Flows with a Free Surface

Q1. **Choose the correct answer**

(i) Which one of the following statements is appropriate for the free surface, the hydraulic gradient line and energy gradient line in an open channel flow?
(a) Parallel to each other but they are different lines
(b) Such that only the first two coincide
(c) Such that they are all inclined to each other
(d) All coinciding

[*Ans.(*b)]

(ii) For a critical flow in an open channel
(a) the specific energy is minimum for a given flow
(b) the specific energy is maximum for a given flow
(c) the flow is minimum for a given specific energy
(d) the shear stress is maximum at the bed surface

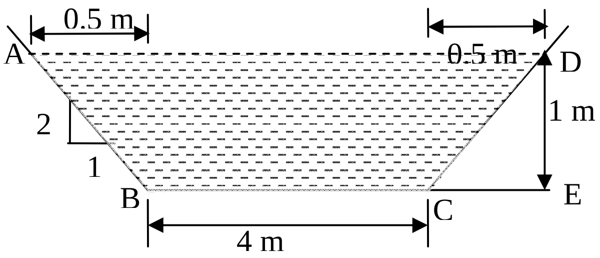
[*Ans.(*a)]

Q2.

A trapezoidal channel with a base width 2 m and side slope 1 horizontal to 2 vertical carries water 1 m deep on a slope of 0.001. Find the rate of uniform discharge. Assume roughness coefficient as 0.03.

Solution

The channel is schematically shown in the figure below.



Cross-sectional area of flow is

$$A = \frac{1}{2}\left[4 + \left(4 + 2\times 0.5\right)\right]\times 1 = 4.5 \text{ m}^2$$

Wetted perimeter is

$$P = 4 + 2\sqrt{0.5^2 + 1^2} = 6.236 \text{ m}$$

Hydraulic radius is

$$R_h = \frac{A}{P} = \frac{4.5}{6.236} = 0.722 \text{ m}$$

Discharge in the channel is given by

$$Q = \frac{1}{n} A R_h^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$= \frac{1}{0.03}\times 4.5\times\left(0.722\right)^{\frac{2}{3}}\times\left(0.001\right)^{\frac{1}{2}} = 3.818 \text{ m}^3/\text{s}$$

Q3.

Water is flowing through a rectangular channel of 4 m in width. When the depth of water is 1.6 m, the flow is12 m^3/s . Calculate the specific energy head, the critical depth and the minimum specific energy head. State whether the flow is tranquil or rapid.

Solution

The cross-sectional area of flow is

$$A = 4 \times 1.6 = 6.4 \text{ m}^2$$

Specific energy head is given by

$$E_s = h + \frac{1}{2g}\left(Q/A\right)^2$$

where h is the depth of flow and Q is the volumetric flow rate.

Substituting the respective values, we obtain

$$E_s = 1.6 + \frac{1}{2 \times 9.81}\left(\frac{12}{6.4}\right)^2 = 1.779 \text{ m}$$

The depth of flow corresponding to minimum specific energy is called as critical depth and is obtained as

$$\frac{\partial E_s}{\partial h} = 1 + \frac{q^2}{2g}\left(-\frac{2}{h^3}\right) = 0$$

or
$$h_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

where q is the discharge per unit width.

Substituting the respective values, we get

$$h_c = \left(\frac{\left(12/4\right)^2}{9.81}\right)^{\frac{1}{3}} = 0.972 \text{ m}$$

Minimum specific energy head is given by

$$E_{\min} = \frac{3}{2} h_c$$

$$= \frac{3}{2} \times 0.972 = 1.458 \text{ m}$$

Since the actual depth exceeds the critical one, the flow is tranquil.

Q4.

In a rectangular channel of 0.6 m wide, a hydraulic jump occurs where the Froude number is 3. The depth after the jump is 0.6 m. What is the height of the hydraulic jump? Estimate the loss of head and the power dissipated due to the jump.

Solution

The depths of flow on both sides of a hydraulic jump can be expressed as

$$\frac{h_2}{h_1} = \frac{1}{2}\left(\frac{1}{2}\sqrt{1 + 8Fr_1^2} - 1\right)$$

Substituting the respective values, we get

$$\frac{0.6}{h_1} = \frac{1}{2}\left[\left\{1+8(3)^2\right\}^{1/2} - 1\right]$$

from which $h_1 = 0.16$ m

The height of hydraulic jump is found to be

$$h_2 - h_1 = 0.6 - 0.16 = 0.44 \text{ m}$$

The loss of energy head is given by

$$h_j = \frac{(h_2 - h_1)^3}{4h_1h_2}$$

$$= \frac{0.44^3}{4\times 0.16\times 0.6} = 0.22 \text{ m}$$

From the expression of the Froude number, one can write

$$Fr_1 = \frac{V_1}{(gh_1)^{1/2}}$$

from which $V_1 = 3.76$ m/s

Volumetric flow rate is given by

$$Q = V_1h_1b \qquad (\ b \text{ is the width of the channel})$$

$$= 3.76\times 0.16\times 0.6 = 0.36 \text{ m}^3/\text{s}$$

The loss of total energy per second is found to be

$$P = \rho Qgh_j$$

$$= \frac{10^3\times 0.36\times 9.81\times 0.22}{10^3} = 0.78 \text{ kW}$$