Fluid Statics

Q1. **Choose the correct answer**

(i) The normal stress is the same in all directions at a point in a fluid
(a) only when the fluid is frictionless
(b) only when the fluid is frictionless and incompressible
(c) only when the fluid is incompressible
(d) when the fluid is at rest, regardless of its nature

[*Ans.(*d)]

(ii) In an isothermal atmosphere, the pressure
(a) remains constant
(b) decreases linearly with elevation
(c) decreases exponentially with elevation
(d) varies in the same way as the density.

[*Ans.(*c)]

(iii) The line of action of buoyancy force acts through the
(a) centre of gravity of any submerged body
(b) centroid of the volume of any floating body
(c) centroid of the displaced volume of fluid
(d) centroid of the volume of fluid vertically above the body

[*Ans.(*c)]

(iv) How is the metacentric height, MG expressed?

(a) $MG = BG + \frac{I}{\forall}$ (b) $MG = \frac{\forall}{I} - BG$

(c) $MG = \frac{I}{\forall} - BG$ (d) $MG = BG - \frac{\forall}{I}$

where I = Moment of inertia of the plan of the floating body at the water surface
$\forall$ = Volume of the body submerged in water
BG = Distance between the centre of gravity (G) and the centre of buoyancy (B).

[*Ans.(*c)]

Q2.

In construction, a barometer is a graduated inverted tube with its open end dipped into the measuring liquid contained in a trough opened to atmosphere. Estimate the height of liquid column in the barometer where the atmosphere pressure is 100 kN/m^2 (a) when the liquid is mercury and (b) when the liquid is water. The measuring temperature is 50°C, the vapour pressure of mercury and water at this temperature are respectively 0.015×10^4 N/m^2 and 1.23×10^4 N/m^2 and the densities are 13500 and 980 kg/m^3 respectively. What would be the percentage error if the effect of vapour pressure is neglected.

Solution

From the principle of barometer, we have

$$\rho g h = p_{atm} - p_v$$

where ρ is the barometric liquid, h is the height of liquid column in the barometer, p_{atm} is the atmospheric pressure and p_v is the vapor pressure of the barometric liquid.

(a) Substituting the respective values, for mercury as barometric liquid, we get

$$13500 \times 9.81 \times h_{mercury} = 100 \times 10^3 - 0.015 \times 10^4$$

or $$h_{mercury} = 0.754 \text{ m}$$

(b) Substituting the respective values, for water as barometric liquid, we get

$$980 \times 9.81 \times h_{water} = 100 \times 10^3 - 1.23 \times 10^4$$

or $$h_{water} = 9.12 \text{ m}$$

When the effect of vapour pressure is neglected, we get

$$\rho g h = p_{atm}$$

Substituting the respective values, for mercury as barometric liquid, we have

$$13500 \times 9.81 \times h_{mercury} = 100 \times 10^3$$

or $$h_{mercury} = 0.755 \text{ m}$$

Substituting the respective values, for water as barometric liquid, we have

$$980 \times 9.81 \times h_{water} = 100 \times 10^3$$

or $$h_{water} = 10.4 \text{ m}$$

Percentage error in the height of liquid column for mercury as barometric liquid when the effect of vapour pressure is neglected is

$$\frac{0.755 - 0.754}{0.754} \times 100 = 0.133\%$$

Percentage error in the height of liquid column for water as barometric liquid when the effect of vapour pressure is neglected is

$$\frac{10.14 - 9.12}{9.12} \times 100 = 14\%$$

Q3.

Calculate the atmospheric pressure at the end of troposphere, which extends upto a height of 9 km from sea level. Consider a temperature variation in the troposphere as $T = 288 - 0.006y$, where y is the elevation in m and T is temperature in K. The atmospheric pressure and temperature at sea level are 101.3 kN/m^2 and 288 K respectively. The characteristic gas constant for air is 287 J/kg-K.

Solution

The fundamental equation of fluids at rest can be written as

$$\frac{dp}{dz} = -\rho g$$

For an ideal gas, the thermodynamic equation of state is given by

$$p = \rho R T$$

where R is the characteristic gas constant.

