}$$

After Lecture 40

Q1. **Choose the correct answer**

(i) A supersonic flow while passing through an oblique shock wave

(a) will always be subsonic

(b) will always be supersonic

(c) may be subsonic or supersonic depending upon the shock wave angle

(d) none of the above

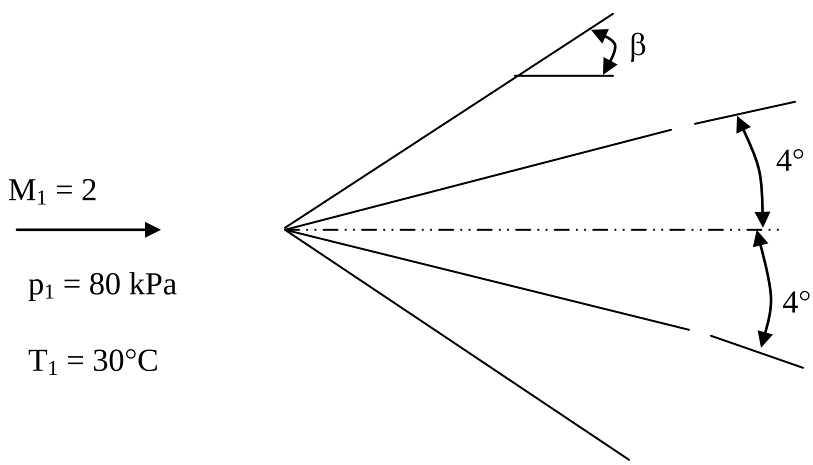
[*Ans.* (c)]

Q2.

Air flowing at Mach 2 with a pressure of 80 kPa and a temperature of 30°C passes over an wedge with an included angle of 8° that is aligned with the flow. The flow is turned by both the upper and lower surfaces of the wedge through an angle of 4°, leading to the generation of oblique shock wave. Find the pressure acting on the surface of the wedge.

Solution

The flow situation being considered is shown in the figure below.



Here $\delta\left(\text{ the turning angle of flow}\right) = 4°$ and the Mach number upstream of the shock wave is $M_1 = 2.0$.

For $M_1 = 2.0$ and $\delta = 4°$, we get from oblique wave chart

$$\beta = 33.4°$$

$$\mathrm{M}_{N1} = \mathrm{M}_1 \sin\beta = 2 \times \sin 33.4° = 1.10$$

Normal shock tables give for an upstream Mach number of 1.10 (M_{N1})

$$\frac{p_2}{p_1} = 1.245$$

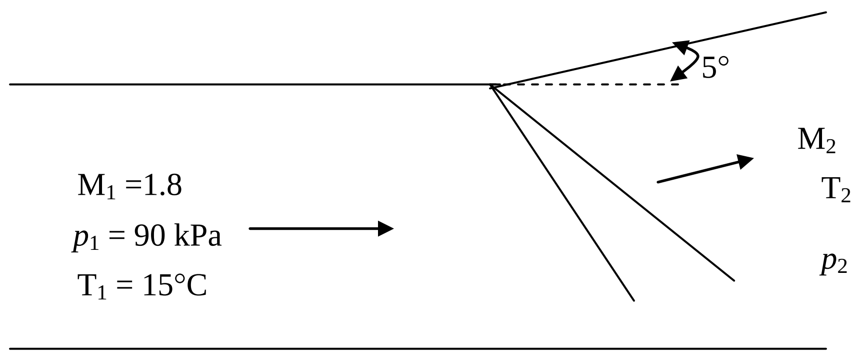
Hence, the pressure acting on the surface of the wedge $p_2 = 1.245 \times 80 = 99.6$ kPa

Q3.

Air flows at Mach 1.8 with a pressure of 90 kPa and a temperature of 15°C down a wide channel. The upper wall of this channel turns through an angle of 5° away from the flow leading to the generation of an expansion wave. Find the pressure, Mach number, and temperature behind this expansion wave.

Solution

The flow situation being considered is shown in the figure below.