From the above two equations, we obtain

$$\frac{dp}{dz} = -\frac{p}{RT} g$$

It is given that the temperature decreases linearly with the altitude z as given by

$$T = T_0 - \alpha z$$

where $T = 288$ K and $\alpha = 0.006$.

Substituting the value of T in the above equation, we have

$$\frac{dp}{dz} = -\frac{p}{R\left(T_0 - \alpha z\right)} g$$

or
$$\frac{dp}{p} = -\frac{g}{R} \frac{dz}{\left(T_0 - \alpha z\right)}$$

Integrating the above equation, we have

$$\ln \frac{p}{p_0} = \frac{g}{R\beta} \ln \frac{T_0 - \alpha z}{T_0}$$

or
$$p = p_0 \left(1 - \frac{\alpha z}{T_0}\right)^{\frac{g}{R\beta}}$$

Substituting the respective values, we obtain

$$p = 101.3 \left(1 - \frac{0.006 \times 9000}{288}\right)^{\frac{9.81}{287 \times 0.006}}$$

$$= 30.998 \text{ kN/m}^2$$

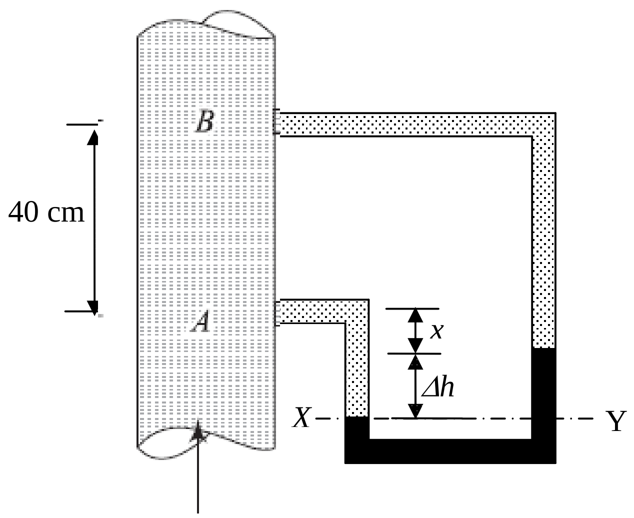
The atmospheric pressure at the end of troposphere is 30.998 kN/m^2

Q4.

A differential U-tube mercury manometer is used to measure the pressure difference between two sections of a vertical pipe through which oil of specific gravity 0.8 flows upwards. The distance between the two sections is 40 cm. If the difference of pressure between the two sections is 30 kN/m^2, find the deflection in the mercury manometer. Assume specific gravity of mercury as 13.6 and density of water as 1000 kg/m^3.

Solution

The differential U-tube mercury manometer is shown in the figure below.



Let the difference in height of meniscus between two limbs be Δh m and the vertical height between point A in the pipe and the level of meniscus in the right limb be x m.

For the left limb of the manometer, pressure along the plane XY is

$$p_X = p_A + \rho_{oil} g(x + \Delta h)$$

where p_A is the pressure at section A and ρ_{oil} is the density of oil.

For the right limb of the manometer, pressure along the plane XY is

$$p_Y = p_B + \rho_{oil} g(0.4 + x) + \rho_m g \Delta h$$

where p_B is the pressure at section B and ρ_m is the density of manometric fluid.

Equating the pressures of both the limb along the horizontal plane XY, we have

$$p_X = p_Y$$

or $$p_A + \rho_{oil} g(x + \Delta h) = p_B + \rho_{oil} g(0.4 + x) + \rho_m g \Delta h$$

or $$p_A - p_B = 0.4\rho_{oil} g + \rho_m g \Delta h - \rho_{oil} g \Delta h$$

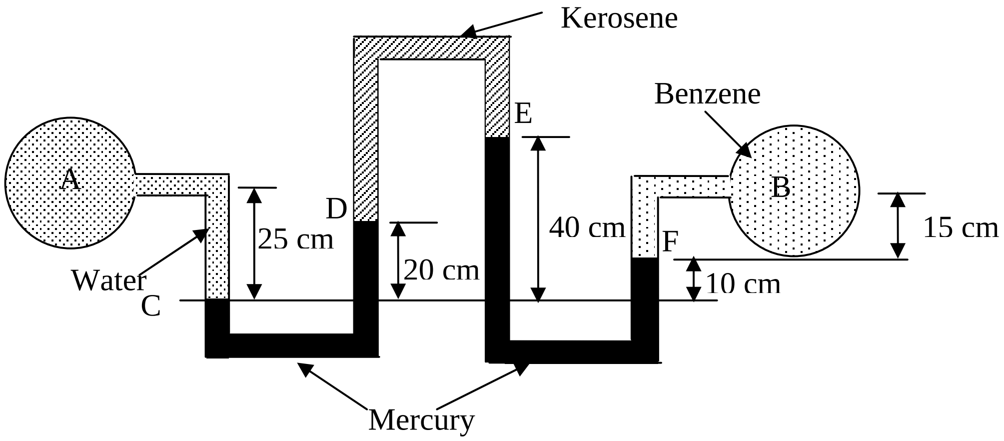
Substituting the respective values, we obtain

$$30 \times 10^3 = 800 \times 9.81 \times 0.4 + 13600 \times 9.81 \times \Delta h - 800 \times 9.81 \times \Delta h$$

or $$\Delta h = 0.2139 \text{ m} = 21.39 \text{ cm}$$

Q5.

A multi-tube manometer is used to determine the pressure difference between points A and B as shown in the figure below. For the given values of heights, determine the ressure difference between points A and B. Specific gravities of benzene, kerosene and mercury are 0.88, 0.82 and 13.6 respectively.



Solution

We know that pressure variation between any two points in any static fluid is described by the relation

$$p_2 - p_1 = \rho g \Delta h$$

where Δh is the vertical distance between the points.

Beginning at point A and applying the equation between successive points along the manometer (see the above figure) gives

$$p_C - p_A = \rho_w g\left(h_A - h_C\right)$$

$$p_D - p_C = -\rho_{mercury} g\left(h_D - h_C\right)$$

$$p_E - p_D = -\rho_{kerosene} g\left(h_E - h_D\right)$$

$$p_F - p_E = \rho_{mercury} g \left(h_E - h_F\right)$$
$$p_B - p_F = -\rho_{benzene} g \left(h_B - h_F\right)$$

Multiplying each equation by minus one and then adding, we get

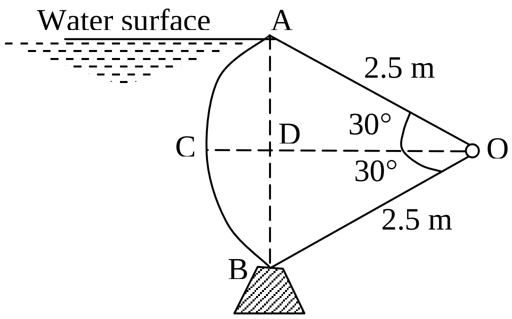
$$p_A - p_B = -\rho_w g\left(h_A - h_C\right) + \rho_{mercury} g\left(h_D - h_C\right)$$
$$+\rho_{kerosene} g\left(h_E - h_D\right) - \rho_{mercury} g\left(h_E - h_F\right) + \rho_{benzene} g\left(h_B - h_F\right)$$

Substituting the values of ρ_w, $\rho_{mercury}$, $\rho_{kerosene}$, and $\rho_{benzene}$ and the respective heights, we obtain

$$p_A - p_B = -1000\times 9.81\times 0.25 + 13600\times 9.81\times 0.2 + 820\times 9.81\times 0.2$$
$$-13600\times 9.81\times 0.3 + 880\times 9.81\times 0.05$$
$$= -13.75\times 10^3\,\text{Pa} = -13.75\text{ kPa}$$

Q6.

Find the horizontal and vertical forces per metre width on the tainter gate which is a sector of a circle of radius 2.5 m as shown in the figure below. Density of water is 1000 kg/m^3.