For $M_1 = 1.8$ isentropic flow tables give

$$\theta_1 = 20.73^\circ\,,\ \frac{p_{01}}{p_1} = 5.746\,,\ \frac{T_{01}}{T_1} = 1.648$$

Downstream of the expansion wave

$$\theta_2 = \theta_1 + 5^\circ = 20.73^\circ + 5^\circ = 25.73^\circ$$

For $\theta_2 = 25.73^\circ$, isentropic flow tables give

$$\mathrm{M}_2 = 1.98\,,\ \frac{p_{02}}{p_2} = 7.585\,,\ \frac{T_{02}}{T_2} = 1.784$$

Since, the flow through the expansion wave is isentropic, $p_{02} = p_{01}$ and $T_{02} = T_{01}$

Hence,

$$T_2 = \frac{T_{01}}{T_1}\frac{T_2}{T_{02}}T_1 = \frac{1.648}{1.784}\times 288 = 266\ \mathrm{K} = -7^\circ\mathrm{C}$$

$$p_2 = \frac{p_{01}}{p_1}\frac{p_2}{p_{02}}p_1 = \frac{5.746}{7.585}\times 90 = 68.2\ \mathrm{kPa}$$

Introduction to Fluid Machinery and Compressible Flow, Time-3 hours, Full Marks-100

Q1.

(a) In a vertical shaft inward-flow reaction turbine, the sum of the pressure and kinetic head at the entrance to the spiral casing is 120 m and the vertical distance between this section and the tail race level is 3 m. The peripheral velocity of the runner at the entry is 30 m/s, the radial velocity of water is constant at 9 m/s and discharge from the runner is without swirl. The estimated hydraulic losses are (i) between turbine entrance and exit from the guide vanes 4.8 m, (ii) in the runner 8.8 m, (iii) in the draft tube 0.79 m, and (iv) kinetic head rejected to the tail race 0.46 m. calculate the guide vane angle and the runner blade angle at the inlet and the pressure heads at the entry to and the exit from the runner.

(b) A Francis turbine discharges radially at the outlet and the velocity of flow through the runner is constant. Show that the hydraulic efficiency can be expressed as

$$\eta_h = \frac{1}{1+\left(\dfrac{0.5\tan^2\alpha_1}{1-\tan\alpha_1/\tan\beta_1}\right)}$$

where α_1 and β_1 are respectively the guide vane angle and runner blade angle at inlet.
If the vanes are radial at inlet, then show that

$$\eta_h = \frac{2}{2+\tan^2\alpha_1}$$

(c) Air at a stagnation temperature of 27°C enters the impeller of a centrifugal compressor in the axial direction. The rotor which has 15 radial vanes, rotates at 20000 rpm. The stagnation pressure ratio between the diffuser outlet and the impeller inlet is 4 and the isentropic efficiency is 85%. Determine (i) the impeller tip radius and (ii) power input to the compressor when the mass flow rate is 2 kg/s. Assume a power input factor of 1.05 and a slip factor $\sigma = 1 - 2/n$, where n is the number of vanes. For air, $\gamma = 1.4$, $R = 287$ J/kg K .

[9+9+7=25 Marks]

Q2.

(a)For a rotodynamic hydraulic machine, the parameters head (H), discharge (Q), and power (P) depend on the following: rotor diameter (D), rpm(N), fluid density(ρ), fluid viscosity(μ), and acceleration due to gravity(g). Considering these functional dependences, obtain the important dimensionless parameters depicting the respective dependences of H, Q, and P on the other parameters, using Buckingham pi theorem. Also, obtain the expression for the specific speed (dimensionless) of a centrifugal pump and a hydraulic turbine as a combination of some of those dimensionless parameters.

(b) Following data are obtained during the testing of a centrifugal pump at constant speed:

Parameter	Inlet section (suction)	Outlet section (delivery)
Gage pressure in kPa	95.2	412
Elevation above datum in m	1.25	2.75

Average speed in m/s	2.35	3.62

The measured flow rate is 11.5 m^3/hr and the measured input torque to the impeller is 3.68 N.m. Mechanical efficiency of the pump is 85%. Determine:

(i) The hydraulic power input to the fluid.

(ii) Hydraulic efficiency

(iii) Electrical power input.

(c) A centrifugal pump is required to work against a head of 20 m while rotating at the speed of 700 rpm. If the blades are curved back to an angle of 30° to the tangent at outlet tip and velocity of flow through impeller is 2 m/s, calculate the impeller diameter when (i) all the kinetic energy at impeller outlet is wasted and (ii) when 50% of this energy is converted into pressure energy in pump casing.

[10+7+8=25 Marks]

Q3.

A convergent-divergent nozzle is designed to expand air from a reservoir in which the pressure is 800 kPa and the temperature is $40°\text{C}$ to give a Mach number of 2.7. The throat area of the nozzle is $0.08\ \text{m}^2$. Find:

(a) The exit area of the nozzle.

(b) The flow rate through the nozzle under design conditions.

(c) The design back pressure

(d) The lowest back pressure for which there is only subsonic flow in the nozzle.

(e) The back pressure at which a normal shock wave occurs on the exit plane of the nozzle

(f) The back-pressure below which there are no shock waves in the nozzle

(g) The range of back-pressures over which there are oblique shock waves in the exhaust from the nozzle

(h) The range of back-pressures over which there are expansions waves in the exhaust from the nozzle.

[3+3+3+5+5+2+2+2=25 Marks]

Q4.

(a)When a body is placed in a stream which at infinite distance upstream is in uniform flow with free-stream conditions V_∞, p_∞, M_∞, etc., the local pressures in the neighborhood of the body are usually reported in terms of the dimensionless pressure coefficient, C_p:

$$C_p \equiv \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$$

Show that the value of the pressure coefficient corresponding to the appearance of the critical velocity is given by

$$C_p^* = \frac{\left[\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2}M_\infty^2}$$

where γ is the ratio of specific heats. Consider the fluid to be an ideal gas.

(b)An oblique shock wave occurs in an air flow in which the Mach number upstream of the shock is 2.6. The shock wave turns the flow through 10°. The shock wave impinges on a free boundary along which the pressure is constant and equal to that existing upstream of the shock wave. The shock is reflected from this boundary as an expansion wave. Find the Mach number downstream of this expansion wave. The shock wave angle for $M = 2.6$ and $\delta = 10^\circ$ is 30°.

(c)A simple wing may be modeled as a 0.25 m wide flat plate set at an angle of 3° to an air flow at Mach 2.5, the pressure in this flow being 60 kPa. Assuming that the flow over the wing is two-dimensional, estimate the lift force per meter span due to the wave formation on the wing.

[7+8+10 = 25 marks]

Solution

1.

(a)

Total head at entry = 120+3 =123 m

Total loss of head = 4.8+8.8+0.79+0.46 = 14.85 m

Work equivalent head

$$\frac{V_{w1}U_1}{g} = 123 - 14.85 = 108.15 \text{ m}$$

Therefore $$V_{w1} = \frac{108.15 \times 9.81}{30} = 35.36 \text{ m/s}$$

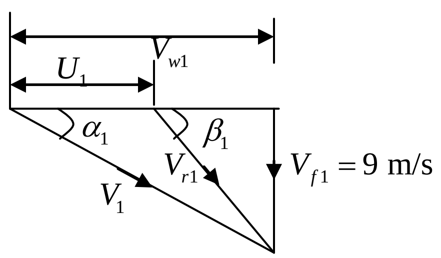
From inlet velocity triangle,

$$\tan \alpha_1 = \frac{9}{35.36}$$

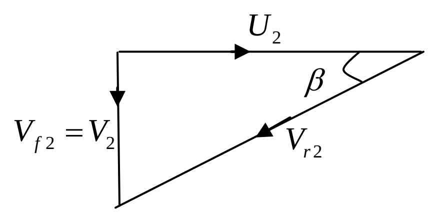
which gives $$\alpha_1 = 14.28°$$

$$\tan \beta_1 = \frac{9}{(35.36 - 30)}$$

which gives $$\beta_1 = 59.22°$$



Inlet velocity triangle



Outlet velocity triangle

From the inlet velocity triangle, we have

$$V_1^2 = V_{w1}^2 + V_{f1}^2$$

$$V_1^2 = (35.36)^2 + 9^2 = 1331.33 \text{ m}^2/\text{s}^2$$

Therefore, pressure head at entry

$$= 120 - \frac{1331.33}{2 \times 9.81} - 4.8 = 47.34 \text{ m}$$

From outlet velocity triangle

$$V_2 = V_{f2} = V_{f1} = 9 \text{ m/s}$$

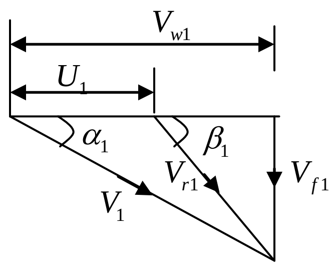
(since the flow velocity is constant)

Let p_2 be the pressure (above atmospheric) at exit from the runner. Applying energy equation between the inlet and outlet of the draft tube, we have

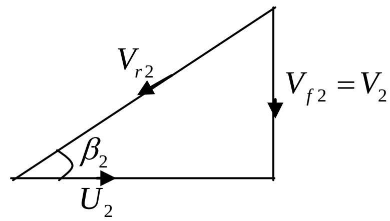
$$\frac{p_2}{\rho g}+\frac{9^2}{2\times 9.81}+3=0+0.46+0.79$$

which gives $$\frac{p_2}{\rho g}=-5.88 \text{ m}$$

(b) The inlet and outlet velocity triangles are shown in the figure below.