Solution

From the geometry of the above figure, we have

$$AD = AO\sin 30^\circ = 2.5\times\frac{1}{2} = 1.25\text{ m}$$
$$OD = AO\cos 30^\circ = 2.5\times\frac{\sqrt{3}}{2} = 2.165\text{ m}$$
$$AB = 2AD = 2\times 1.25 = 2.5\text{ m}$$

Horizontal component of the force acting on the gate is given by

$$F_H = \text{Force of water on area ADB}$$
$$= \rho g A h_c$$

where $\quad A = AB\times\text{width of gate}$

$$= 2.5\times 1 = 2.5\text{ m}^2$$
$$h_c = \frac{AB}{2} = \frac{2.5}{2} = 1.25\text{ m}$$

Substituting the respective values, we obtain

$$F_H = 1000\times 9.81\times 2.5\times 1.25 = 30656\text{ N} = 30.656\text{ kN}$$

Vertical component of the force acting on the gate is given by

$$F_V = \text{Weight of water of volume ADBCA}$$
$$= \rho g \times \text{area of ADBCA} \times \text{width of gate}$$
$$= \rho g \times \left(\text{area of ACBOA} - \text{area of } \Delta\text{AOB}\right) \times 1$$
$$= \rho g \times \left(\frac{\pi R^2}{6} - 2 \times \text{area of } \Delta\text{AOD}\right)$$
$$= \rho g \times \left(\frac{\pi R^2}{6} - 2 \times \frac{AD \times OD}{2}\right) = \rho g \times \left(\frac{\pi R^2}{6} - AD \times OD\right)$$

Substituting the respective values, we obtain

$$F_V = 1000 \times 9.81 \times \left[\frac{\pi}{6} 2.5^2 - 1.25 \times 2.165\right]$$
$$= 9810 \times (3.272 - 2.706) = 5552 \text{ N} = 5.552 \text{ kN}$$

Q7.

A wooden block of rectangular cross section of width *b*, length *L* and a submerged depth of *H* has its centre of gravity at the waterline. Find the metacentric height, and the ratio b/H for which the block is stable.

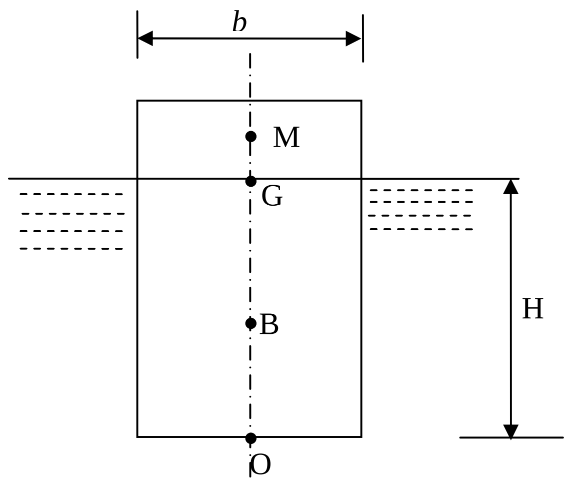
Solution

Let *B*, *G* and *M* be the centre of buoyancy, centre of gravity and metacentre of the block (shown in the figure below) respectively.

Distance of centre of gravity from the base $OG = H$

Distance of centre of buoyancy from the base $OB = \frac{H}{2}$

$\therefore$ $$BG = OG - OB = H - \frac{H}{2} = \frac{H}{2}$$



Then, metacentric height is given by

$$MG = BM - BG = \frac{I_{YY}}{\forall} - BG$$

where I_{YY} is the moment of inertia of the plane of floatation about the centroidal axis perpendicular to the plane of rotation and is given by

$$.\ I_{YY} = \frac{1}{12} L b^3$$

where, L be the length of the block in a direction perpendicular to the plane of the figure and $\forall$ is the volume of submerged potion and is given by

$$\forall = L \times b \times H$$

Thus, $$BM = \frac{I_{YY}}{\forall} = \frac{\frac{1}{12} L b^3}{L \times b \times H} = \frac{b^2}{12H}$$

Therefore, $$MG = \frac{I_{YY}}{\forall} - BG$$

$$= \frac{\frac{1}{12} L b^3}{L \times b \times H} - \frac{H}{2} = \frac{b^2}{12H} - \frac{H}{2}$$