Inlet velocity triangle



Outlet velocity triangle

From the inlet velocity triangle, we have

$$\tan\alpha_1=\frac{V_{f1}}{V_{w1}}$$

or $$V_{f1}=V_{w1}\tan\alpha_1$$

Again, from the inlet velocity triangle, we have

$$\tan\beta_1=\frac{V_{f1}}{V_{w1}-U_1}$$

or $$\left(V_{w1}-U_1\right)\tan\beta_1=V_{w1}\tan\alpha_1$$

or $$U_1=V_{w1}\left(1-\frac{\tan\alpha_1}{\tan\beta_1}\right)$$

When water flows through the vanes, we get

$$H-\frac{V_{f2}^{\ 2}}{2g}=\frac{1}{g}V_{w1}U_1$$

or $$H=\frac{1}{g}V_{w1}U_1+\frac{V_{f1}^{\ 2}}{2g} \qquad [\because V_{f2}=V_{f1}]$$

Hydraulic efficiency of a turbine is defined as the power developed by the rotor to the power supplied at the inlet and is given by

$$\eta_h=\frac{V_{w1}U_1}{gH}$$

$$=\frac{V_{w1}U_1}{g\left[\frac{1}{g}V_{w1}U_1+\frac{V_{f1}^{\ 2}}{2g}\right]}$$

$$= \frac{V_{w1}V_{w1}\left(1-\frac{\tan\alpha_1}{\tan\beta_1}\right)}{V_{w1}V_{w1}\left(1-\frac{\tan\alpha_1}{\tan\beta_1}\right)+\frac{V_{w1}{}^2\tan^2\alpha_1}{2}}$$

or $$\eta_h = \frac{1}{1+\left(\frac{0.5\tan^2\alpha_1}{1-\tan\alpha_1/\tan\beta_1}\right)}$$

When the vanes are radial at inlet, $\beta_1 = 90°$

Hydraulic efficiency is then

$$\eta_h = \frac{1}{1+\left(\frac{0.5\tan^2\alpha_1}{1-\tan\alpha_1/\tan 90°}\right)}$$

$$= \frac{1}{1+0.5\tan^2\alpha_1}$$

or $$\eta_h = \frac{2}{2+\tan^2\alpha_1}$$

(c) (i)The overall stagnation pressure ratio can be written as

$$\frac{p_{3t}}{p_{1t}} = \left(1+\frac{\eta_c\left(T_{3t}-T_{1t}\right)}{T_{1t}}\right)^{\frac{\gamma}{\gamma-1}}$$

or $$T_{3t}-T_{1t} = T_{1t}\frac{\left[\left(\frac{p_{3t}}{p_{1t}}\right)^{\frac{\gamma}{\gamma-1}}-1\right]}{\eta_c}$$

Further, the stagnation temperature rise across the impeller can be wriiten as

$$T_{2t}-T_{1t} = \frac{\psi\sigma U_2^2}{c_p}$$

From the above two equations and noting that $T_{3t} = T_{2t}$, we have

$$U_2^2 = \frac{c_pT_{1t}\left[\left(\frac{p_{3t}}{p_{1t}}\right)^{\frac{\gamma}{\gamma-1}}-1\right]}{\eta_c\sigma\psi}$$

Here, $p_{3t}/p_{1t} = 4$, $T_{1t} = 300$ K

$$c_p = \frac{\gamma R}{\gamma-1} = \frac{1.4\times 287}{1.4-1} = 1005 \text{ J/kgK}$$

Slip factor $\sigma = 1-\frac{2}{15} = 0.867$

Power input factor $\psi = 1.05$

Therefore, $$U_2^2 = \frac{1005\times 300\left[(4)^{\frac{1.4}{1.4-1}} - 1\right]}{0.85\times 0.867\times 1.05}$$

or $$U_2 = 435 \text{ m/s}$$

Thus the impeller tip radius

$$r_2 = \frac{435\times 60}{2\pi\times 20000} = 0.21 \text{ m/s}$$

(ii) Power input to the air

$$= 2\times 1.05\times 0.867\times (435)^2 = 344.52\times 10^3 \text{ W} = 344.52 \text{ kW}$$

2.

(a)

The problem is described by 7 variables as

$$F\left(P, N, D, \rho, Q, gH, \mu\right) = 0$$

These variables are expressed by 3 fundamental dimensions M, L, and T. Therefore, the number of π terms $= 7 - 3 = 4$

Using D, ρ and N as repeating variables, π terms can be written as

$$\pi_1 = D^{a_1}\rho^{b_1}N^{c_1}Q \qquad (1)$$

$$\pi_2 = D^{a_2}\rho^{b_2}N^{c_2}gH \qquad (2)$$

$$\pi_3 = D^{a_3}\rho^{b_3}N^{c_3}P \qquad (3)$$

$$\pi_4 = D^{a_4}\rho^{b_4}N^{c_4}\mu \qquad (4)$$

Substituting the variables of Eqs (1-4) in terms of their fundamental dimensions M, L, and T, we get

$$\text{M}^0\text{L}^0\text{T}^0 = (\text{L})^{a_1}\left(\text{ML}^{-3}\right)^{b_1}\left(\text{T}^{-1}\right)^{c_1}\text{L}^3\text{T}^{-1} \qquad (5)$$

$$\text{M}^0\text{L}^0\text{T}^0 = (\text{L})^{a_2}\left(\text{ML}^{-3}\right)^{b_2}\left(\text{T}^{-1}\right)^{c_2}\text{L}^2\text{T}^{-2} \qquad (6)$$

$$\text{M}^0\text{L}^0\text{T}^0 = (\text{L})^{a_3}\left(\text{ML}^{-3}\right)^{b_3}\left(\text{T}^{-1}\right)^{c_3}\text{ML}^2\text{T}^{-3} \qquad (7)$$

$$\text{M}^0\text{L}^0\text{T}^0 = (\text{L})^{a_4}\left(\text{ML}^{-3}\right)^{b_4}\left(\text{T}^{-1}\right)^{c_4}\text{ML}^{-1}\text{T}^{-1} \qquad (8)$$

Equating the exponents of M, L and T from Eq.(5), we have

$$b_1 = 0$$

$$a_1 + 3 = 0$$

`

$$-c_1 - 1 = 0$$

which give $a_1 = -3,\ b_1 = 0,\ c_1 = -1$

Substituting these values into Eq. (1), we have

$$\pi_1 = \frac{Q}{ND^3}$$

Similarly, from Eq. (6), we get

$$b_2 = 0$$

$$a_2 - 3b_2 + 2 = 0$$

$$-c_2 - 2 = 0$$

which give $a_2 = -2,\ b_2 = 0,\ c_2 = -2$

Substituting these values into Eq. (2), we have

$$\pi_2 = \frac{gH}{N^2 D^2}$$

Equating the exponents of M, L and T from Eq.(7), we have

$$b_3 + 1 = 0$$

$$a_3 - 3b_3 + 2 = 0$$

$$-c_3 - 3 = 0$$

which give $a_3 = -5,\ b_3 = -1,\ c_3 = -3$

Substituting these values into Eq. (3), we have

$$\pi_3 = \frac{P}{\rho N^3 D^5}$$

Similarly, from Eq. (8), we get

$$b_4 + 1 = 0$$

$$a_4 - 3b_4 - 1 = 0$$

$$-c_4 - 1 = 0$$

which give $a_4 = -2,\ b_4 = -1,\ c_4 = -1$

Substituting these values into Eq. (4), we have

$$\pi_4 = \frac{\mu}{\rho N D^2}$$

Therefore, the problem can be expressed in terms of independent dimensionless parameters as

$$f\left(\frac{Q}{ND^3}, \frac{gH}{N^2 D^2}, \frac{P}{\rho N^3 D^5}, \frac{\mu}{\rho N D^2}\right) = 0$$

Dimensionless specific speed for the hydraulic turbine is obtained by eliminating D and relates it with H and P as

$$K_{ST} = \frac{\pi_3^{1/2}}{\pi_2^{5/4}} = \frac{N\sqrt{Q}}{\rho^{1/2}\left(gH\right)^{5/4}}$$

Dimensionless specific speed for the centrifugal pump is obtained by eliminating D and relates it with H and Q as

$$K_{SP} = \frac{\pi_1^{1/2}}{\pi_2^{3/4}} = \frac{N\sqrt{Q}}{\left(gH\right)^{3/4}}$$

(b)