For stable equilibrium of the block, metacentric height should be positive. That means

$$GM \geq 0$$

$$\frac{b^2}{12H} - \frac{H}{2} \geq 0$$

or $$\frac{H}{2}\left[\frac{1}{6}\left(\frac{b}{H}\right)^2 - 1\right] \geq 0$$

or $$\frac{b}{H} \geq \sqrt{6}$$

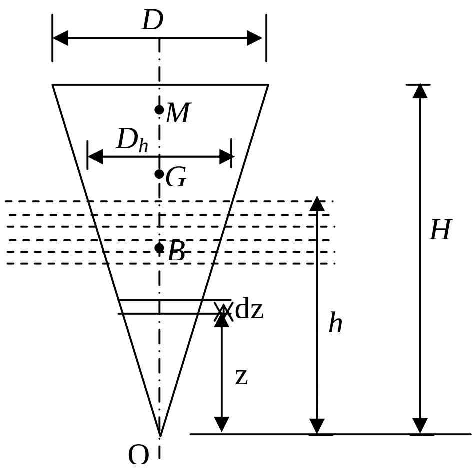
Q8.

A solid cone of base diameter D and vertical height H floats in water with its apex downwards. If the specific gravity of the cone is S, show that for stable equilibrium

$$H^2 < \frac{1}{4}\left(\frac{D^2 S^{1/3}}{1 - S^{1/3}}\right)$$

Solution

Let the height of the cone under water be h and the diameter of cone at the water surface be D_h as shown in the figure below.



. From the geometry of the above figure, we have

$$D_h = D\frac{h}{H}$$

For the equilibrium, we have
Weight of the cone = Total buoyant force

$$\frac{1}{3}\frac{\pi D^2}{4}HS = \frac{1}{3}\frac{\pi D_h^2}{4}h$$

or $$HSD^2 = D^2\frac{h^2}{H^2}h \qquad \left[\because D_h = D\frac{h}{H}\right]$$

or $$h = HS^{1/3}$$

The centre of gravity G of the cone is found out by considering the mass of cylindrical element of height dz and diameter Dz/H, and its moment about the apex o in the following way

$$OG = \frac{\int_0^H \pi\frac{D^2z^2}{4H^2}zdz}{\frac{1}{3}\pi\frac{D^2}{4}H} = \frac{3}{4}H$$

The centre of buoyancy B is the centre of volume of the submerged part of the cone and hence

$$OB = \frac{3}{4}h$$

Therefore, from the above figure, we have

$$BG = OG - OB = \frac{3}{4}H - \frac{3}{4}h$$

$$= \frac{3}{4}H\left(1 - S^{1/3}\right) \qquad \left[\because h = HS^{1/3}\right]$$

Let *M* be the metacentre. Then , we can write

$$BM = \frac{I_{YY}}{\forall}$$

where I_{YY} is the moment of inertia of the plane of floatation about the centroidal axis perpendicular to the plane of rotation and is given by

$$.\ I_{YY} = \frac{\pi}{64}D_h^4$$

and $\forall$ is the volume of submerged potion and is given by

$$\forall = \frac{1}{3}\frac{\pi D_h^2}{4}h$$

$\therefore$ $$BM = \frac{\frac{\pi}{64}D_h^4}{\frac{1}{3}\frac{\pi D_h^2}{4}h} = \frac{3}{16}\frac{D_h^2}{h} = \frac{3}{16}D^2\frac{h}{H^2}$$

$$= \frac{3}{16}\frac{D^2}{H}S^{1/3}$$

The metacentric height is given by

$$MG = BM - BG$$

$$= \frac{3}{16}\frac{D^2}{H}S^{1/3} - \frac{3}{4}H\left(1 - S^{1/3}\right)$$

$$= \frac{3}{16H}\left[D^2S^{1/3} - \frac{1}{4}H^2\left(1 - S^{1/3}\right)\right]$$

For stable equilibrium, metacentric height should be positive. That means

$$MG > 0$$

or $$\frac{3}{16H}\left[D^2S^{1/3} - \frac{1}{4}H^2\left(1 - S^{1/3}\right)\right] > 0$$

or $$D^2S^{1/3} > \frac{1}{4}H^2\left(1 - S^{1/3}\right)$$

or $$H^2 < \frac{1}{4}\left(\frac{D^2S^{1/3}}{1 - S^{1/3}}\right)$$