Head developed by the pump = Head at delivery – head at suction

$$= \left(\frac{p_d}{\rho g} + \frac{\bar{V}_d^{\,2}}{2g} + z_d\right) - \left(\frac{p_s}{\rho g} + \frac{\bar{V}_s^{\,2}}{2g} + z_s\right)$$

$$= \left(\frac{412 \times 10^3}{10^3 \times 9.81} + \frac{3.62^2}{2 \times 9.81} + 2.75 \right) - \left(\frac{95.2 \times 10^3}{10^3 \times 9.81} + \frac{2.35^2}{2 \times 9.81} + 1.25 \right)$$

$$= 34.18 \text{ m}$$

(i) Hydraulic power input to the fluid $= \rho QgH$

$$= 10^3 \times \frac{11.5}{3600} \times 9.81 \times 34.18 = 1.07 \times 10^3 \text{ W} = 1.07 \text{ kW}$$

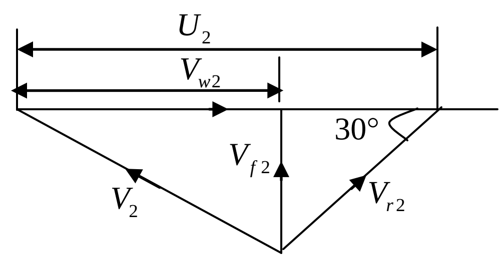
(ii) Mechanical power input to the fluid $= T\omega$

$$= 3.68 \times \frac{2\pi \times 3500}{60} = 1.35 \times 10^3 \text{ W} = 1.35 \text{ kW}$$

Hydraulic efficiency $\eta_h = \frac{\rho QgH}{T\omega} = \frac{1.07}{1.35} = 79.26\%$

(iii) Electrical power input $= \frac{\text{Mechanical power input to the fluid}}{\eta_{mech}} = \frac{1.35}{0.85} = 1.59 \text{ kW}$

(c)



From the outlet velocity triangle

$$V_{w2} = U_2 - V_{f2} \cot \beta_2$$

$$= U_2 - 2 \cot 30° = U_2 - 3.46$$

Energy given to the fluid per unit weight

$$= \frac{V_{w2} U_2}{g} = \frac{(U_2 - 3.46) U_2}{g}$$

(i) Under the situation when the entire kinetic energy at impeller outlet is wasted

$$20 = \frac{(U_2 - 3.46) U_2}{g} - \frac{V_2^2}{2g}$$

From the outlet velocity triangle

$$V_2^2 = V_{f2}^2 + V_{w2}^2 = 4 + (U_2 - 3.46)^2$$

Therefore,

$$20 = \frac{(U_2 - 3.46) U_2}{g} - \frac{4 + (U_2 - 3.46)^2}{2g}$$

or $\quad U_2^2 - 18.97 = 2 \times 9.81 \times 20$

which gives $\quad U_2 = 20.21 \text{ m/s}$

Impeller diameter is

$$D_2 = \frac{60 U_2}{\pi N} = \frac{60 \times 20.21}{\pi \times 700} = 0.55 \text{ m}$$

(ii) When 50% of the kinetic energy at impeller outlet is converted into pressure energy in pump casing, we can write

$$20 = \frac{(U_2 - 3.46) U_2}{g} - \frac{1}{2} \left\{ \frac{4 + (U_2 - 3.46)^2}{2g} \right\}$$

or $$3U_2^2 - 6.92U_2 - 800.77 = 0$$

The feasible solution is $U_2 = 17.53$ m/s

Hence, $$D_2 = \frac{60 \times 17.53}{\pi \times 700} = 0.48 \text{ m}$$

3.(a)

For $M = 2.7$, isentropic flow tables give

$$\frac{A}{A^*} = 3.183$$

Hence, $$A_e = 3.183 \times 0.08 = 0.255 \text{ m}^2$$

(b) The flow rate through the nozzle under design conditions can be written as

$$\dot{m} = \rho^* V^* A^*$$

However $$\rho_0 = \frac{p_0}{RT_0} = \frac{800 \times 10^3}{287 \times 313} = 8.91 \text{ kg/m}^3$$

Isentropic flow tables give for $M = 1$

$$\frac{\rho^*}{\rho_0} = 0.63394, \qquad \frac{T^*}{T_0} = 0.83055$$

Therefore,

$$T^* = 0.83055 \times 313 = 260 \text{ K}$$

Further,

$$V^* = a^* = \sqrt{\gamma R T^*} = \sqrt{1.4 \times 287 \times 260} = 323.2 \text{ m/s}$$

and

$$\rho^* = 0.63394 \times 8.91 = 5.65 \text{ kg/m}^3$$

Hence,

$$\dot{m} = \rho^* V^* A^* = 5.65 \times 323.2 \times 0.08 = 146 \text{ kg/s}$$

(c) Isentropic flow tables give for $M = 2.7$

$$\frac{p_0}{p} = 23.283$$

Hence, $$p_{\text{design}} = 800/23.283 = 34.36 \text{ kPa}$$

(d) For $A/A^* = 3.183$, we get from the subsonic regime of the isentropic flow tables

$$p_0/p = 1.025$$

This gives

$$p = \frac{800}{1.025} = 780.5 \text{ kPa}$$

Therefore the lowest back pressure upto which the flow will be entirely subsonic is 780.5 kPa.

(e) In case of a shock wave on the exit plane of the nozzle, the Mach number ahead of the shock is 2.7 and the pressure is 34.36 kPa. For Mach number 2.7, normal shock tables give

$$\frac{p_b}{p_{\text{design}}} = 8.33832$$

Hence, the back-pressure at which there is a shock wave on the nozzle exit plane is

$$p_b = 8.33832 \times 34.36 = 286.5 \text{ kPa}$$

(f) When the back-pressure has dropped below 286.5 kPa, the shock wave moves out of the nozzle and hence, there are no shock waves in the nozzle.

(g) There are oblique shock waves in the exhaust when

$$34.36 \text{ kPa} < p_b < 286.5 \text{ kPa}$$

(h) Expansion waves will occur when $p_b < p_{\text{design}}$, i.e., when $p_b < 34.36$ kPa

4.(a)

It is given that $$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$$

We know that $$\frac{p_0}{p_\infty} = \left[1 + \frac{(\gamma-1)}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_0}{p^*} = \left[1 + \frac{(\gamma-1)}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

Therefore, $$\frac{p^*}{p_\infty} = \frac{\left[1 + \frac{(\gamma-1)}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{(\gamma-1)}{2}\right]^{\frac{\gamma}{\gamma-1}}}$$

or $$\frac{p^*}{p_\infty} = \frac{\left[2 + (\gamma-1) M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}}{[\gamma+1]^{\frac{\gamma}{\gamma-1}}}$$

or $$\frac{p^* - p_\infty}{p_\infty} = \frac{\left[2 + (\gamma-1) M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}}{[\gamma+1]^{\frac{\gamma}{\gamma-1}}} - 1$$

or $$\frac{p^* - p_\infty}{\frac{1}{2}\rho V_\infty^2} = \frac{\left[\frac{2 + (\gamma-1) M_\infty^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}} - 1}{\frac{1}{2}\frac{1}{p_\infty}\rho V_\infty^2}$$

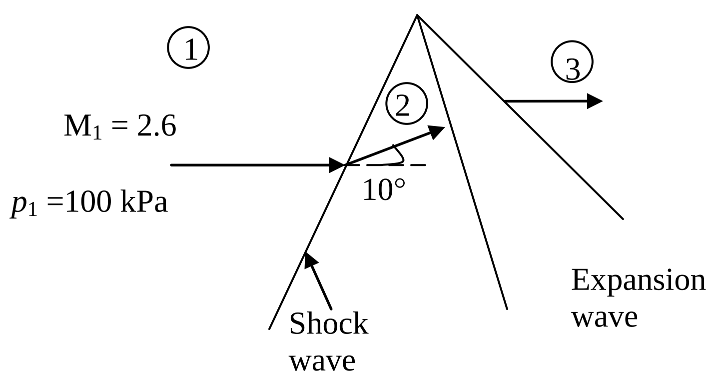
or $$C_p^* = \frac{\left[\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}}{\frac{\gamma}{2}\frac{1}{a_\infty^2}V_\infty^2} - 1 \qquad \left[\because \frac{\gamma p}{\rho} = a^2\right]$$

or $$C_p^* = \frac{\left[\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2}M_\infty^2}$$

(b)It is given

$$\mathrm{M}_1 = 2.6\ ,\ \delta = 10°\ ,\ \beta_1 = 30°$$

The flow situation being considered is shown in the figure below.



$$\mathrm{M}_{N1} = \mathrm{M}_1 \sin\beta_1 = 2.6 \times \sin 30° = 1.3$$

From normal shock table for $\mathrm{M}_1 = 1.3$, we have

$$\mathrm{M}_{N2} = 0.765\ ,\ \frac{p_2}{p_1} = 1.805\ ,\ \frac{T_2}{T_1} = 1.13$$

Again, $$\mathrm{M}_{N2} = \mathrm{M}_2 \sin(\beta_1 - \delta)$$

Thus, $$\mathrm{M}_2 = \frac{\mathrm{M}_{N2}}{\sin(30° - 10°)} = 2.295$$

From isentropic flow table, for $\mathrm{M}_2 = 2.295$,

$$\theta_2 = 34.03°$$

Expansion wave will deflect the air by $10°$ and make the pressure $p_3 = p_1$.

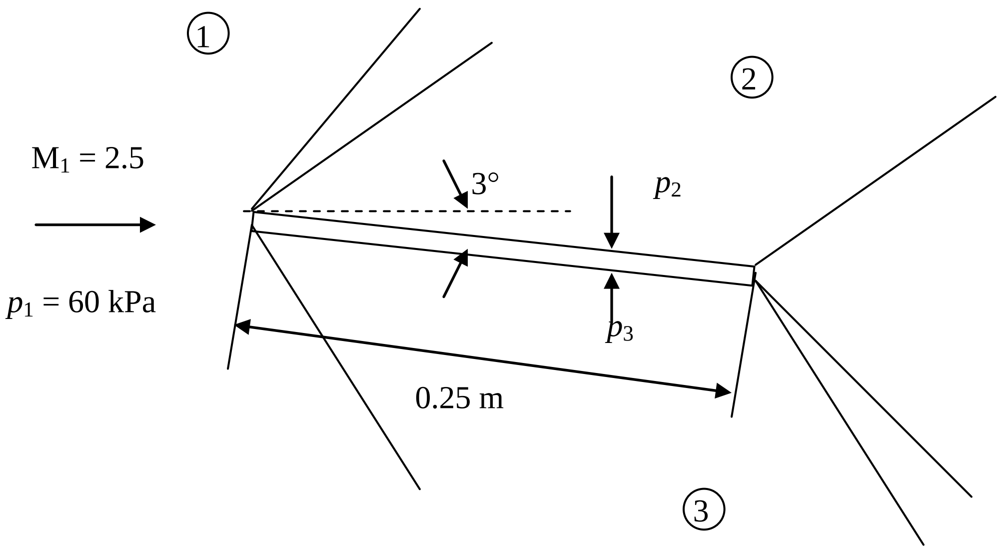
Thus, $$\theta_3 = 34.03° + 10° = 44.03°$$

From isentropic flow table, for $\theta_3 = 44.03°$

$$\mathrm{M}_3 = 2.72$$

(c)

The flow situation being considered is shown in the figure below.



An expansion wave forms on the upper surface at the leading edge, while an oblique shock wave forms on the lower surface at the leading edge. Both the waves turn the flow parallel to the plate. Waves formed at the trailing edge of the plate have been neglected.

Let us designate the state of air after expansion wave at the upper surface as 2 and that after oblique shock at the lower surface as 3 as shown in the figure.
Let us consider the expansion wave at the upper surface,
For $\mathrm{M}_1 = 2.5$, from isentropic flow tables, we get

$$\frac{p_{01}}{p_1} = 17.09, \ \theta_1 = 39.13°$$

Since the flow is turned through 3° by the expansion wave, we get

$$\theta_2 = 39.13° + 3° = 42.13°$$

Using $\theta_2 = 42.13°$, isentropic tables give

$$\mathrm{M}_2 = 2.63, \ \frac{p_{02}}{p_2} = 20.92$$

Since the flow through the expansion wave is isentropic i. e., $p_{02} = p_{01}$, we have

$$p_2 = \frac{p_2}{p_{02}} \frac{p_{01}}{p_1} p_1 = \frac{17.09}{20.92} \times 60 = 49.02 \text{ kPa}$$

Hence, the pressure acting on the upper surface of the plate is 49.02 kPa.

Let us consider the oblique shock wave at the lower surface.

For $M_1 = 2.5$ and $\delta = 3°$, we get from oblique shock wave charts

$$\beta = 26°$$

Then,

$$M_{N1} = M_1 \sin\beta = 2.5 \times \sin 26° = 1.096$$

For a value of Mach number 1.096, we get from normal shock tables

$$\frac{p_3}{p_1} = 1.23$$

Therefore,

$$p_3 = \frac{p_3}{p_1} p_1 = 1.23 \times 60 = 74 \text{ kPa}$$

Hence, the pressure acting on the lower surface of the plate is 74 kPa.
Therefore the lift force per meter span

$$= \left(p_3 - p_2\right) A \cos 3° = \left(74 - 49\right) \times 0.25 \times \cos 3° = 6.23 \text{ kN/ m span}$$